MECHANICS

STATICS

Power moment $M = F\,l$

M	power moment	N · m
F	force/power	N
l	length of arm	m

Conditions for equilibrium

$\Sigma F_x = 0$

$\Sigma F_y = 0$

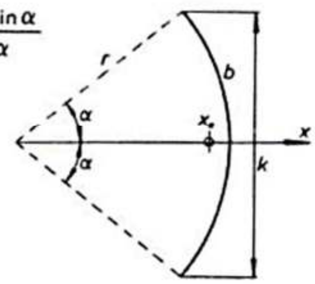
$\Sigma M = \Sigma(F\,l) = 0$

F_x	horizontal component of force	N
F_y	vertical component of force	N
M	power moment	N · m

Center of gravity

Arc of circle

$$x_0 = \frac{k\,r}{b} = \frac{r \sin \alpha}{\alpha}$$



x_0	distance	m
k	chord	m
r	radius	m
b	arc	m
α	angle	rad

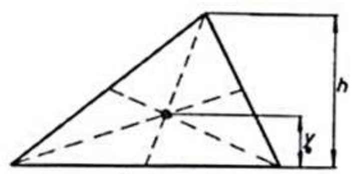
Arc of half circle

$$x_0 = \frac{2r}{\pi}$$

Area

Triangle

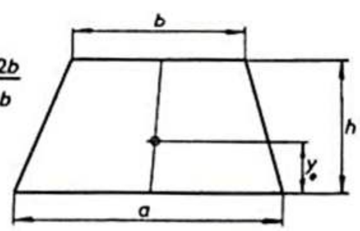
$$y_0 = \frac{h}{3}$$



y_0	distance	m
h	height/altitude	m

Trapezoid

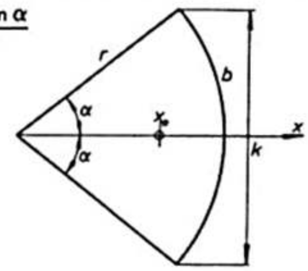
$$y_0 = \frac{h}{3} \cdot \frac{a + 2b}{a + b}$$



y_0	distance	m
h	height/altitude	m
a and b	length	m

Circle sector

$$x_o = \frac{2rk}{3b} = \frac{2r\sin\alpha}{3\alpha}$$



x_o	distance	m
k	chord	m
r	radius	m
b	arc	m
α	angle	rad

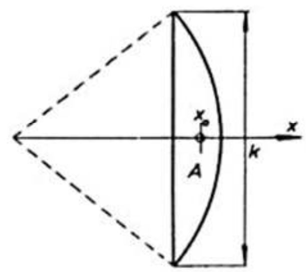
Half circle

$$x_o = \frac{4r}{3\pi} = 0.424\,r$$

x_o	distance	m

Segment

$$x_o = \frac{k^3}{12A}$$



x_o	distance	m
A	area	m^2

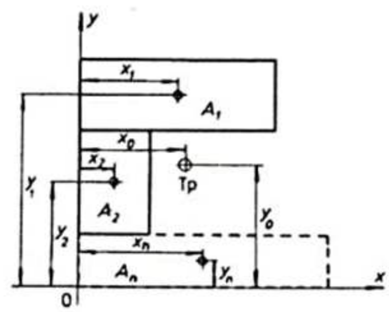
Any area

$$x_o = \frac{A_1x_1 + A_2x_2 + \cdots + A_nx_n}{A_1 + A_2 + \cdots + A_n} = \frac{\Sigma(A_x)}{\Sigma A}$$

x_o	distance	m
A	area	m^2
x	distances	m

$$y_o = \frac{A_1y_1 + A_2y_2 + \cdots + A_ny_n}{A_1 + A_2 + \cdots + A_n} = \frac{\Sigma(A_y)}{\Sigma A}$$

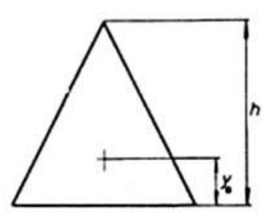
y_o	distance	m
y	distances	m



Volumes

Pyramid and cone

$$y_o = \frac{h}{4}$$

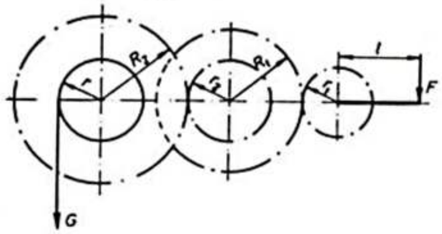


y_o	distance	m
h	height/altitude	m

(See also chapter on **Mathematics**)

Double gear exchange – Power/force gained

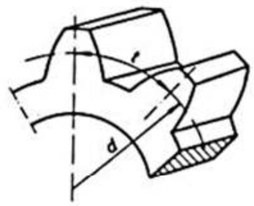
$$K_t = \frac{G}{F} = \frac{R_1}{r_1} \cdot \frac{R_2}{r_2} \cdot \frac{l}{r}$$



K_t	theoretical force gained	1
G	gravity (cargo)	N
F	force	N
R_1, R_2, r_1, and r_2	radii of cogwheels	m
l	length of crank	m

Dividing cogs

$$t = \pi m = \frac{\pi d}{z}$$



t	tooth division	mm
m	module	mm

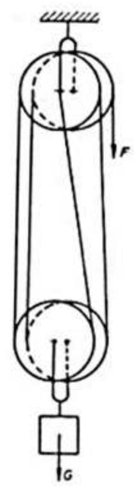
Diameter

$$d = \frac{t z}{\pi} = m z$$

d	part of the diameter of the circle	mm

Blocks and tackles

$$F = (G + \frac{Gfn}{100}) : n_1$$



F	force	N
G	gravity (cargo)	N
n	number of discs	1
f	friction in percent of gravity per disc	1
n_1	number of force-gaining discs	1

$$l = h n$$

l	length of the pulling part	m
h	height of lift	m

$$K_p = \frac{G}{F}$$

K_p	practical force gained	1

$$F = \frac{G}{\alpha + \alpha^2 + \alpha^3 + \dots + \alpha^n}$$

α	block coefficient	1

$$F_l = \frac{G}{\frac{1}{\alpha} + (\frac{1}{\alpha})^2 + (\frac{1}{\alpha})^3 + \dots + (\frac{1}{\alpha})^n}$$

F_l	force when lowering	N

$$F_o = \frac{G}{n}$$

F_o	pulling force without friction	N

$$\eta = \frac{F_o}{F} = \frac{\alpha + \alpha^2 + \alpha^3 + \dots + \alpha^n}{n}$$

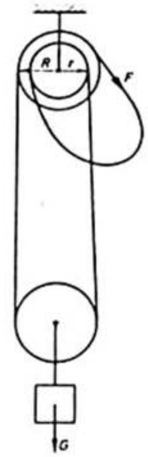
η	efficiency	1

$$\eta = \frac{Gh}{Fl} = \frac{G}{Fn}$$

Differential tackle

$$F\,R = \frac{G}{2}\,(R - r)$$

$$h = \pi n\,(R - r)$$



F	force	N
R	radius	m
r	radius	m
h	height of lift	m
n	estimated rotation of the tackle	1

Efficiency multiplied by force when hoisting and by gravity (cargo) when lowering.

The lifting screw

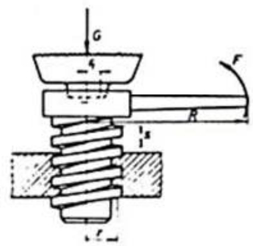
$$F_0 = \frac{sG}{2\pi R}$$

$$F = \frac{s + 2\pi(\mu r + \mu_1 r_1)}{2\pi R}\,G$$

$$K_t = \frac{G}{F_0} = \frac{2\pi R}{s}$$

$$K_p = \frac{G}{F} = \frac{2\pi R}{s + 2\pi(\mu r + \mu_1 r_1)}$$

$$\eta = \frac{F_0}{F} = \frac{Gs}{2\pi R F} = \frac{s}{s + 2\pi(\mu r + \mu_1 r_1)}$$

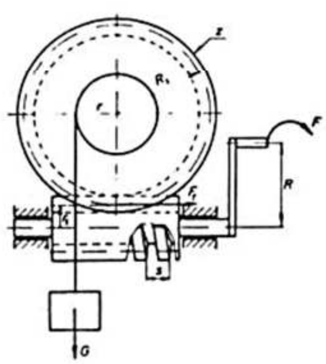


F_0	force on crank without friction	N
s	rise	m
R	radius	m
F	force on crank with friction	N
μ	friction coefficient	1
r	radius	m
K_t	theoretical force gained	N
K_p	practical force gained	1
η	efficiency	1

Endless screw

$$F R z = G r x$$

$$G r = F_T R_1$$



F	force on crank	N
z	number of teeth	1
x	number of threads	1
F_T	tooth pressure	N
R_1	radius of endless wheel	m

$$2\pi F R = F_T s + 2\pi r F_T \mu$$

$$2\pi F R = \frac{G r_1 s}{R_1} + \frac{2\pi F G r_1 \mu}{R_1}$$

r_1 mean radius of screw — m

$$n_1 = \frac{h}{2\pi r}$$

n_1 frequency of rotation of the cylinder — s^{-1}
h lifting height — m

$$n = \frac{2\pi R_1 n_1}{s}$$

n frequency of rotation of the screw — s^{-1}

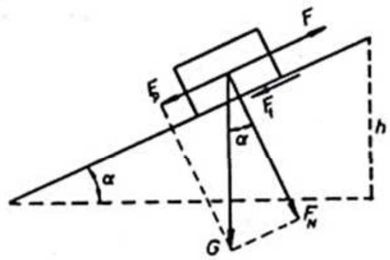
Inclined plane

$$F_N = G \cos\alpha$$

$$F_p = G \sin\alpha$$

$$F_f = F_N \mu$$

$$F = F_f + F_p$$



F_N normal force — N

F_p force down and parallel to the inclined plane — N

F_f friction force — N

F force up and parallel to the inclined plane — N

$$\mu = \tan\alpha$$

μ Friction coefficient — 1
α angle of rise — rad

The formula applies also when $F_p = F$

$$K_t = \frac{G}{F_p} = \frac{l}{h}$$

l length of inclined plane

$$K_p = \frac{G}{F}$$

h height/altitude of inclined plane

DYNAMICS

Constant speed/velocity

Rectilinear motion

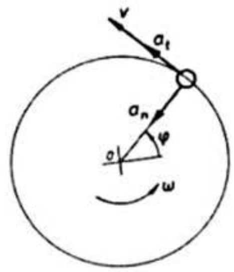
$$d = vt$$

d	distance	m
v	velocity	m/s
t	time	s

Circular movement

$$v = \pi\, d\, n = 2\, \pi\, r\, n$$

v	peripheral velocity	m/s
d	diameter	m
n	frequency of rotation (rotation/second or rotation/minute)	s^{-1}



$$\omega = \frac{v}{r} = 2\, \pi\, n$$

ω	rotation of angle	s^{-1}
r	radius	m

$$\varphi = \omega t$$

φ	distance of angle	rad

$$a_n = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = \omega^2 r$$

a_n	centripetal acceleration	m/s^2
T	period	s

$$n = \frac{1}{T}$$

Constant acceleration

Rectilinear motion

$$v_f = v_i + at$$

v_f	final velocity	m/s
v_i	initial velocity	m/s
a	acceleration	m/s^2
t	time	s

$$d = \frac{v_f + v_i}{2}\, t = v_i\ t + \frac{1}{2} a t^2$$

d	distance	m

$$v_f^2 - v_i^2 = 2ad$$

Free fall

$$v_f = v_o + Agt$$

v_f	end velocity	m/s
Ag	acceleration of gravity	m/s^2

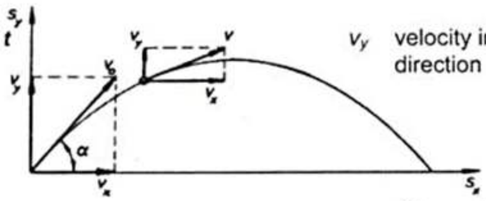
$$d = \frac{v_f + v_i}{2}\, t = v_i\ t + \frac{1}{2} Ag \cdot t^2$$

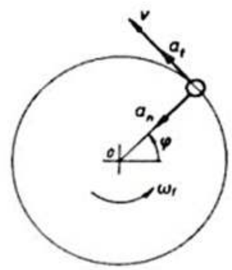
d	distance	m

$$v_f^2 - v_i^2 = 2Agh$$

h	height	

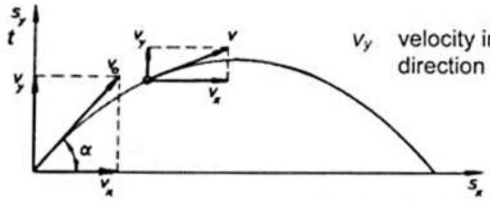
Projectile Motion	$v_x = v_0 \cos\alpha$	v_x	velocity in x-direction	m/s
		α	angle	rad
	$v_y = v_0 \sin\alpha - Ag \cdot t$	v_y	velocity in y-direction	m/s
	$s_x = v_0 t \cos\alpha$	s_x	distance in x-direction	m
	$s_y = v_0 t \sin\alpha - \frac{Ag \cdot t^2}{2}$	s_y	distance in y-direction	m
Circular movement	$\omega_f = \omega_i + \alpha t$	ω_f	velocity of angle (end)	rad/s
		ω_i	velocity of angle (start)	rad/s
		α	angle acceleration	rad/s^2
		a_n	centripetal acceleration	m/s^2
	$\alpha = \frac{\omega_f - \omega_i}{t}$			
	$\varphi = \frac{\omega_i + \omega_f}{2} t$	φ	distance of angle	rad
	$\varphi = \omega_i t + \frac{\alpha t^2}{2} = \frac{\omega_f^2 - \omega_i^2}{2\alpha}$			
	$a_t = \alpha r$	a_t	tangential acceleration	m/s^2
		r	radius	m
Weight and force	$W = mAg$	W	weight	N
		m	mass	kg
		Ag	acceleration due to gravity	m/s^2
	$F = ma$	F	force	N
		a	acceleration	m/s^2



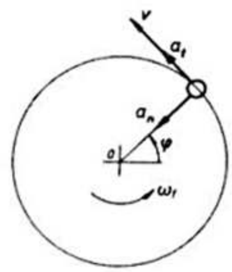


Work	$W = Fd$	W F d	work force distance	J N m
Energy	$E_p = mAgh$	E_p h	potential energy (position energy) height/altitude	 J m
	$E = Fd$	d	distance	m
	$E_k = \frac{mv^2}{2}$	E_k v	kinetic energy (energy of motion) velocity	 J m/s
Effect	$P = \frac{W}{t} = \frac{Fd}{t} = Fv = T\omega$	P W t F d v T ω	effect work time force distance velocity torsion moment velocity of angle	W J s N m m/s N · m rad/s
Efficiency	$\eta = \frac{P_a}{P_t}$	η P_a P_t	efficiency effect delivered (useful) effect supplied	1 W W
Rotation around a fixed axis	$T = I\alpha$	T I α	torsion moment inertia moment of mass angle acceleration	N · m kgm^2 rad/s^2
	$W = T\varphi$	W φ	work angle of torsion	J rad
	$E_k = \frac{I\omega^2}{2}$	E_k ω	kinetic energy angle of velocity	J rad/s

Projectile Motion	$v_x = v_o \cos\alpha$	v_x velocity in x-direction	m/s
		α angle	rad
	$v_y = v_o \sin\alpha - Ag \cdot t$	v_y velocity in y-direction	m/s



	$s_x = v_o t \cos\alpha$	s_x distance in x-direction	m
	$s_y = v_o t \sin\alpha - \frac{Ag \cdot t^2}{2}$	s_y distance in y-direction	m
Circular movement	$\omega_f = \omega_i + \alpha t$	ω_f velocity of angle (end)	rad/s
		ω_i velocity of angle (start)	rad/s
		α angle acceleration	rad/s^2
		a_n centripetal acceleration	m/s^2

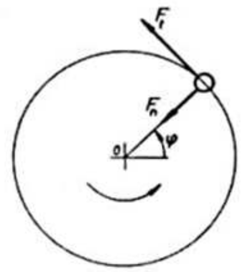


	$\alpha = \frac{\omega_f - \omega_i}{t}$		
	$\varphi = \frac{\omega_i + \omega_f}{2} t$	φ distance of angle	rad
	$\varphi = \omega_i t + \frac{\alpha t^2}{2} = \frac{\omega_f^2 - \omega_i^2}{2\alpha}$		
	$a_t = \alpha r$	a_t tangential acceleration	m/s^2
		r radius	m
Weight and force	$W = mAg$	W weight	N
		m mass	kg
		Ag acceleration due to gravity	m/s^2
	$F = ma$	F force	N
		a acceleration	m/s^2

Work	$W = Fd$	W	work	J
		F	force	N
		d	distance	m
Energy	$E_p = mAgh$	E_p	potential energy (position energy)	J
		h	height/altitude	m
	$E = Fd$	d	distance	m
	$E_k = \frac{mv^2}{2}$	E_k	kinetic energy (energy of motion)	J
		v	velocity	m/s
Effect	$P = \frac{W}{t} = \frac{Fd}{t} = Fv = T\omega$	P	effect	W
		W	work	J
		t	time	s
		F	force	N
		d	distance	m
		v	velocity	m/s
		T	torsion moment	N · m
		ω	velocity of angle	rad/s
Efficiency	$\eta = \frac{P_a}{P_t}$	η	efficiency	1
		P_a	effect delivered (useful)	W
		P_t	effect supplied	W
Rotation around a fixed axis	$T = I\alpha$	T	torsion moment	N · m
		I	inertia moment of mass	kgm^2
		α	angle acceleration	rad/s^2
	$W = T\varphi$	W	work	J
		φ	angle of torsion	rad
	$E_k = \frac{I\omega^2}{2}$	E_k	kinetic energy	J
		ω	angle of velocity	rad/s

Centripetal force $F_n = m\,a_n$

Tangential force $F_t = m\,a_t$

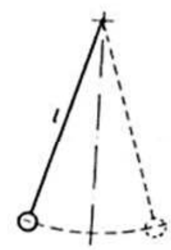


F_n	centripetal force	N
m	mass	kg
a_n	centripetal acceleration	m/s^2
F_t	tangential force	N
a_t	tangential aceleration	m/s^2

Harmonic swings

Mathematic pendulum (small swings)

$$T = 2\pi\sqrt{\frac{l}{Ag}}$$

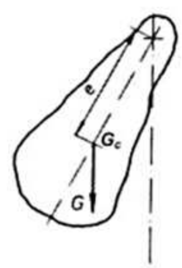


T	swing time	s
l	length	m
Ag	acceleration of gravity	m/s^2

Physic pendulum

$$T = 2\pi\sqrt{\frac{I}{mge}}$$

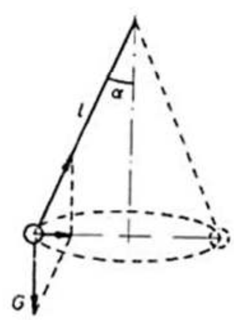
$$T = 2\pi\sqrt{\frac{m}{c}}$$



I	inertia moment of mass	kgm^2
m	mass	kg
e	distance to center of gravity	m
G_c	center of gravity	
c	constant	N/m

Conic pendulum

$$T = 2\pi\sqrt{\frac{l\cos\alpha}{Ag}}$$

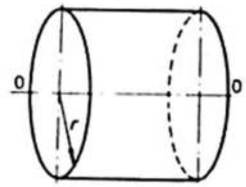


α	angle	rad

Inertia Moment of Mass

Solid cylinder

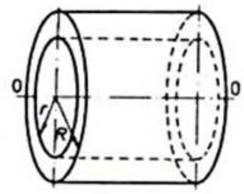
$$I_o = \frac{mr^2}{2}$$



I_o	inertia moment	kgm^2
m	mass	kg
r	radius	m

Hollow cylinder

$$I_o = \frac{m(R^2+r^2)}{2}$$



I_o	inertia moment	kgm^2
R	outer radius	m
r	inner radius	m

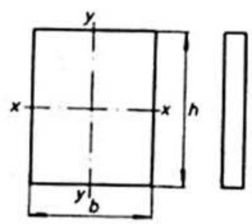
$$I_o = m\, r_m^2$$

r_m	mean radius (shall be used only when the difference between the radii is small)	m

Rectangular thin plate

$$I_x = \frac{mh^2}{12}$$

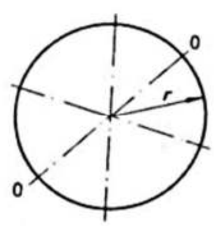
$$I_y = \frac{mb^2}{12}$$



I_x	inertia moment on x-axis	kgm^2
h	side	m
I_y	inertia moment on y-axis	kgm^2
b	side	m

Sphere

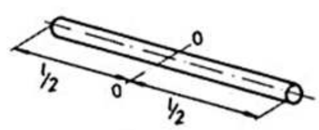
$$I_o = \frac{2}{5}mr^2$$



I_o	inertia moment	kgm^2

Straight bar

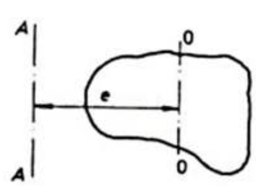
$$I_o = \frac{ml^2}{12}$$



l	length	m

Steiner's formula

$$I_A = I_o + me^2$$



I_A	inertia moment on A axis	kgm^2
e	distance between the axises	m

Centric impact

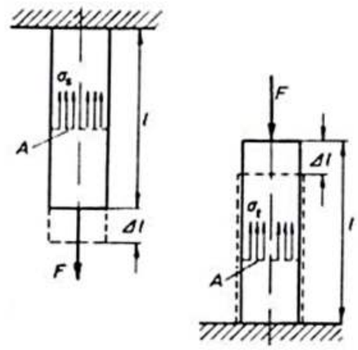
Inelastic impact	$m_1v_1 + m_2v_2 = (m_1 + m_2)u$	m_1	mass of body no. 1	kg
		m_2	mass of body no. 2	kg
		v_1	velocity before impact no. 1	m/s
		v_2	velocity before impact no. 2	m/s
		u	joint velocity after impacts	m/s
Elastic impact	$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$	u_1	velocity after impact no. 1	m/s
		u_2	velocity after impact no. 2	m/s
	$u_1 - u_2 = c(v_2 - v_1)$	c	coefficient of impact	1
			Elastic impact (k = 1), inelastic impact (k = 0), and partly elastic impact (0<k<1)	
Loss of energy at impact	$E = \frac{1}{2}[m_1(v_1^2 - u_1^2) + m_2(v_2^2 - u_2^2)]$	E	loss of energy	J

SOLIDITY/COMPACTNESS/ELASTICITY

Centric stress and pressure

$\sigma_s = \frac{F}{A}$

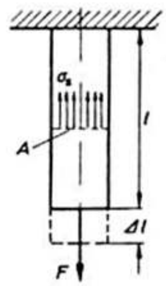
$\sigma_t = \frac{F}{A}$



σ_s	tension of stress	Pa
F	force of stress	N
A	area	m^2
σ_t	tension of pressure	Pa
F	force	N

Hooke's law

$\epsilon = \frac{\Delta l}{l} = \frac{\sigma}{E}$



ϵ	relative change of length	1
l	original length	m
Δl	extension	m
σ	tension (stress/ pressure)	Pa
E	module of elasticity	Pa

$\Delta l = \frac{\sigma l}{E} = \frac{F l}{E A}$

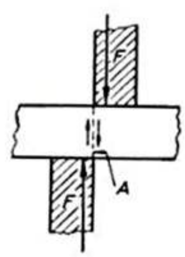
Δl	extension	m
F	force (stress/ pressure)	N
A	area	m^2

$F = -kx$ *(ideal spring)*

F	force (stress/ pressure)	N
k	spring constant	Nm^{-1}
x	amount by which the spring is stretched	m
(-)	negative sign means that the spring force is directed oppositely to the stretching compression	

Cutting

$\tau = \frac{F}{A}$

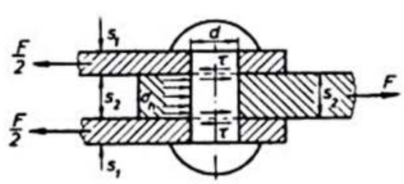


τ	cutting tension	Pa
F	force	N
A	area	m^2

Rivets and screw connections

Hole pressure tension

$\sigma_h = \frac{F}{nds}$



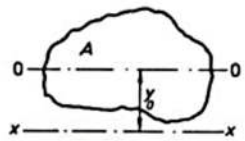
σ_h	tension of hole pressure	Pa
F	force	N
d	diameter	m
s	thickness of plate	m
n	number of areas	1

Cutting tension $\tau = \frac{F}{n \frac{\pi d^2}{4}}$

τ	cutting tension in rivet	Pa

Inertia and resistan moments when bending

Steiner's formula $I_x = I_o + A y_0^2$



I_x	inertia moment of area on x-axis	m^4
I_o	inertia moment of area on axis of gravity	m^4
A	area	m^2
Y_o	distance	m

Table

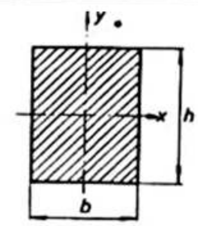
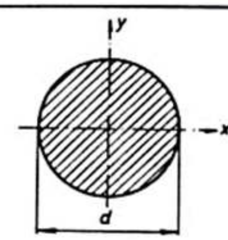
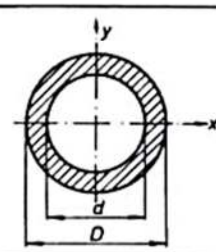
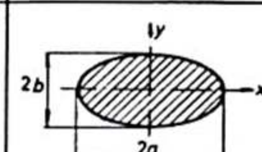
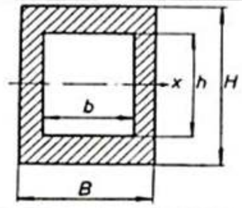
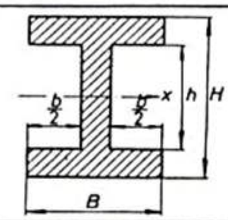
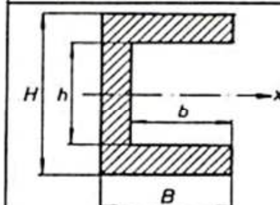
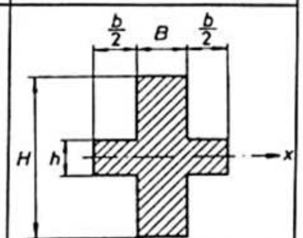
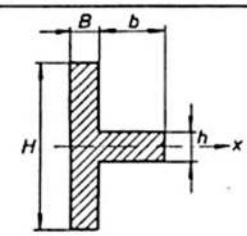
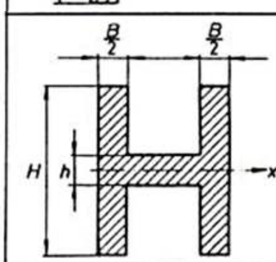
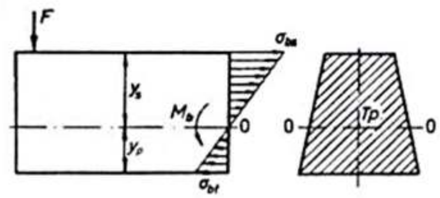
Inertia Moment	Resistance moment	Edge of the cut
$I_x = \frac{bh^3}{12}$ $I_y = \frac{hb^3}{12}$	$W_x = \frac{bh^2}{6}$ $W_y = \frac{hb^2}{6}$	
$I_x = I_y = \frac{\pi d^4}{64}$	$W_x = W_y = \frac{\pi d^3}{32}$	
$I_x = I_y = \frac{\pi}{64}(D^4 - d^4)$	$W_x = W_y = \frac{\pi}{32} \cdot \frac{D^4 - d^4}{D}$	
$I_x = \frac{\pi a b^3}{4}$ $I_y = \frac{\pi b a^3}{4}$	$W_x = \frac{\pi a b^2}{4}$ $W_y = \frac{\pi b a^2}{4}$	

Table continued

Inertia Moment	Resistance moment	Edge of the cut
		
$I_x = \frac{BH^3 - bh^3}{12}$	$W_x = \frac{BH^3 - bh^3}{6H}$	
		
		
$I_x = \frac{BH^3 + bh^3}{12}$	$W_x = \frac{BH^3 + bh^3}{6H}$	
		

Bending

$$\sigma_b = \frac{M_b}{W}$$



$$W = \frac{I}{y}$$

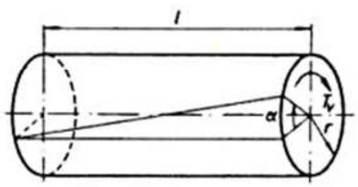
σ_b	tension of bending	Pa
M_b	moment of bending	N · m
W	moment of resistance when bending	m^3
I	moment of inertia	m^4
Y	distance from neutral axis to the point of the edge of the cut where the tension is sought	m

Index s means stress, p means pressure

Torsion

Tension of torsion

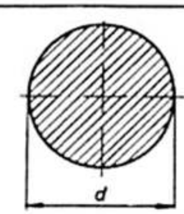
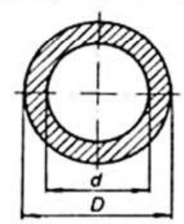
$$\tau_v = \frac{T}{W_p}$$



$$W_p = \frac{I_p}{r}$$

τ_v	tension of torsion	Pa
T	moment of torsion (torsion moment)	N · m
W_p	polar resistance moment	m^3
I_p	polar inertia moment	m^4
r	radius	m

Table

Polar inertia moment	Polar resistance moment	Edge of cut
$I_p = \frac{\pi d^4}{32}$	$W_p = \frac{\pi d^3}{16}$	
$I_p = \frac{\pi}{32}(D^4 - d^4)$	$W_p = \frac{\pi}{16}\left(\frac{D^4 - d^4}{D}\right)$	

Torsion deformation

$$\alpha = \frac{T\,l}{W_p G r} = \frac{T\,l}{G I_p}$$

α	angle of torsion	rad
l	length of axis	m
G	cutting module	Pa

Connection between effect, rotation figure, and moment of torsion

$$T = \frac{60\,P}{2\pi n} \approx 9{,}55\,\frac{P}{n_1}$$

T	moment of torsion	$N \cdot m$
P	effect	W
n_1	rotation figure	rev/min

$$T = \frac{P}{2\pi n} \approx 0{,}159\,\frac{P}{n}$$

n	rotation frequency	s^{-1}
ω	velocity of angle	rad/s

$$T = \frac{P}{\omega}$$

Cracking/bending

Radius of inertia

$$i = \sqrt{\frac{I_0}{A}}$$

i	radius of inertia	m
I_0	Moment of inertia	m^4
A	cross cut/section/ area	m^2

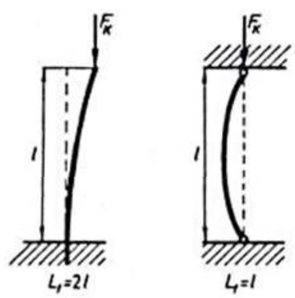
Slenderness ratio

$$\lambda = \frac{L_f}{i}$$

λ	slenderness ratio (slimness)	1
L_f	free length of bending/cracking	m

Euler's formula

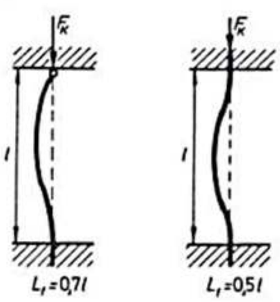
$$F_K = \frac{\pi^2 E I_0}{L_f^2}$$



F_K	cracking/bending force	N
E	elasticity module	Pa

Firmness of cracking/ bending

$$\sigma_K = \frac{F_K}{A}$$



σ_K	firmness of cracking/bending	Pa
A	cross cut/section area	m^2

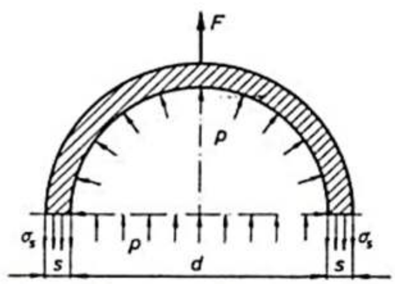
Allowed bending tension	$\sigma_k = \frac{\sigma_K}{n}$	σ_k	allowed cracking/ bending tension	Pa
		n	bending safety factor	1.
Allowed load	$F = \frac{F_K}{n}$	F	allowed load	N
Tetmajer's formula	$F_K = \sigma_K A$	F_K	bending/cracking force	N
		σ_K	bending firmness	Pa
		A	area, cross cut	m^2

Connection between slenderness and bending firmness related to Tetmajer			
Material	E N/mm²	σ_K N/mm²	Slenderness
Wood (pine and spruce)	$1 \cdot 10^3$	$29 - 0{,}19\,\lambda$	$\lambda < 100$
Cast iron	$100 \cdot 10^3$	$760 - 11{,}8 + 0{,}052\,\lambda^2$	$\lambda < 80$
Steel (mild)	$210 \cdot 10^3$	$304 - 1{,}12\,\lambda$	$\lambda < 105$
Steel (hard)	$210 \cdot 10^3$	$328 - 0{,}608\,\lambda$	$\lambda < 90$

Pressurized containers

Thin plated cylindrical container

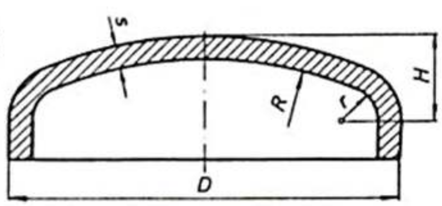
$$s = \frac{pd}{2\sigma}$$



s	thickness of plates (walls)	m
p	inner high pressure	Pa
d	inner diameter	m
σ	tension of stress	Pa

Arched bottoms

$$s = \frac{3pD}{4\sigma_s}$$



s	thickness of plates (walls)	m
D	outer diameter	m

Prerequisites for the formula, $H = 0{,}2\,D$, $R \geq D$, $r = 0{,}1\,D$, length of the cylindrical part of the bottom > 4s