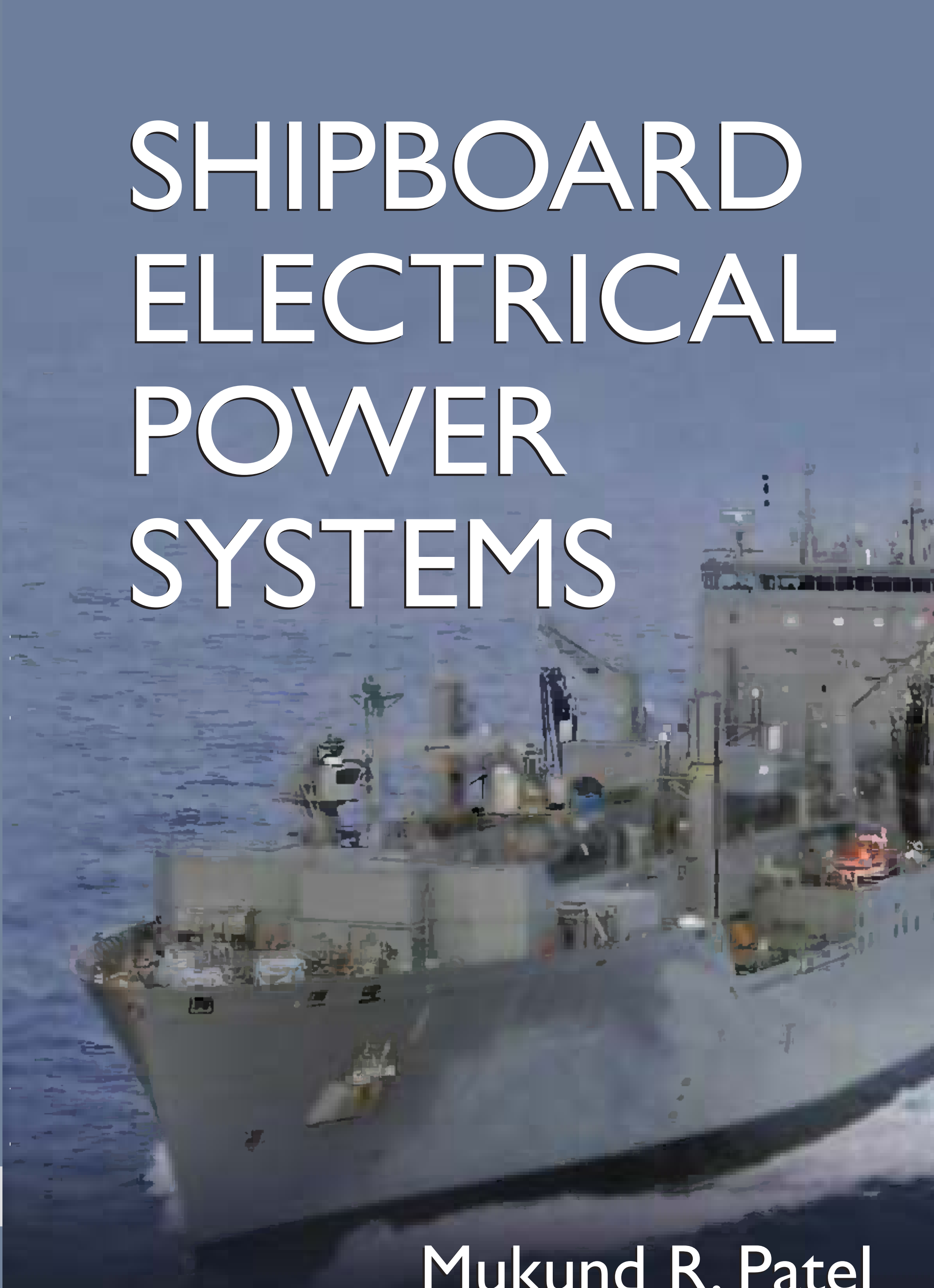
SHIPBOARD ELECTRICAL POWER SYSTEMS



Mukund R. Patel

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Dedication

to …

the young sailor and the sea.

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Chapter 5 AC and DC Motors

Preface

The United States, Canada, the United Kingdom, and many other countries are presently experiencing a shortage of power engineers. The shortage is expected to get worse in the U.S. where about 45% of the U.S. utility power engineers will become eligible to retire in the next 5 years. They would require about 7000 new power engineering graduates to replace them, and about equal number in the supporting industries. At the same time, many countries are making huge investment in building new power plants and power grid that will require even more power engineers to serve the industry. It is because of this trend that many university students are now once again becoming attracted to electrical power programs.

In the shipping industry, the demand for larger cargo and cruise ships with higher speed, lower life-cycle cost, low environmental impact, and greater maneuverability, reliability, and safety has been rapidly growing. The conventional ship today can deliver transatlantic cargo in 2–3 weeks at average freight rates approaching $100,000 per day. A fast ship being designed now can deliver transatlantic cargo freight within a week at about 1/5th the cost of air freight. Moreover, high emissions from land and air transport on congested routes are now favoring marine transportation.

In the past decade, there has been a strong trend for more electrification of naval and commercial ships. Large passenger cruise lines that are being designed today may be 350 m long and 40 m wide to serve about 4500 passengers with 1500 crew members. The diesel–electric propulsion for such ships may need about 150 MW electrical power, which compares in size with some new land-based power plants. These trends indicate that electrification in the shipping industry is a growth area. This book is focused on preparing young engineers in shipboard power system design, control, protection, economic use of power, and power electronics–based variable frequency motor drives.

Ship builders around the world have added requirements of minimizing the noise and vibrations and maximizing the usable space. This allows larger combat weapons in navy warships and more paying passengers in passenger cruise ships, where the premium on space is high. Navy ships with integrated power systems for propulsion and all other service loads can also become reconfigurable for greater survivability. The U.S. Office of Naval Research, therefore, has been providing research funding for developing the integrated electric ship. The research is conducted by the Electric Ship Research and Development Consortium that includes Florida State University, Massachusetts Institute of Technology, Mississippi State University, Purdue University, the U.S. Naval Academy, University of South Carolina and the University of Texas–Austin.

The shipboard power systems have undergone significant new developments in the last decade and will continue to do so even at a faster rate in the current decade. Considering that large- and medium-size ships at 20 knots travel a mere 50–100 feet per gallon (4–8 meters per liter) of oil, the electrical power generation and utilization efficiency becomes increasingly important. Today's shipbuilders of both commercial and navy ships, along with their support industries, are now taking an active part in the research and development to advance shipboard electrical power systems.

It is in this light that modern commercial and military ship builders are looking for electrical power engineers to meet their rapidly growing need for large, fast, efficient, and reconfigurable ships to compete in the international market. There is no single book available today that covers the entire scope of shipboard power systems. The marine engineering students and industry professionals have been relying on limited publications in bits and pieces presented at various conferences and a few books having short sections with sketchy coverage on this wide subject. This book is the first comprehensive volume of its kind that focuses on all aspects of shipboard electrical power systems.

It is hoped that the book is seen as a timely addition to the literature, and makes an excellent resource for students at various marine and naval academies around the world and a range of industry professionals.

Mukund R. Patel
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Acknowledgments

The book of this nature on shipboard electrical power system that is rapidly growing in size to incorporate new technologies in commercial and navy ships cannot possibly be written without the help from many sources. I have been extremely fortunate to receive full support from many organizations and individuals in the field. They encouraged me to write the book on this timely subject and also provided valuable suggestions during the development of the book.

At the U.S. Merchant Marine Academy, I am grateful to Professor Jose Femenia, Director of the Graduate Program; Dr. David Palmer, Engineering Department head; and Dr. Shashi Kumar, Academic Dean; for supporting my research and publications that led to writing this book. I have benefited from many midshipmen at the academy, both the undergraduate and graduate students coming from all over the world, who contributed to my learning by pointed discussions based on their professional experience. They are David Condron, Arthur Faherty, Bill Frost, James Hogan, Derrick Kirsch, Enrique Melendez, Dana Walker, Bill Veit, Raul Osigian, and others.

Several ship builders and organizations provided current data and reports on the shipboard power technologies and gladly provided all the help I asked for. Additionally, Jonathan Plant, senior editor at Taylor and Francis/CRC Press LLC, patiently encouraged me to complete the book even with many interruptions. My gratitude also goes to Lt. Anthony J. Indelicato, Jr., U.S. Navy Reserve (Ret.) and Professor John Hennings of Webb Institute of Naval Architecture for reviewing the book proposal and providing valuable comments.

My wife Sarla, and grandchildren Rayna and Dhruv, cheerfully contributed the time they would have otherwise spent with me.

I heartfully acknowledge the valuable support and encouragement from all.

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Yardley, Pennsylvania

About This Book

This book has evolved from the author's 30 years of industry experience at the General Electric, Lockheed Martin, and Westinghouse Electric corporations, and 15 years of teaching experience at the U.S. Merchant Marine Academy, Kings Point, New York. It is the first of its kind that covers all aspects of the electrical power system—design, analyses, and operation—with details not found in any other single textbook. Although the coverage is general, the book has a focus on the shipboard power system topics suitable for undergraduate and graduate students in marine engineering. It is also written as a reference book for a range of professional engineers working for commercial and military shipbuilders, ship users, port operators, classification societies, machinery and equipment manufacturers, research institutes, universities, and others.

The topics covered in 13 chapters are applicable to electrical power in ships, ports, offshore industries, factories, chemical plants, refineries, or industrial plants. They review the basic theory of electrical machines, power cables, fault current analyses, system relaying and protection, batteries, and economic use of power. To that extent, the book provides basic transferable skills in the electrical power field anywhere—on ship or on land. The book is, therefore, more comprehensive in coverage for turning out well-rounded electrical power engineers than with the traditional electrical machine course required in many marine engineering programs. The author ventures to suggest replacing the electrical machine course with the electrical power system course that comprehensively covers all aspects of electrical power that professional engineers deal with, which includes much more than the machines.

Each chapter has numerous examples integrated within the text, exercise problems, and questions to reflect back on the basic concepts that can engage students in group discussions on the topics covered. The book assumes a basic background in ac circuits, but their important aspects are briefly reviewed before moving on to more advance topics.

Additional features of the books are these: (i) It's the only book that covers all topics required for power engineers under one cover; (ii) batteries, essential in all practical power systems, are described in a depth not found in traditional books on power systems, (iii) difficult electrical concepts are explained with mechanical and hydraulic analogies that marine engineers can easily relate to, (iv) marine industry standards are also briefly introduced to widen the student's horizons, and (v) long visible suffixes in equations to make their use fluent, unlike hard-to-remember one-letter symbols often used in many books.

Both systems of units—international and British—are used in the book to present data, as they came from various sources. An extensive conversion table connecting the two systems of units is therefore given in the front, along with a list of acronyms commonly used in the power industry and also in the book.

It is hoped that the book is found to fill a gap in current knowledge in the field, and becomes regarded as an excellent resource for students and professionals engaged in the shipboard power systems on commercial and navy ships.

The Author

Mukund R. Patel, Ph.D., P.E., is a Professor of Engineering at the U.S. Merchant Marine Academe in Kings Point, NY. He has about 50 years of hands-on involvement in research, development, and design of the state-of-the-art electrical power equipments and systems. He has served as a Principal Engineer at the General Electric Company in Valley Forge, PA; Fellow Engineer at the Westinghouse Research & Development Center in Pittsburgh, PA; Senior Staff Engineer at the Lockheed Martin Corporation in Princeton, NJ; Development Manager at Bharat Bijlee (Siemens) Limited, Mumbai, India; and 3M McKnight Distinguished Visiting Professor at the University of Minnesota, Duluth.

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Dr. Patel is an Associate Editor of Solar Energy, the journal of the International Solar Energy Society; and a member of the review panels for the government-funded research projects on renewable energy in the State of California and the Emirate of Qatar. He has authored three books, and major chapters in two international handbooks. He has taught 3-day courses to practicing engineers in the electrical power industry for over 15 years, has presented and published over 50 papers at national and international conferences and journals, holds several patents, and has earned NASA recognition for exceptional contribution to the power system design for the Upper Atmosphere Research Satellite. He can be reached at patelm@usmma.edu or patelm30@gmail.com.

Acronyms and Abbreviations (Upper case or lower case)

AC	Alternating current
Ah	Ampere-hour of battery
CB	Circuit breaker
DoD	Depth of discharge of battery
DC	Direct current
EMF	Electromotive force
HP	Horsepower
K	Constant of proportionality
LV	Low voltage
MV	Medium voltage
HV	High voltage
PF	Power factor
PM	Permanent magnet
RPM	Revolutions per minute
SoC	State of charge of battery
VR	Voltage regulation
kVAR	Kilovolt Ampere Reactive
$\sqrt{(\ )}$	Square root of the parenthesis
KVL	Kirchhoff's Voltage Law
RMS	Root Mean Square
Th	Thevenin (source voltage or impedance)
OC	Open-circuit (load terminals open)
SC	Short-circuit (load terminals shorted)
SS	Steady-state (alternating or dc)
Syn	Synchronous
PF	Power Factor (cosθ)
$\tilde{A}$	Sinusoidal phasor of magnitude A
$\angle\theta^{o}$	Angle of the phasor

Conventions: The following notations are generally used for the variables in this book:

Lower case letter represents time-varying variable (current, voltage, power, etc.)

Upper case letter represents fixed (dc or ac rms) value of current, voltage, power, etc.

Letter with wavy hat sign (e.g. $\tilde{V}$ or $\tilde{I}$) represents sinusoidally varying ac phasor with magnitude (rms) and phase difference of angle θ with respect to a reference sinusoidal wave (usually the voltage.)

FREQUENTLY CITED ORGANIZATIONS

ABS	American Bureau of Shipping
IEC	International Electrocommission
IEEE	Institute of Electrical and Electronics Engineers
NEC	National Electrical Code
NEMA	National Electrical Manufacturers Association
NFPA	National Fire Protection Association
SNAME	Society of Naval Architects and Marine Engineers

SYSTEMS OF UNITS AND CONVERSION FACTORS

Both the international units (SI or MKS system) and the British units are used in this book. The table below relates the International units with the British units commonly used in the U.S.

Category	SI Unit =	Factor Below X	English Unit
Length	meter	0.304 8	foot
	mm	25.4	inch
	micron	25.4	mil
	km	1.609 3	mile
	km	1.852	nautical mile
Area	m^2	0.092 9	foot squared
	μm^2	506.7	circular mil
Volume	liter (dm^3)	28.316 8	cubic foot
	liter	0.016 39	cubic inch
	cm^3	16.387 1	cubic inch
	m^3/s	0.028 31	cubic foot /hr
	liter	3.785 3	gallon (US)
	liter/sec	0.063 09	gallon/minute
Mass	Kg	0.453 59	pound mass
	Kg	14.593 9	Slug mass
Density	Kg/m^3	16.020	pound mass/ft^3
	Kg/cm^3	0.027 68	pound mass/in^3
Force	N	4.448 2	pound force
Pressure	kPa	6.894 8	pound/in^2 (psi)
	kPa	100.0	bar
	kPa	101.325	std atm (760 torr)
	kPa	0.132 84	1mm Hg at 20 °C
Torque	Nm	1.355 8	pound-force foot
Power	W	1.355 8	foot pound/sec
	W	745.7	horsepower
Energy	J	1.355 8	foot pound-force
	kJ	1.055 1	Btu International
	kWh	3412	Btu International
	MJ	2.684 5	horsepower hour
	MJ	105.506	therm

Category	SI Unit =	Factor Below X	English Unit
Temperature	°C	(°F–32)·5/9	°F
	°K	(°F+459.67)·5/9	°R
Heat	W	0.293 1	Btu(Int.) / hour
	kW	3.517	Ton Refrigeration
	W/m²	3.154 6	Btu/(ft² hr)
	W/(m²°C)	5.678 3	Btu/(ft² hr °F)
	MJ/(m³°C)	0.067 1	Btu/(ft³ °F)
	W/(m °C)	0.144 2	Btu inch /(ft² hr °F)
	W/(m °C)	1.730 4	Btu ft/(ft² hr °F)
	J/kg	2.326	Btu / pound
	MJ/m³	0.037 3	Btu / ft³
	J/(kg °C)	4.186 8	Btu/(pound °F)
Velocity	m/s	0.304 8	foot/sec
	m/s	0.447 04	mile/hour
	m/s	0.514 46	knot
Magnetics	weber	10^{-8}	line
(tesla)	wb/m²	0.0155	kiloline/inch²

PREFIXES TO UNITS

μ	micro	10^{-6}	m	mili	10^{-3}
k	kilo	10^{3}	M	mega	10^{6}
G	giga	10^{9}	T	tera	10^{12}

OTHER CONVERSIONS

1 nautical mile = 1.15081 mile
1 calorie (CGS unit) = 4.1868 J
1 kg cal (SI unit) = 4.1868 kJ
1 horsepower = 550 ft-lb/s
1 tesla magnetic flux density = 10,000 gauss (lines/cm^2)
Absolute zero temperature = 273.16 °C = 459.67° F
Acceleration due to Earth gravity = 9.806 7 m/s^2 (32.173 5 ft/s^2)
Permeability of free space $\mu_o = 4\pi \times 10^{-7}$ henry/m
Permittivity of free space $\varepsilon_o = 8.85 \times 10^{-12}$ farad/m

ENERGY CONTENT OF FUELS

1 tip of match-stick = 1 Btu (heats 1 Lb water by 1°F)
1 Therm = 100,000 Btu

1 Quad = 10^{15} Btu
1 ft^3 of natural gas = 1000 Btu (1055 kJ)
1 gallon of LP gas = 95,000 Btu
1 gallon of gasoline = 125,000 Btu
1 gallon of No. 2 oil = 140,000 Btu
1 gallon of oil (U.S.) = 42 kWh
1 barrel = 42 gallons (U.S.)
1 barrel of refined oil = 6×10^6 Btu
1 barrel of crude oil = 5.1×10^6 Btu
1 ton of coal = 25×10^6 Btu
1 cord of wood = 30×10^6 Btu
1 million Btu = 90 Lb coal, or 8 gallons gasoline, or 11 gallons propane
1 quad (10^{15} Btu) = 45 million tons coal, or 109 cft natural gas, or 170 million barrels oil
1 Lb of hydrogen = 52,000 Btu = 15.24 kWh of primary energy (requires 8 Lb of oxygen)
World's total primary energy demand in 2010 was about 1 Quad (10^{15} Btu) per day = 110×10^{16} J/day = 305×10^{12} kWh/day
About 10,000 Btu of primary thermal energy input at the power generating plant produces 1 kWh of electrical energy at the user's outlet.

1 AC Power Fundamentals

Thomas Edison's Pearl Street low-voltage dc generating station opened in 1882 to serve the New York City market. Its low-voltage power had to be consumed close to the generating station to keep conductor I^2R loss to an economically viable level. Many neighborhood power stations closed to the users were built by Edison and his competitors. Then came Westinghouse's ac power systems, with large steam and hydro power plants built where it was economical to do so, such as on the Niagara Falls. The high-voltage ac power was brought to the load centers, which was stepped down by transformers before feeding to the end users. The ac system proved to be much more economical and flexible, and soon drove away the dc competitors. The New Yorkers paid an inflation-adjusted price of about $5 per kWh for dc power in 1890 compared to the average of about $0.12 per kWh we pay for ac power today in the United States. With the development of transformer and Nicola Tesla's induction motor, ac soon became universally adopted for electric power all around the word.

The fundamentals of power flow in alternating current (ac) circuits—usually covered in an undergraduate course in electrical engineering—are reviewed in this chapter. A clear understanding of these fundamentals will prepare the student for the chapters that follow, and is also essential for working in the electrical power field.

1.1 CURRENT VOLTAGE POWER AND ENERGY

The basis of electricity is electric charge (measured in coulombs) moving between two points at different electrical potentials, either absorbing or releasing energy along its way. The basic electrical entities in power engineering are defined below with their generally used symbols and units.

Current I (ampere) = Flow rate of electrical charge (1 A = 1 C/sec)
Voltage V (volt) = Electrical potential difference, that is, energy absorbed or released per coulomb of charge moving from one point to another (1 V = 1 J/coulomb)
Power P (watt) = Rate of energy flow (1 W = 1 J/sec)

$$\therefore P = \frac{Joules}{Second} = \frac{Joules}{Coulomb} \times \frac{Coulombs}{Second} = V \times I \quad \text{watts} \tag{1.1}$$

With time-varying voltage and current, the instantaneous power $p(t) = v(t) \times i(t)$, and energy = power × time duration. With time-varying power, the energy used between 0 and T seconds is given by the integral (i.e., area under the power versus time curve)

$$Energy = \int_0^T v(t) \cdot i(t)\, dt \qquad \text{watt-seconds (or joules)} \tag{1.2}$$

Inversely, the power is time differential of energy, that is

$$Power = \frac{d}{dt}(Energy) \tag{1.3}$$

Example 1.1

The electrical potential of point A is 200 V higher than that of point B, and 30 C of charge per minute flows from A to B. Determine the current and power flow from A to B, and the energy transferred in 1 min.

SOLUTION

Current flow from point A to B = 30 ÷ 60 = 0.50 A
Power flow from A to B = Voltage × Current = 200 × 0.5 = 100 W
Energy transferred in 60 sec = 100 × 60 = 6000 W-sec (J)

It is worth repeating here that 1 watt = 1 joule/sec or 1 joule = 1 watt-sec. The commercial unit of electrical energy is the kilowatt-hour (kWh). One kWh = 1000 watts × 3600 sec = 3,600,000 watt-sec = 3.6 MJ. The utility bill is based on kWh electrical energy used over the billing period. The average urban U.S. customer using 1000 kWh per month pays about \$0.15 per kWh, out of which about 40% is for generation, 5% for transmission, 35% for distributions, and 20% for account service and profit.

For example, if a heater consumes 1500 W for 4 h and 750 W for 8 h every day in a winter month of 30 days, then the electrical bill at 15 cents/ kWh tariff will be (1.5 × 4 + 0.750 × 8) kWh /day × 30 days/month × \$ 0.15 per kWh = \$54 for that month.

1.2 ALTERNATING CURRENT

Alternating current is used all over the world for electrical power. It varies sinusoidally with time t as

$$i(t) = I_m \sin\omega t \text{ or } i(t) = I_m \cos\omega t \tag{1.4}$$

Although its representation by either the sine or cosine function is called sinusoidal, the cosine function shown in Figure 1.1 is more common, where

I_m = maximum value, or amplitude, or peak value of the sinusoid, amperes
$\omega = 2\pi f$ = angular speed (or angular frequency) of the alternations, rad/sec

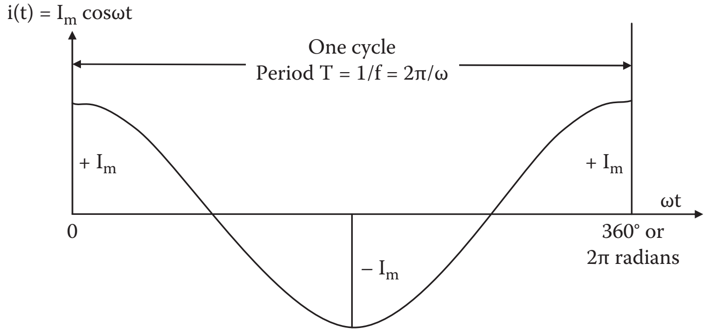


FIGURE 1.1 Sinusoidal alternating current over one cycle represented by the cosine function.

T = period of repetition, sec/cycle
$f = 1/T = \omega/2\pi$ = frequency, cycles/sec (or Hz)

In either representation, the ac current completes one cycle of alternation in $\omega t = 360°$ or 2π radians. For this reason, one cycle of ac is customarily displayed with respect to ωt, and not with time t. As an example, for sinusoidal current $i(t) = 170 \cos 377t$,

- Peak value (amplitude) $I_m = 170$ A (front number)
- Angular frequency $\omega = 377$ rad/sec (number in front of t)
- Numerical frequency $f = \omega/2\pi$ cycles/sec (Hz)
- Period of repetition $T = 1/f = 2\pi/\omega$ sec/cycle

We recall that the direct current (dc) needs only one number to specify its value, but we see in Equation (1.4) that the ac needs three numbers to specify its value at any instant of time t, namely, the peak value I_m, the angular frequency ω, and the time t. This makes the mathematics in ac circuits complex.

1.2.1 RMS VALUE AND AVERAGE POWER

Although the ac varies sinusoidally with time with no real fixed value, we often speak in terms of a fixed ac value, for example, ac current of 10 A or ac voltage of 120 V. We see below what a fixed number in ac means, taking the example of a resistor carrying a current $i(t)$. The power absorbed by the resistor at any time t is given by $p(t) = i(t)^2R$, which varies in time as a square function of the current. It is always positive even when the current is negative during one-half cycle. Therefore, the average of i^2 is always a positive nonzero value, although the average of sinusoidal current is always zero. To find the average power, we must therefore use (average of i^2), not (average of i)^2, carefully noticing the placement of the parenthesis in each case. We also note that since i_{avg} is always zero, $(i_{avg})^2$ is also always zero. However, since i^2 is always positive in both positive and negative half-cycles, $(i^2)_{avg}$ is never zero, unless $i(t) = 0$ at all times. Therefore, $P_{avg} = \{i(t)^2\}_{avg} \times R$. The square root of $\{i(t)^2\}_{avg}$ is called the root-mean-squared (rms) value of the current $i(t)$.

In all practical applications, it is the average power that matters. For example, we are mostly interested in how much a room heater, a fan motor, or a pump motor produces at the end of an hour or any other time duration. Since ac repeats every cycle, the average power over one cycle is the same as that over one minute or one hour or one day, as long as the power is *on*. Therefore, the effective value of the current for determining the average power is the square root of (average of i^2) over one cycle, that is, $I_{eff} = \sqrt{Mean\,of\,i^2} = I_{\text{root-mean-squared}} = I_{rms}$. It is the equivalent dc value that would result in the same *average power*. For any wave shape in general, that is, cosine, sine, square, rectangular, triangular, etc.,

$$I_{rms} = \sqrt{\frac{\int_0^T i^2(t)\cdot dt}{T}} \tag{1.5}$$

For a sinusoidal current, $I_{rms} = \sqrt{\dfrac{\int_0^T (I_{pk}\cos\omega t)^2\cdot dt}{T}} = \dfrac{I_{pk}}{\sqrt{2}}$, and similarly

for a sinusoidal voltage, $V_{rms} = \sqrt{\dfrac{\int_0^T (V_{pk}\cos\omega t)^2\cdot dt}{T}} = \dfrac{V_{pk}}{\sqrt{2}}$ (1.6)

The divisor √2 above is for the sinusoidal ac only. It is different for different wave shapes. Using the basic calculus of finding the average, the student is encouraged to derive the divisor √2 for a sinusoidal wave, 1.0 for a rectangular wave, and √3 for a triangular wave.

1.2.2 POLARITY MARKING IN AC

Although ac circuit terminals alternate their + and – polarities every one-half cycle, we still mark them with fixed + and – polarities, as if they were of fixed polarities, as in dc. Such polarity marking in ac has the following meanings:

With multiple voltage sources (generators and transformers) in parallel, all + marks are + at the same time, and all – marks are – at the same time. This information is needed to connect multiple voltage sources in parallel to share a large load.

With multiple voltage sources or loads in series, the (+ –) and (+ –) sequence indicates an additive voltage pair, whereas the (+ –) and (– +) sequence indicates a subtractive voltage pair.

In a single-voltage source circuit, the + terminal is connected to the load and the – terminal is connected to the ground.

To eliminate such technical contradictions in using the + and – marks, the modern polarity marking is sometimes done with dots • for positive terminals and no mark for the negative terminals.

1.3 AC PHASOR

The ac current $i(t) = I_m \cos \omega t$ can be represented by an arm of length I_m rotating at an angular speed (frequency) ω rad/sec as shown in Figure 1.2. The actual instantaneous value of $i(t)$ at any time t is $I_m \cos \omega t$, which is the projection on the reference axis. It alternates between $+ I_m$ and $-I_m$, going through zero twice at $\omega t = 90°$ and 270° every cycle. Since the actual value of $i(t)$ depends on the phase angle of the rotating arm, the rotating arm is called the *phasor*. Decades ago, it used to be called a vector, since the algebra dealing with it works like vector algebra.

The ac voltage and current phasors in general can have phase difference between their peaks, that is, their peaks can occur at different instances of time. A voltage *V* and current *I* phasors shown in Figure 1.3(a) have the phase difference of angle θ, the current peak lagging the voltage peak by angle θ. Two phasors of the same frequency with a phase difference of θ between their peaks will also have the same phase difference θ between their zeros. The wavy hat sign ~ on *V* and *I* signifies the sinusoidal variations with respect to time.

In the electrical power industry, since the voltage is given by the generator or the utility company, the power engineer always takes the voltage as the reference phasor, and then designates the current as leading or lagging the voltage. In all practical power circuits, the current lags the voltage, so we say that the current lags in most practical power circuits.

Drawing a neat sine wave by hand is not easy. Power engineers generally circumvent this difficulty by drawing the phasor diagram as shown in Figure 1.3(b), where *V* and *I* phasors are rotating at the same angular speed ω, keeping their phase difference θ fixed. The actual instantaneous values of both *V* and *I* are their respective projections on the reference axis at any given instant of time.

The phasor diagram can be drawn using the arm's length equal to the peak value or the rms value. Since the power engineer is always interested in the average power, he or she always draws the phasor diagram using the rms values. The rms

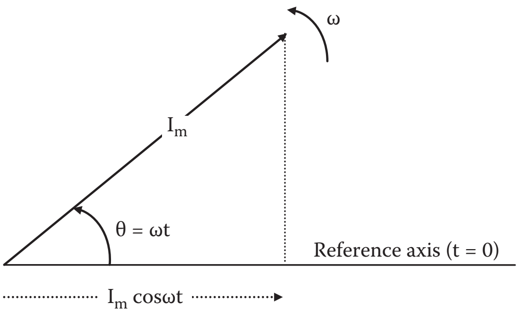
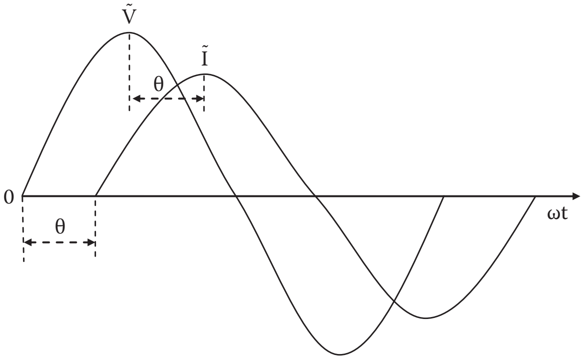
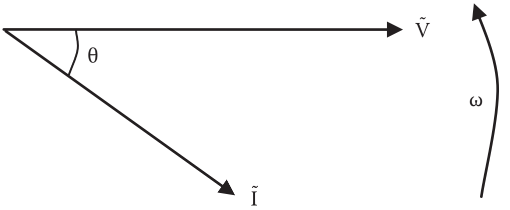


FIGURE 1.2 Rotating phasor $\tilde{I}$ representing alternating current.



(a) Time diagram



(b) Phasor diagram

FIGURE 1.3 Two sinusoidal phasors out of phase by angle θ.

values are customarily implied in the power field, and we will do the same in this book as well.

For average power, we recognize that the voltage and current that are out of phase would produce less average power than those in phase with their peaks occurring at the same time. If V and I are in phase ($\theta = 0$), their product $P = V \times I$ is always positive even when both V and I are negative during one half cycle. However, when $\theta \neq 0$, the instantaneous power is negative when either V or I is negative and the other is positive. If positive power means power flowing from the source to the load, then negative power means power flowing backward to the source from the energy stored in the load inductance or capacitance. The average power in such cases is always less than the maximum power V and I would produce if they were in phase. The average power of voltage V and current I lagging the voltage by a phase angle θ is given by the time-average of $p(t) = v(t) \cdot i(t)$ over one cycle of period T, that is,

$$P_{avg} = \frac{1}{T}\int_0^T V_{pk}\cos(\omega t)\cdot I_{pk}\cos(\omega t - \theta)\,dt = \frac{V_{pk}I_{pk}}{2}\cos\theta = V_{rms}I_{rms}\cos\theta \quad (1.7)$$

The voltage and current, if in phase with θ = 0°, would produce the maximum possible power equal to $V_{rms} \times I_{rms}$. When not in phase, they produce less average power. The power reduction factor cosθ is called the power factor (pf). Obviously, pf = 1.0 (unity) when θ = 0°, and pf = 0 when θ = 90°.

Example 1.2

A circuit element has a sinusoidal voltage of 300 cos314t V across its terminals and draws 80 cos(314t – 25°) A. Determine the average power delivered to the element.

SOLUTION

The current is lagging the voltage by 25°, so the power factor of this element is cos25°. For the average power, we use rms values and power factor, that is,

$$P_{avg} = (300 \div \sqrt{2}\ ) \times (80 \div \sqrt{2}) \times \cos 25° = 10{,}876 \text{ W}$$

1.3.1 OPERATOR J FOR 90° PHASE SHIFT

An uppercase letter with wavy hat sign (e.g., $\tilde{I} = I\angle\theta$) in this book represents a sinusoidally varying ac phasor with rms magnitude I and a phase difference of angle θ with respect to a reference sinusoidal wave (usually the voltage.) If $\tilde{A}$ in Figure 1.4 represents any voltage or current phasor, then another phasor $\tilde{B}$ that is of the same magnitude as $\tilde{A}$ but with 90° phase shift in the positive (counterclockwise) direction can be written in long-hand as $\tilde{B} = \tilde{A}$ with an added phase shift of +90°, or $\tilde{B} = \tilde{A}\ \angle 90° = \tilde{A}\ j$ or $j\ \tilde{A}$ in short-hand where the *operator j* represents a phase shift of $\tilde{A}$ by +90° in the positive (counterclockwise) direction.

Shifting $\tilde{B}$ further by +90°, we get another phasor $\tilde{C} = j\ \tilde{B} = j\ (j\ \tilde{A}) = j^2\ \tilde{A}$. We graphically see in Figure 1.4 that $\tilde{C} = j^2 \tilde{A} = -\tilde{A}$, and therefore deduce that $j^2 = -1$ or $j = \sqrt{-1}$. Thus, j is an imaginary number generally denoted by the letter i in the mathematics of complex numbers (we use the letter j to avoid confusion with current i in electrical circuits).

$$\text{So, in mathematical operations, } j = \sqrt{-1} \text{ represents a +90° phase shift.} \tag{1.8}$$

From $j^2 = -1$ or $1 = -j^2$, we get $\frac{1}{j} = -j$, which is a useful relation to remember.

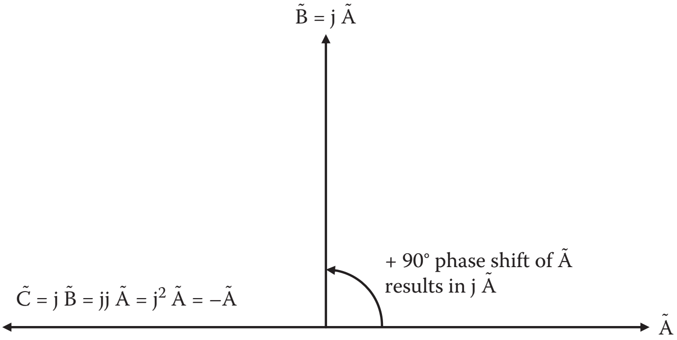


FIGURE 1.4 Operator j representing 90° phase shift in the positive (counterclockwise) direction.

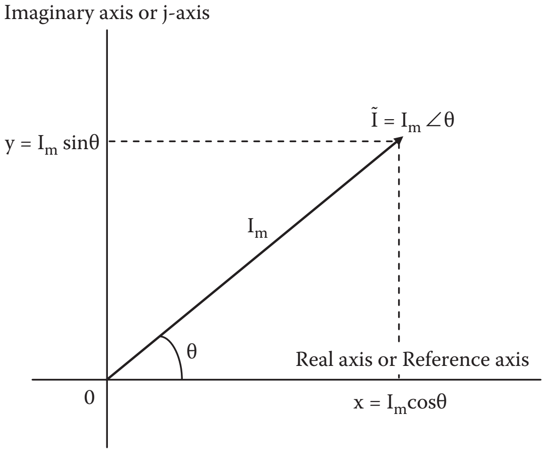


FIGURE 1.5 Polar and rectangular components of phasor $\tilde{I}$.

1.3.2 THREE WAYS OF WRITING PHASORS

Alternative ways of writing a phasor $\tilde{I}$ are depicted in Figure 1.5.

In polar form (*we call it* θ-*form*), we write a phasor as $\tilde{I} = I_m\angle\theta$, where I_m = magnitude of the phasor (can be peak, but rms magnitude is used in this book, as is customary in power engineering), and θ = phase angle of the phasor. Here, the hat sign ~ signifies a sinusoidal phasor. In routine use, we often drop the hat sign ~ and write $I = 3\angle{-20°}$, meaning an ac current of 3 A rms value lagging the voltage by 20°.

In rectangular form (*we call it j-form*), we write $\tilde{I} = x + j\,y = I_m \cos\theta + j\,I_m \sin\theta$, where x and y are the phasor's rectangular components on the real and imaginary axes, respectively.

In exponential form (*we call it e-form*), we use Euler's trigonometric identity $e^{j\theta} = \cos\theta + j\sin\theta$ to write the phasor in yet another form: $\tilde{I} = I_m\cos\theta + j\,I_m\sin\theta = I_m \times (\cos\theta + j\sin\theta) = I_m \cdot e^{j\theta}$

The three alternative ways of representing a phasor are summarized as follows:

$$\tilde{I} = I_m\angle\theta = I_m\cos\theta + j\,I_m\sin\theta = I_m e^{j\theta} \text{ where } \theta = \omega t \tag{1.9}$$

It is important to note here that two phasors $\tilde{A} = A_m\angle\theta_1 = x_1 + j\,y_1$ and $\tilde{B} = B_m\angle\theta_2 = x_2 + j\,y_2$ are equal if and only if both their magnitudes and phase angles are equal, that is, $A_m = B_m$ and $\theta_1 = \theta_2$, or $x_1 = x_2$ and $y_1 = y_2$, that is, both their real and imaginary components are individually equal.

1.3.3 PHASOR FORM CONVERSION

The form we choose to represent various phasors depends on the algebraic operation required on a given set of phasors. Certain algebraic operations require the phasors in a certain form, as seen in the next section. Therefore, converting the phasor from one form to another is often necessary, and is done using the trigonometry of Figure 1.5.

Polar to Rectangular (θ *to j*) *Conversion:* Consider a phasor given in θ-form, that is, $\tilde{I} = I_m \angle\theta$, where I_m and θ are known. To convert it in *j*-form, we write $\tilde{I}$ using the rectangular components, that is,

$$\tilde{I} = x + j\,y = I_m \cos\theta + j\,I_m \sin\theta \qquad \therefore\ x = I_m \cos\theta \text{ and } y = I_m \sin\theta \tag{1.10}$$

Rectangular to polar (*j to* θ) *conversion:* Consider a phasor given in *j*-form, that is, $\tilde{I} = x + j\,y$, where *x* and *y* are known. To convert it in θ-form, we write $\tilde{I}$ using the polar components, that is,

$$\tilde{I} = I_m \angle\theta \qquad \text{where} \quad I_m = \sqrt{x^2 + y^2} \ \ \textit{and}\ \theta = \tan^{-1}\left(\frac{y}{x}\right) \tag{1.11}$$

1.4 PHASOR ALGEBRA REVIEW

AC power engineers are routinely required to perform six basic mathematical operations on phasors, namely, the addition, subtraction, multiplication, division, differentiation, and integration of phasors. These operations are normally covered in books on algebra of complex numbers and also on ac circuits. This section is a brief summary of such operations.

Consider two phasors $\tilde{A}$ and $\tilde{B}$ given by

$$\tilde{A} = A_m \angle\theta_1 = x_1 + j\,y_1 = A_m\, e^{j\theta 1} \qquad \text{and} \qquad \tilde{B} = B_m \angle\theta_2 = x_2 + j\,y_2 = B_m e^{j\theta 2}$$

We add or subtract two phasors in the rectangular form, that is,

$$\tilde{A} + \tilde{B} = (x_1 + j\,y_1) + (x_2 + j\,y_2) = (x_1 + x_2) + j\,(y_1 + y_2) \tag{1.12}$$

$$\tilde{A} - \tilde{B} = (x_1 + j\,y_1) - (x_2 + j\,y_2) = (x_1 - x_2) + j\,(y_1 - y_2) \tag{1.13}$$

We multiply or divide two phasors in the polar and exponential forms, that is,

$$\tilde{A} \cdot \tilde{B} = A_m \angle\theta_1 \cdot B_m \angle\theta_2 = A_m e^{j\theta_1} \cdot B_m e^{j\theta_2} = A_m B_m e^{j(\theta_1 + \theta_2)} = A_m B_m \angle(\theta_1 + \theta_2) \tag{1.14}$$

$$\frac{\tilde{A}}{\tilde{B}} = \frac{A_m \angle\theta_1}{B_m \angle\theta_2} = \frac{A_m e^{j\theta_1}}{B_m e^{j\theta_2}} = \frac{A_m}{B_m} e^{j(\theta_1 - \theta_2)} = \frac{A_m}{B_m} \angle(\theta_1 - \theta_2) \tag{1.15}$$

We differentiate or integrate phasor $\tilde{A}$ with respect to time *t* in the exponential form in time domain, that is, $\tilde{A} = A_m \angle\theta = A_m e^{j\theta} = A_m e^{j\omega t}$. Then,

$$\frac{d\tilde{A}}{dt} = \frac{d}{dt} A_m e^{j\omega t} = j\omega A_m e^{j\omega t} = j\omega \tilde{A} \tag{1.16}$$

$$\int \tilde{A}\, dt = \int A_m e^{j\omega t}\, dt = A_m \frac{e^{j\omega t}}{j\omega} = \frac{\tilde{A}}{j\omega} \tag{1.17}$$

The summary of the foregoing phasor operations in words follows:

- To add two phasors, add their x and y components separately.
- To subtract two phasors, subtract their x and y components separately.
- To multiply two phasors, multiply their magnitudes and add their angles.
- To divide two phasors, divide their magnitudes and subtract their angles.
- To differentiate a phasor, multiple it by $j\omega$, that is, $d/dt = j\omega$.
- To integrate a phasor, divide it by $j\omega$, that is, $\int dt = 1/j\omega = -j\omega$.

Tip-to-tail method is the graphical method of adding or subtracting two phasors as illustrated in Figure 1.6. To add $\tilde{A}$ and $\tilde{B}$ in (a), first draw $\tilde{A}$. Then, at the tip of $\tilde{A}$, place the tail of $\tilde{B}$ and draw $\tilde{B}$. The end point from the origin is then $\tilde{A} + \tilde{B}$. To subtract $\tilde{B}$ from $\tilde{A}$ in (b), first draw $\tilde{A}$. Then, at the tip of $\tilde{A}$, place the tail of $\tilde{B}$ and draw $-\tilde{B}$ (i.e., $\tilde{B}$ in the negative direction). The end point from the origin is then $\tilde{A} - \tilde{B}$.

Example 1.3

Given the two phasors $\tilde{A} = 60\angle 30°$ and $\tilde{B} = 40\angle 60°$, determine (i) $\tilde{A} + \tilde{B}$, (ii) $\tilde{A} - \tilde{B}$, (iii) $\tilde{A} \times \tilde{B}$, and (iv) $\tilde{A} / \tilde{B}$. Express each result in *j*-form and also in θ-form.

General Note: For simplicity in writing, all angles we write following ∠ signs in this book are in degrees, whether or not expressly shown with superscripts°.

SOLUTION

For adding and subtracting $\tilde{A}$ and $\tilde{B}$, we must express phasors in *j*-form, that is,

$$\tilde{A} = 60\,(\cos 30° + j \sin 30°) = 51.96 + j\,30,$$

and

$$\tilde{B} = 40\,(\cos 60° + j \sin 60°) = 20 + j\,34.64$$

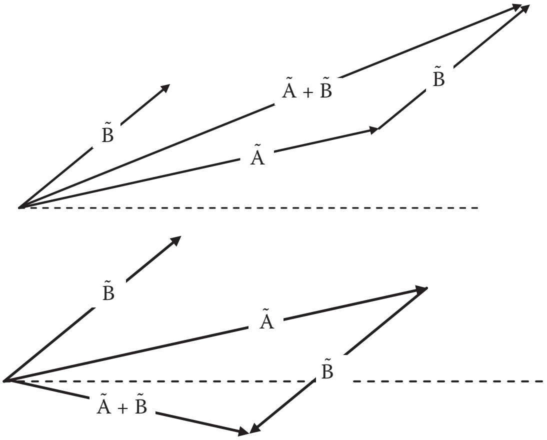


FIGURE 1.6 Tip-to-tail method of adding and subtracting two phasors.

Then

$$\tilde{A} + \tilde{B} = (51.96 + 20) + j\,(30 + 34.64) = 71.96 + j\,64.64 = 96.73\ \angle 41.93^\circ$$

$$\tilde{A} - \tilde{B} = (51.96 - 20) + j\,(30 - 34.64) = 31.96 - j\,4.64 = 32.30\ \angle{-8.26^\circ}$$

For multiplying and dividing, we must use $\tilde{A}$ and $\tilde{B}$ in θ-form as given, that is,

$A \times \tilde{B} = 60\angle 30^\circ \times 40\angle 60^\circ = 60{\times}40\angle 30{+}60 = 2400\angle 90^\circ = 2400\ (\cos 90 + j\sin 90) = 0 + j\,2400$

$\tilde{A} / \tilde{B} = 60\angle 30^\circ \div 40\angle 60^\circ = (60 \div 40)\angle 30 - 60 = 1.5\angle{-30^\circ} = 1.5\ (\cos 30 - \sin 30) = 1.3 - j\,0.75$

Example 1.4

Determine $\dfrac{2}{j1.5}$ in θ-form.

SOLUTION

All ac voltages, currents, impedances, and powers are phasors, that is, complex numbers having $x + j\,y$ components or the rms magnitude and phase angle θ. When we write 2 in ac, it really means $x = 2$ and $y = 0$, or magnitude 2 and $\theta = 0^\circ$. And, when we write $j\,1.5$, it really means $x = 0$ and $y = 1.5$, or magnitude 1.5 and $\theta = 90^\circ$.

The algebra of dividing two phasors requires first converting them in θ-form, and then dividing the two as follows:

$$\frac{2}{j1.5} = \frac{2 + j0}{0 + j1.5} = \frac{2\angle 0}{1.5\angle 90} = \frac{2}{1.5}\angle 0 - 90 = 1.33\angle - 90^\circ$$

Recognizing that a real number alone always means $\theta = 0^\circ$ and an imaginary number alone always means $\theta = 90^\circ$, we can skip the formal *j*-form to θ-form conversion, and write directly as

$$\frac{2}{j1.5} = \frac{2\angle 0}{1.5\angle 90} = \frac{2}{1.5}\angle 0 - 90 = 1.33\angle - 90^\circ$$

Or, recalling that $1 / j = -j = \angle{-90^\circ}$, we can simplify the algebra further as

$$\frac{2}{j1.5} = -j\frac{2}{1.5} = -j1.33 = 1.33\angle - 90^\circ$$

Understanding all of the foregoing three ways of arriving at the same results adds fluency in the complex algebra of phasors in ac power circuits, which is required for professional electrical power engineers.

1.5 SINGLE-PHASE AC POWER CIRCUIT

In analyzing any power circuit (ac or dc), we use the following basic circuit laws:

Kirchhoff's' voltage law (*KVL*): In any closed loop, the phasor sum of all source voltages = phasor sum of all voltage drops in load elements. It applies in every loop, covering every segment along the loop, in ac or dc. In a voltage-driven circuit—usually, the case in power engineering—it is KVL that determines the current, that is, the circuit draws current that will satisfy KVL in every closed loop of the circuit.

Ohms law: It basically gives the voltage drop across two terminals of the *R*, *L*, or *C* element as summarized in Table 1.1. It is local, it applies only between two terminals of *R*, *L*, or *C* (but not of a source).

Kirchhoff's current law (*KCL*): At any junction node (point or a closed box), the phasor sum of all currents going inward = phasor sum of all currents going outward. It is even more local than Ohm's law; it applies only at a junction node.

1.5.1 Series R-L-C Circuit

The basic power circuit is made of one or more of the three basic load elements, namely, the resistor R, inductor L, and capacitor C, powered generally by a voltage source (e.g., ship generator or utility grid). Since the R, L, and C elements are distinctly different in units and also in behavior, they cannot be combined with each other using any series-parallel combination formulas until they are converted into their respective ohm values with phase shifts.

TABLE 1.1
Voltage Drop, Current, and Energy Storage Relations in R, L, and C Elements in Ac or Dc Circuits

Parameter between two terminals of the element	**Resistance *R* of a wire (Ω)**	**Inductance *L* of a coil (H)**	**Capacitance *C* between two conductors (F)**
Voltage drop in the element in dc or ac (Ohm's law or its equivalent)	$v = R \times i$	$v = L\frac{di}{dt}$	$v = \frac{\text{Charge } Q}{C}$ $= \frac{\int i \cdot dt}{C}$ or $i = C\frac{dv}{dt}$
Voltage drop in steady state dc (i.e., when di/dt = 0 and dv/dt = 0)	$V = R \cdot I$	$V_{drop} = 0$ (i.e., coil in steady dc behaves like an ideal wire shorting the coil)	$I_{cap} = 0$ after fully charged (i.e., capacitor in steady dc behaves like an open circuit)
Energy stored in the element	None (dissipates energy in heat, which leaves the circuit)	½ L I^2 (stores energy in magnetic flux through the coil)	½ C V^2 (stores energy in electrical charge on the capacitor conductors)
Energy conservation	Dissipates the absorbed energy into heat	Tends to hold on stored energy by retaining its current I	Tends to hold on stored energy by retaining its voltage V

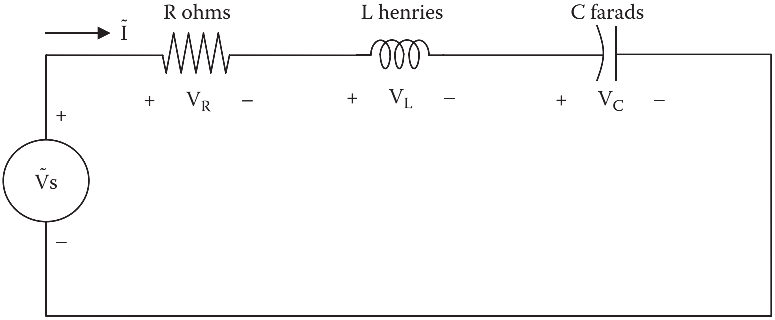


FIGURE 1.7 Series R-L-C circuit powered by sinusoidal source voltage $\tilde{V}_s$.

Consider a series R-L-C circuit of Figure 1.7 excited by a sinusoidal voltage source $\tilde{V}_s$. Since the driving voltage in this circuit is sinusoidal, the current must also be sinusoidal of the same frequency, say, $\tilde{I} = I\angle\theta$, where I = rms magnitude (note that we now drop the suffix m from I_m and imply that I without the hat sign means the rms magnitude).

In the series loop of Figure 1.7, we write KVL using Ohm's law relations given in Table 1.1, and Equations (1.16) and (1.17):

$$\tilde{V}_s = \tilde{V}_R + \tilde{V}_L + \tilde{V}_c = R\tilde{I} + L\frac{d\tilde{I}}{dt} + \frac{\int \tilde{I}dt}{C} = R\tilde{I} + Lj\omega\tilde{I} + \frac{\tilde{I}}{Cj\omega}$$

Recalling that $j = -1/j$, we rewrite this equation as

$$\tilde{V}_s = R\,\tilde{I} + j\omega L\,\tilde{I} - \frac{j}{\omega C}\,\tilde{I} = R\,\tilde{I} + j\,(\omega L - \frac{1}{\omega C})\,\tilde{I} \tag{1.18}$$

where $j = \sqrt{-1} = \angle 90° = 90°$ phase shift in the positive direction. We note here the following phasor relations in the R, L, and C elements:

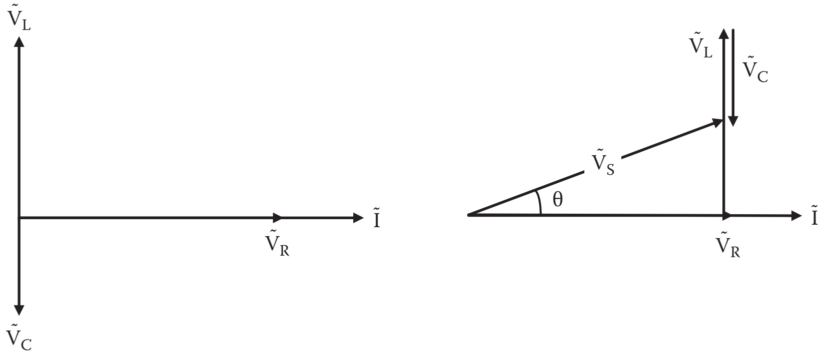
$\tilde{V}_R = R\,\tilde{I}$, that is, V_R is in phase with I with $\theta = 0°$, meaning a pure R always draws current at pf = 1.0 (unity), absorbing average power that is not zero in R.

$\tilde{V}_L = j\omega L\,\tilde{I}$, that is, V_L leads I by 90°, or I lags V_L by 90°, meaning that a pure L always draws current at pf = 0 lagging, giving zero average power absorbed in L.

$\tilde{V}_c = -j\frac{1}{\omega C}\tilde{I}$, that is, V_c lags I by 90° or I leads V_c by 90°, meaning that a pure C always draws current at pf = 0 leading, again giving zero average power absorbed in C.

The voltmeter measures only the rms magnitude without the phase angle. Therefore, three voltmeter readings across R, L, and C (i.e., V_R, V_L, and V_c) will not add up to the total source voltage magnitude V_S. We must add the three load voltages taking their phase differences into account (called the phasor sum) to obtain the total source voltage. Figure 1.8 shows the phasor sum using the tip-to-tail method, where

$$\tilde{V}_s = \tilde{V}_R + \tilde{V}_L + \tilde{V}_c = \tilde{V}_s\angle\theta \tag{1.19}$$



(a) Individual phasors $\tilde{I}$, $\tilde{V}_R$, $\tilde{V}_L$, and $\tilde{V}_C$ (b) Tip-to-trail addition $\tilde{V}_R + \tilde{V}_L + \tilde{V}_C = \tilde{V}_S$

FIGURE 1.8 Phasor $\tilde{V}_R$, $\tilde{V}_L$, $\tilde{V}_C$, and their phasor sum $\tilde{V}_s$ in a series R-L-C circuit.

We next write Equation (1.18) as

$$\tilde{V}_s = \{R + j\,(\omega L - \frac{1}{\omega C})\} \times \tilde{I} = \tilde{Z} \times \tilde{I} \tag{1.20}$$

where $\tilde{Z} = R + j\,(\omega L - \frac{1}{\omega C}) = R + j\,X$ ohms.

The R and X both have units of ohms, and so does $\tilde{Z}$. Therefore, Equation (1.20) gives ac voltage as a product of ampere and ohm, as in dc. Equation (1.20) is Ohm's law in ac, where $\tilde{Z}$ is called the total circuit *impedance*. The $\tilde{Z}$ is generally written as $\tilde{Z} = Z\angle\theta = R + j\,X$, where $X = (\omega L - 1/\omega C)$ is called the *reactance*, which comes from L and C. Since the reactance has two parts, we write $X = X_L - X_C$, where

$X_L = \omega L$ = Reactance from inductance (ohms)
$X_C = 1/(\omega C)$ = Reactance from capacitance (ohms)

The L and C contributions in the total reactance X are subtractive; that is, they tend to neutralize each other. As a result, X can be zero in some circuits even if L and C are not individually zero.

KVL in a closed loop in any power circuit with multiple voltage sources and impedances in series can be expressed in a simple form as follows:

$$\tilde{I}_{\text{loop}} = \frac{\textit{Phasor sum of all source voltages}}{\textit{Phasor sum of all impedances}} = \frac{\sum \tilde{V}_{source}}{\sum \tilde{Z}\angle\theta} \tag{1.21}$$

The current lags the voltage if the phase angle θ of all load impedances combined is positive (i.e., when the loop is more inductive than capacitive). So, the overall pf of a circuit is determined by the nature of the load; it has nothing to do with the source voltage.

Example 1.5

A one-loop circuit has a source voltage of 120 V and a load impedance 10∠ + 84.3° Ω. Determine the current and average power delivered to the load. Can you guess the nature of the load—could it be a heater, capacitor, motor ….?

SOLUTION

The 120 V with no angle stated implies that it is the reference phasor at 0°. It is also implied in power engineering that it is the rms value. The current phasor in this loop will then be

$$\tilde{I}_{loop} = \frac{\tilde{V}_{loop}}{\tilde{Z}_{loop}} = \frac{120\angle 0^\circ}{10\angle 84.3^\circ} = \left(\frac{120}{10}\right)\angle 0^\circ - 84.3^\circ = 12\angle -84.3^\circ \ A$$

The 12 A is the rms value, since we used 120 V_{rms} to derive the current, which lags the voltage by 84.3°, giving a pf of cos (84.3°) = 0.10 lagging. The average power delivered by the source and absorbed by the load is given by $P_{avg} = V_{rms}\ I_{rms} \cos\theta = 120 \times 12 \times 0.10 = 144$ W (versus 1440 W 120 V and 12 A can produce if they were in phase with $\theta = 0°$). This is an example of an extremely poor power factor.

As for the nature of the load, since it draws current with a large lag angle close to 90°, it must have a large inductive reactance with a small resistance. Therefore, it is most likely to be a large coil.

Example 1.6

Determine the average real power drawn by a coil having an inductance of 10 mH and resistance of 1.5 Ω connected to a 120 V, 60 Hz source.

SOLUTION

For a 60 Hz source, $\omega = 2\pi\ 60 = 377$ rad/sec; hence, the coil impedance $\tilde{Z} = 1.5 + j\,377 \times 0.010 = 1.5 + j\,3.77\ \Omega = 4.057\angle 68.3°\ \Omega$.

Taking 120 V as the reference phasor, the current drawn by the coil is

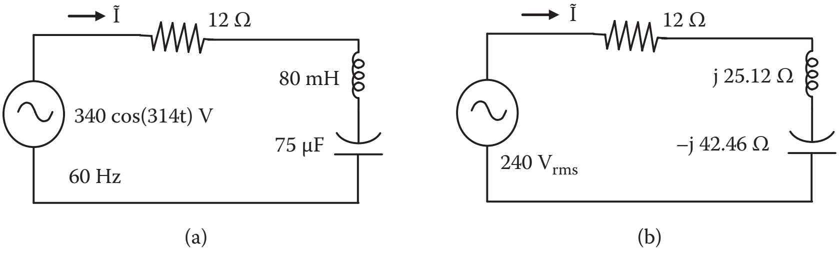
$\tilde{I} = 120\angle 0° \div 4.057\angle 68.3° = 29.58\angle -68.3°$ A, which lags the voltage by 68.3°
$\therefore$ Real power $P = 120 \times 29.58 \times \cos 68.3° = 1312.45$ W

Alternatively, the average real power is absorbed only by the circuit resistance, and the power absorbed by the inductance or capacitance always averages out to zero over one cycle. So, the average real power can also be derived simply from

Real power $P = I^2\ R = 29.58^2 \times 1.5 = 1312.45$ W, which is the same as the foregoing.

Example 1.7

Express the time-domain circuit in Figure E1.7(a) in the phasor domain with impedances in ohms. Then, determine the current phasor $\tilde{I}$ and the average power.



SOLUTION

First, the voltage phasor's rms value = 340 ÷ √2 = 240∠0° V (Reference for angle).

Angular frequency ω = 314, which gives $f = \omega/2\pi = 50$ Hz.

The loop impedance of the series R-L-C circuit in (a) is

$$Z = 12 + j\left(314 \times 0.080 - \frac{1}{314 \times 75 \times 10^{-6}}\right)$$

$$= 12 + j(25.12 - 42.46) = 12 - j17.34 = 21.09\angle -55.3 \text{ ohms}$$

The phasor-domain circuit is shown in (b), from which we derive the circuit current

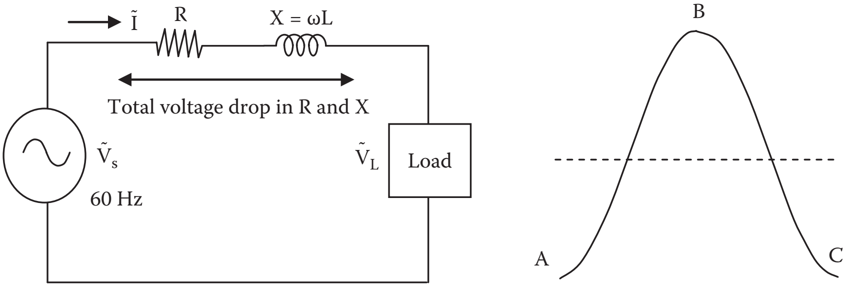
$$I = \frac{V}{Z} = \frac{240\angle 0}{21.09\angle -55.3} = \left(\frac{240}{21.09}\right)\angle 0 - (-55.3) = 11.38\angle +55.3^\circ \text{ A}$$

The 11.38 A current phasor leads the voltage by 55.3°, meaning that the capacitance dominates the inductance in this circuit.

Average power P = 240 × 11.38 × cos55.3° = 1555 W

1.5.2 IMPEDANCE TRIANGLE

As seen in Equation (1.20), we write the total circuit impedance in the form $Z = R + jX$, where the operator $j = \sqrt{-1}$ in algebra or 90° phase shift in a phasor diagram. For engineers and managers not fully versed with the operator *j*, the reason for assigning the inductive reactance *X* with 90° phase shift from *R* can be simply explained as follows in view of the circuit shown in Figure 1.9(a). The voltage drop in *R* is *IR*, which always subtracts from the source voltage. The voltage drop in the inductor, however, is equal to $L \times di/dt$. This drop is positive for ½ cycle of rising current from point A to B under a sinusoidal current as in (b) when the inductor is absorbing the energy that causes the voltage to drop. It is negative for the other ½ cycle of falling current from point B to C when the inductor is supplying the energy that adds into the circuit voltage. The negative voltage drop means it subtracts from the positive

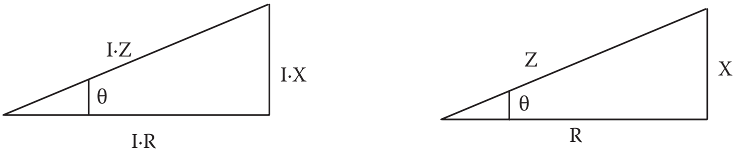


(a) R-L circuit

(b) One cycle of ac

(c) Drops add during A to B under rising current

(d) Drops subtract during B to C under falling current



(e) Effective rms drop I·Z over one cycle

(f) Effective impedance Z

FIGURE 1.9 Simple explanation of voltage drop in an R-L circuit and impedance triangle.

voltage drop in the resistance. Therefore, for ½ cycle of rising current, the total voltage drop is $IR + IX$ as in (c), whereas for the following ½ cycle of falling current, the net voltage drop is $IR - IX$ as in (d). We graphically show the effective rms value of the voltage drop over one cycle as in (e), not positive nor negative but an in-between position with 90° phase shift. Analytically, the effective rms value of the total voltage drop over one cycle is

$$\text{Total } V_{drop.rms} = \sqrt{\frac{(IR+IX)^2 + (IR-IX)^2}{2}} = \sqrt{(IR)^2 + (IX)^2} = I\sqrt{R^2 + X^2} = I \cdot Z$$

where $Z = \sqrt{R^2 + X^2}$, which has the right-angle triangle relation as shown in (f), which is known as the impedance triangle. Therefore, the circuit impedance in general is expressed as a phasor $\tilde{Z} = R + jX$, as drawn in (f) using the tip-to-tail method, where $\cos\theta$ is the circuit pf.

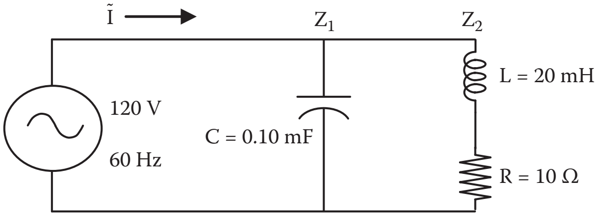
1.5.3 CIRCUIT LAWS AND THEOREMS

All circuit laws and theorems learned in dc apply in the same manner in ac as well, except that all numbers in ac are phasors (complex numbers), each with magnitude and phase angle, making the ac algebra complex. All dc formulas remain valid in ac if we replace R in dc with $\tilde{Z}$ in ac. For example, if the circuit has more than one impedance connected in a series-parallel combination, then the total equivalent $\tilde{Z}$ can be determined by using the corresponding dc formula after replacing R with $Z\angle\theta$, and doing the phasor algebra of the resulting complex numbers, that is,

$$\tilde{Z}_{\text{series}} = \tilde{Z}_1 + \tilde{Z}_2 + \ldots \text{ and } \qquad Z_{\text{parallel}} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \ldots .} \tag{1.22}$$

Example 1.8

A 120 V, 60 Hz, 1-phase source powers a 0.10 mF capacitor in parallel with a 20 mH coil that has 10 Ω winding resistance. Determine the current, power factor, and power delivered by the source. (*Note:* Practical capacitors have negligible resistance, but the inductors have significant resistance due to the long wires needed to wind numerous turns).



SOLUTION

We first determine the circuit impedances (all angles are in°),

Capacitor impedance $Z_1 = -j/\omega C = -j/(2\pi 60 \times 0.10 \times 10^{-3}) = -j\,26.52\ \Omega = 26.52\angle{-90°}\ \Omega$

Coil impedance $Z_2 = R + j\omega L = 10 + j\,2\pi 60 \times 20 \times 10^{-3} = 10 + j\,7.54\ \Omega = 12.52\angle 37°\ \Omega$

Z_1 and Z_2 are in parallel, so they give

$$Z_{\text{Total}} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}} = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2}$$

Notice that the 2nd term $Z_1\ Z_2 \div (Z_1 + Z_2)$ is simple to write and easy to remember as the product divided by the sum, although it is valid only for two impedances in parallel (it does not extend to three impedances in parallel).

$$\therefore Z_{Total} = \frac{26.52\angle -90 \times 12.52\angle 37}{-j26.52+10+j7.54} = \frac{332\angle -53}{21.45\angle -62.2} = 15.48\angle 9.2\ \Omega$$

$$I = \frac{120V}{15.48\angle 9.2} = 7.75\angle -9.2\ A$$

Power factor of the circuit = cos9.2° = 0.987 lagging

$$P_{avg} = 120\ \text{V} \times 7.75\ \text{A} \times 0.987 = 918\ \text{W}$$

The nodal analysis, mesh analysis, superposition theorem, and Thevenin and Norton equivalent source models all are valid in ac as they are in dc. There is a slight change, however, in the maximum power transfer theorem. For maximum power transfer in ac, we must have $\tilde{Z}_{\text{Load}} = \tilde{Z}^*_{Th}$, where Z_{Th} = Thevenin source impedance at the load points, and the superscript * denotes the complex conjugate defined below.

If the phasor $\tilde{Z}_{Th} = Z_{Th}\angle\theta = R_{Th} + j\ X_{Th}$, then its complex conjugate $\tilde{Z}^*_{Th} = Z_{Th}\angle{-\theta} = R_{Th} - j\ X_{Th}$, which is obtained by flipping the sign of the j-part in the rectangular form, or by flipping the sign of angle θ in the polar form. Thus, the complex conjugate of a phasor is its mirror image in the real axis as shown in Figure 1.10. With a

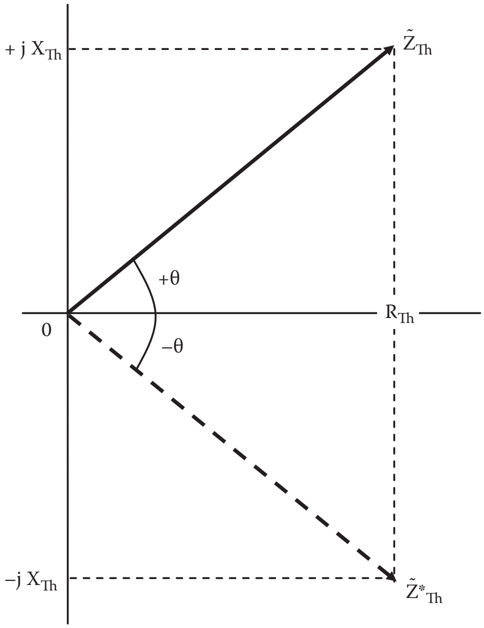


FIGURE 1.10 Complex conjugate of phasor is its own mirror image in reference axis.

load impedance $\tilde{Z}_{\mathrm{Load}} = \tilde{Z}_{Th}^{\,*}$ (called the *load matching*), the maximum power that can be transferred to the load remains the same as in dc, that is,

$$P_{\max} = \frac{V_{Th(\mathrm{rms})}}{4\,R_{Th}} \tag{1.23}$$

1.6 AC POWER IN COMPLEX FORM

For a general form of ac power delivered from a source or absorbed in a load, we consider $V\angle\theta_v$ and $I\angle\theta_i$ phasors shown in Figure 1.11. The ac power is given by the product of the voltage and the complex conjugate of the current phasor. Therefore, the power is also a phasor, a complex number, hence the name *complex power.* For average power, we use the rms values along with their phase angles, that is,

$$\text{Complex Power } \tilde{S} = V_{rms}\angle\theta_v \cdot \{I_{rms}\angle\theta_i\}^* \tag{1.24}$$

where * denotes the complex conjugate, that is, the mirror image of $\tilde{I}$ on the reference axis.

$$\therefore \tilde{S} = V_{rms}\angle\theta_v \cdot I_{rms}\angle{-\theta_i} = V_{rms}\cdot I_{\mathrm{rms}}\angle(\theta_v - \theta_i) = V_{rms}\cdot I_{rms}\angle\theta \tag{1.25}$$

where $\theta = \theta_v - \theta_i =$ phase difference between $\tilde{V}$ and $\tilde{I}$ phasors. In retrospect, if $\tilde{I}$ were used instead of $\tilde{I}^*$ in the power product, θ would be equal to $\theta_v + \theta_i$ giving an incorrect power factor.

In the *j*-form, Equation (1.25) for the complex power becomes

$$\tilde{S} = V_{rms}\cdot(I_{rms}\cos\theta + j\,I_{rms}\sin\theta) = V_{rms}\cdot I_{rms}\cos\theta + j\,V_{rms}\cdot I_{rms}\sin\theta = P + j\,Q \tag{1.26}$$

where the first term is the power on the real axis, called Real Power, $P = V_{rms} \times I_{rms} \cos\theta$, which has the unit of watt, or kW, or MW, depending on the power system size. The second term is the power on the imaginary axis, going in and out of the reactance X, and hence called the Reactive Power $Q = V_{rms} \times I_{rms} \sin\theta$, which has the

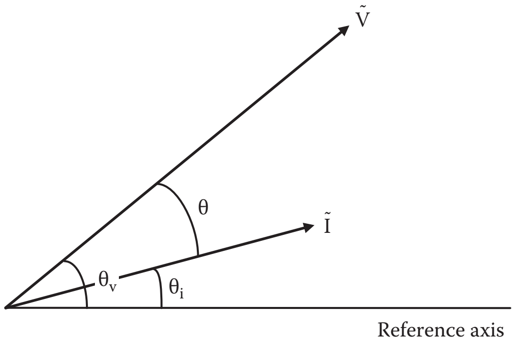


FIGURE 1.11 Product of $\tilde{V}$ and $\tilde{I}^*$ gives complex power in ac.

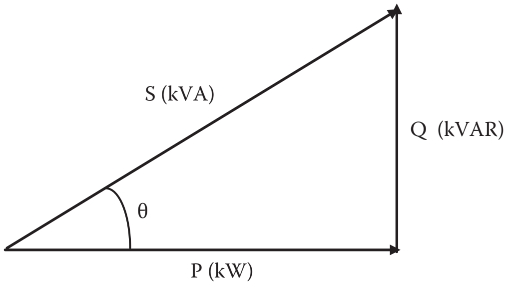


FIGURE 1.12 Power triangle of real power *P*, reactive power *Q*, and apparent power *S*.

unit of volt-ampere reactive (VAR, or kVAR, or MVAR). The power factor cosθ of the load has no units. The complex power $\tilde{S}$, being the product of volts and amperes, has the unit of volt-ampere (VA, or kVA, or MVA).

$\therefore$Complex power $S\angle\theta = P + j\,Q$ is a phasor, with Q out of phase by + 90° from P as shown by the *Power Triangle* in Figure 1.12. The trigonometry of power triangles gives

$$S = \sqrt{P^2 + Q^2} \qquad \text{or} \qquad kVA = \sqrt{kW^2 + kVAR^2}$$

and
$$pf = \frac{P}{S} = \frac{kW}{kVA} \qquad \text{or} \qquad \text{kW} = \text{KVA} \times \text{pf} \tag{1.27}$$

It is noteworthy that the lagging power factor of the load circuit produces a positive Q, and vice versa. Also, in complex power $\tilde{S} = S\angle\theta = V_{rms}\cdot I_{rms}\angle\theta$, and the $S = V_{rms}\cdot I_{rms}$ without θ is known as the apparent power. With θ, $\tilde{S}$ is known as the complex power. Both $\tilde{S}$ and S have units of VA or kVA or MVA.

The real power P in kW delivered by the generator requires fuel (real source of power) to drive the generator engine. The reactive power Q in kVAR on the imaginary axis is the power going in and out of L and C—charging and discharging L and C every half-cycle—with zero average power. It does not require fuel to drive the generator engine, although it loads the generator voltage and current capabilities. Therefore, kVAR adds in the capital cost of power equipments, but does not add in the fuel cost for running the prime mover.

The ac power absorbed in load impedance $\tilde{Z}$ can also be derived as follows. In a given impedance $\tilde{Z} = R + j\,X$, the real power P is absorbed only in R, and the reactive power Q is absorbed only in X. So, the complex power absorbed in $\tilde{Z}$ is

$$\tilde{S} = \tilde{V} \times \tilde{I}^* = (\tilde{I}\tilde{Z}) \times \tilde{I}^* = I^2\tilde{Z} = I^2(R + jX) = I^2R + j\,I^2X = P + j\,Q, \quad \text{where}$$

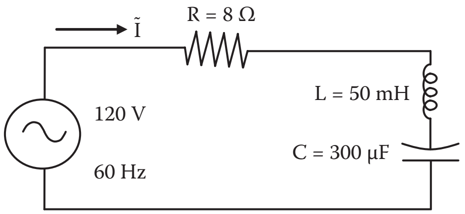
Real power $P = I^2R$ Reactive power $Q = I^2X$ and

$$\text{Apparent power } S = I^2 \cdot Z = \sqrt{P^2 + Q^2} \tag{1.28}$$

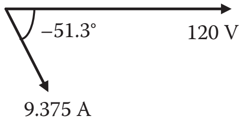
which is the same power triangle relation as in Figure 1.12.

Example 1.9

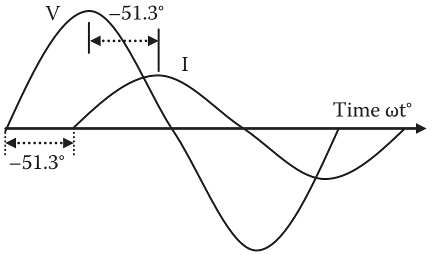
Determine the current and power in circuit shown in (a) below, and draw the phasor and time diagrams and the power triangle.



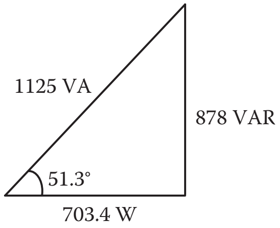
(a) Circuit



(b) Phasor diagram



(c) Time diagram



(d) Power triangle

SOLUTION

We first convert the R-L-C values into the total impedance $\tilde{Z}$ in ohms.

$$\tilde{Z} = R + j\,\omega L - j\,1/\omega C, \text{ where } \omega = 2\pi f = 2\pi \cdot 60 = 377 \text{ rad/sec}$$

$$\therefore\ \tilde{Z} = 8 + j\,377 \times 0.050 - j\,1/(377 \times 300 \cdot 10^{-6}) = 8 + j10\ \Omega$$

$$= \sqrt{(8^2+10^2)}\ \angle\ \tan^{-}(10/8)° = 12.8\ \angle 51.3°$$

In a one-loop circuit, the current $I = \tilde{V}_{\text{loop}} \div \tilde{Z}_{\text{loop}} = 120\ \angle 0°$(Reference) $\div\ (12.8\angle 51.3°)$ which gives $\tilde{I} = (120 \div 12.8)\ \angle\ 0{-}51.3°) = 9.375\ \angle{-51.3°}$ A.

The current lags the reference voltage by 51.3° as shown in the phasor diagram (b) and time diagram (c).

Since 120 V is the implied rms value, 9.375 A is also an rms value. The complex power in any element (source or load) is $\tilde{S} = V \times \tilde{I}^*$ volt-amperes, where $\tilde{V}$ = voltage across and $\tilde{I}$ = complex conjugate of current through the element.

So, in this case, $\tilde{S}_{\text{source}} = 120\angle 0° \times 9.375\angle + 51.3° = (120 \times 9.375)\ \angle\ (0 + 51.3°) = 1125\ \angle\ 51.3° = 1125\ (\cos 51.3° + j\,\sin 51.3°) = 703.4 + j\,878$ VA

Apparent power drawn from the source, S = 1125 VA

Real power P = 703.4 W (absorbed in R and leaving the circuit as heat).

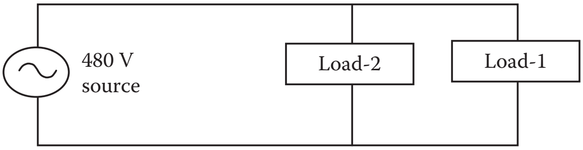
Reactive power Q = 878.0 VAR (charging and discharging L and C, but remaining in the circuit)

The power triangle is shown in (d). The following is another way to determine the ac powers.

Apparent Power = $I^2Z = 9.375^2 \times 12.8 = 1125$ VA, Real Power = $I^2 R = 9.375^2 \times 8 = 703.4$ W, and Reactive Power = $I^2X = 9.375^2 \times 10 = 878$ VAR, all matching with the foregoing values.

Example 1.10

Two parallel loads draw power from a 480 V source as shown below, where Load-1 draws 15 kW and 10 kVAR lagging, and Load-2 draws 7 kW and 5 kVAR leading. Determine the combined power factor and the total kVA drawn from the source.



SOLUTION

In power engineering, the lagging kVAR has a + sign, and the leading kVAR has a – sign. The parallel loads add to make the total, and their real and reactive powers are added individually to make the total real and reactive powers.

$$\therefore\ \tilde{S}_{\text{Total}} = \tilde{S}_1 + \tilde{S}_2 = (15 + j\,10) + (7 - j\,5) = 22 + j\,5 = 22.56\ \angle + 12.8°$$

The total kVA drawn from the source = 22.56, and the combined pf = cos 12.8° = 0.975 lagging, since the combined power has +5 kVAR, the + sign indicating the lagging power factor (or + sign with 12.8° in total complex power $\tilde{S}$ comes from the inductive load, which draws a lagging current).

1.7 REACTIVE POWER

If the current lags the voltage by 90° as shown in Figure 1.13—as in a pure inductor—the instantaneous power shown by the heavy curve is positive in the first ½ cycle and negative in the second ½ cycle. The power is first positive (from source to inductor) and then negative (from inductor to source). Therefore, the average power in the inductor over one full cycle is always zero, although wires are occupied with current all the times, leaving less room for the real power to flow from the source to the load. Similarly, the average power is also zero over one full cycle in a pure capacitor, which draws current leading the voltage by 90°.

On the other hand, if the current and voltage are in phase—as in a pure resistor—the power is always positive even when both the voltage and the current are negative in a ½ cycle. This results in the average power having the same positive value in both ½ cycles, and also over one full cycle.

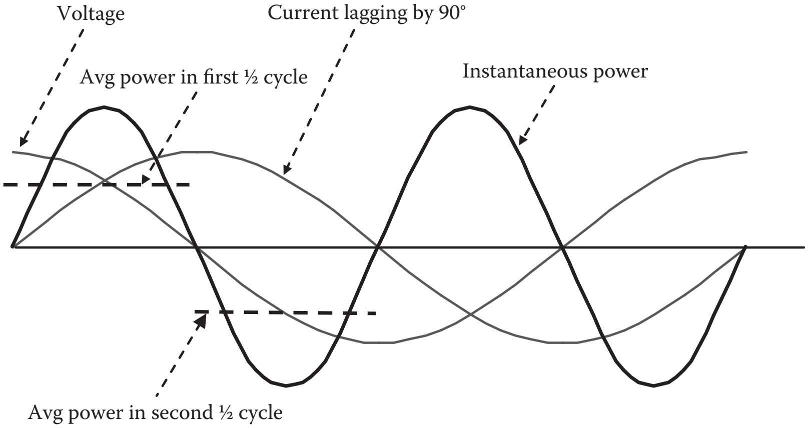


FIGURE 1.13 Voltage, current, and instantaneous and average powers in inductor.

1.8 THREE-PHASE AC POWER SYSTEM

AC power is now universally adopted all over the word because of its easy conversion from one voltage level to another using energy-efficient power transformers. From a large ac power system (power grid), large power users are catered to at high voltage, medium power users at medium voltage, and low power users at a safe low voltage of 120 or 240 V.

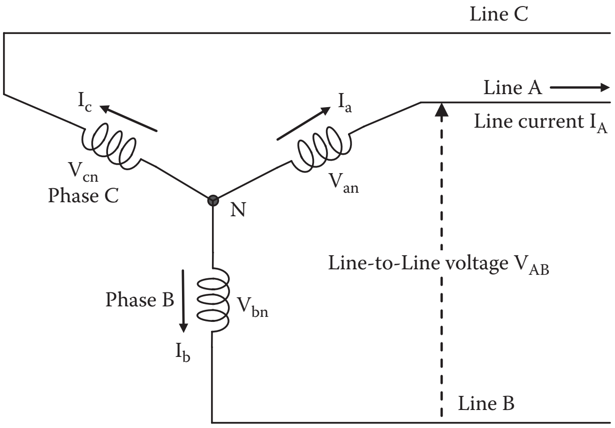
In a single-phase (1-ph) power circuit, the instantaneous power varies between the maximum and minimum values every cycle, with the average power positive. In a balanced three-phase (3-ph) power circuit, when the instantaneous power in one phase is decreasing, the power in the remaining phases is increasing, making the sum of power in all three phases a steady constant value with no time variations. It is for this smooth power flow that the 3-phase ac system has been universally adopted around the world. Since all three phases are balanced and identical, the balanced 3-phase power circuit is generally shown by a 1-line diagram, and analyzed on single-phase (per phase) basis. Then, the *Average 3-phase power* = *3* × *Average 1-phase power*.

The 3-phase source or load can be connected in Y or Δ. The basic voltage and current relations between the line to neutral (called phase) values and the line to line (called line) values are now reviewed.

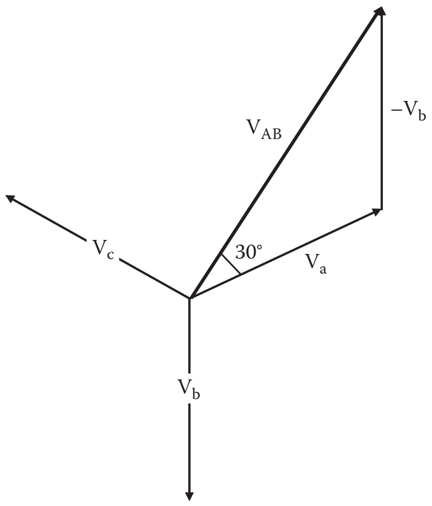
1.8.1 BALANCED Y- AND Δ-CONNECTED SYSTEMS

In a balanced 3-phase system, the magnitudes of line voltage V_L, line current I_L, and 3-phase power in terms of the magnitudes of phase voltage V_{ph} and phase current I_{ph} are derived as follows, where the term *phase voltage* (V_{ph}) also means line-to-neutral voltage (V_{LN}), and the term *line voltage* (V_L) also means line-to-line voltage (V_{LL}).

The *Y-connected system shown in Figure 1.14*(*a*) gives the voltage of line *a* from line *b*, $\tilde{V}_{ab} = \tilde{V}_{an} - \tilde{V}_{bn}$, which is shown in Figure 1.14(b) by a heavy solid line.



(a) Phase and Line Voltage and Current Definitions



(b) Line-to-Line voltage phasor $V_{AB} = V_a - V_b$

FIGURE 1.14 Phase and line voltages in a balanced Y-connected source or load.

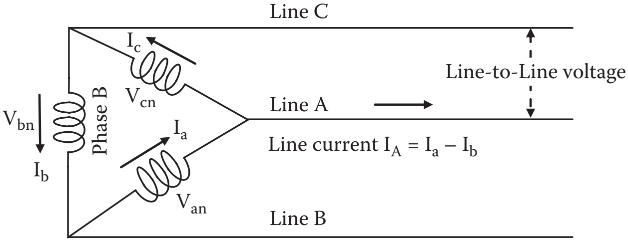
The geometry would give the line-to-line voltage magnitude $V_{LL} = \sqrt{3}\ V_{ph}$. We note that the line voltage $\tilde{V}_{ab}$ leads the phase voltage $\tilde{V}_{an}$ by 30°.

The line current is obviously the same as the phase current, that is, $\tilde{I}_{\mathrm{L}} = \tilde{I}_{ph}$

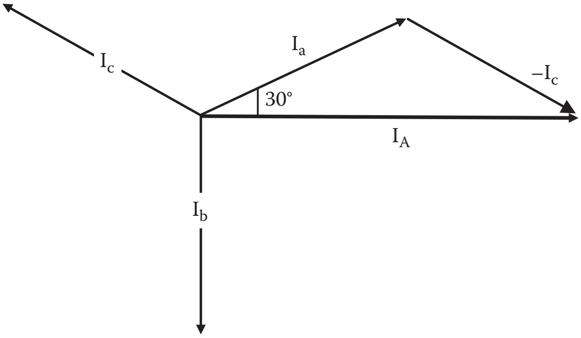
Three-phase power $P_{3\text{-}ph} = 3 \times P_{1\text{-}ph} = 3 \times V_{ph} \times I_{ph} \times \text{pf} = 3 \times (V_{LL}/\sqrt{3}) \times I_{\mathrm{L}} \times \text{pf}$

$$\therefore P_{3\text{-}ph} = \sqrt{3}\ \mathrm{V}_{LL} I_{\mathrm{L}}\ \text{pf watts and} \qquad \mathrm{S}_{3\text{-}ph} = \sqrt{3}\ V_{LL} I_{\mathrm{L}}\ \text{volt-amperes} \qquad (1.29)$$

The Δ-*connected system shown in Figure 1.15*(*a*) *gives* the line current $\tilde{I}_{\mathrm{A}} = \tilde{I}_a - \tilde{I}_c$, which is shown by the heavy line in Figure 1.15(b), the geometry of which leads to $I_{\mathrm{L}} = \sqrt{3}\ I_{ph}$.



(a) Phase and line voltage and current definitions



(b) Line-current phasor $I_A = I_a - I_b$

FIGURE 1.15 Phase and line currents in a balanced Δ-connected source or load.

The line-to-line voltage is obviously the same as the phase voltages, that is, $V_{LL} = V_{ph}$ Three-phase power $P_{3\text{-}ph} = 3 \times P_{1\phi} = 3 \times V_{ph} \times I_{ph} \times \text{pf} = 3 \times V_{LL} \times (I_L/\sqrt{3}) \times \text{pf}$

$$\therefore P_{3\text{-}ph} = \sqrt{3}\ V_{LL} I_L \text{ pf watts and } S_{3\text{-}ph} = \sqrt{3}\ V_{LL} I_L \text{ volt-amperes} \tag{1.30}$$

Note that the expression for 3-phase power works out to be the same in both Y and Δ.

In a 3-phase, 4-wire, Y-connected system, the neutral current is the sum of three line currents. In a balanced Y system, the current in the neutral wire (if used) is always zero, because it is the phasor sum of three equal line currents out of phase by 120° from each other. For this reason, a balanced 3-phase system performs exactly the same with or without the neutral wire, except in unbalanced load conditions.

In a 3-phase, Δ-connected system, no wire exists for the return current, so the phasor sum of all line currents is forced to be zero regardless of a balanced or unbalanced load. With an unbalanced 3-phase load, however, this imposed-zero on the return current produces unbalanced line-to-line voltages.

Example 1.11

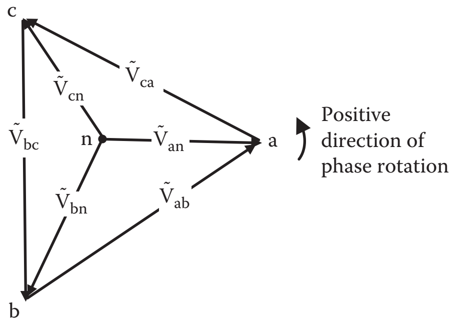
A *Y*-connected generator has the line-to-neutral voltage phasor $\tilde{V}_{an} = 100\angle 0$. Determine the line-to-line voltage phasors $\tilde{V}_{ab}$, $\tilde{V}_{bc}$, and $\tilde{V}_{ca}$ with their phase angles with respect to $\tilde{V}_{an}$.

SOLUTION

The phasor relations between the line and phase voltages are shown in the figure below, the trigonometry of which gives the following:

$\tilde{V}_{ab} = \sqrt{3} \times 100\ \angle + 30°$, $V_{bc} = \sqrt{3} \times 100\ \angle{-90°}$, and $\tilde{V}_{ca} = \sqrt{3} \times 100\ \angle{-210°}$ or $\sqrt{3} \times 100\ \angle{+150°}$.

This means that with reference to the phase voltage $\tilde{V}_{an}$, the line voltage $\tilde{V}_{ab}$ leads $\tilde{V}_{an}$ by 30°, $\tilde{V}_{bc}$ lags $\tilde{V}_{an}$ by 90°, and $\tilde{V}_{ca}$ lags $\tilde{V}_{an}$ by 210° or leads $\tilde{V}_{an}$ by 150°.



1.8.2 Y-Δ EQUIVALENT IMPEDANCE CONVERSION

Almost all 3-phase generators are Y-connected, but some transformers and loads may be Δ-connected. Even so, we always analyze the power system assuming all equipments are Y-connected, since it is much easier to determine the phase values in each phase of the generator, and then convert the results into 3-phase values. For this purpose, if a load is Δ-connected, we first convert it into an equivalent Y-connected load that would absorb the same total power. The equivalence can be derived from rigorous circuit analysis, which would lead to the following final results in view of Figure 1.16 (all Zs are complex phasors in the following equations):

From Δ to Y conversion, that is, from given Z_{ab}, Z_{bc}, and Z_{ca}, we find

$$Z_{an} = \frac{Z_{ab}Z_{ca}}{Z_{ab}+Z_{bc}+Z_{ca}} \qquad Z_{bn} = \frac{Z_{ab}Z_{bc}}{Z_{ab}+Z_{bc}+Z_{ca}} \qquad Z_{cn} = \frac{Z_{bc}Z_{ca}}{Z_{ab}+Z_{bc}+Z_{ca}} \tag{1.31}$$

From Y to Δ conversion, that is, from given Z_{an}, Z_{bn}, and Z_{cn} , we find

$$Z_{ab} = \frac{Z_{an}Z_{bn}+Z_{bn}Z_{cn}+Z_{cn}Z_{an}}{Z_{cn}} \qquad Z_{bc} = \frac{Z_{an}Z_{bn}+Z_{bn}Z_{cn}+Z_{cn}Z_{an}}{Z_{an}}$$

and

$$Z_{ca} = \frac{Z_{an}Z_{bn}+Z_{bn}Z_{cn}+Z_{cn}Z_{an}}{Z_{bn}} \tag{1.32}$$

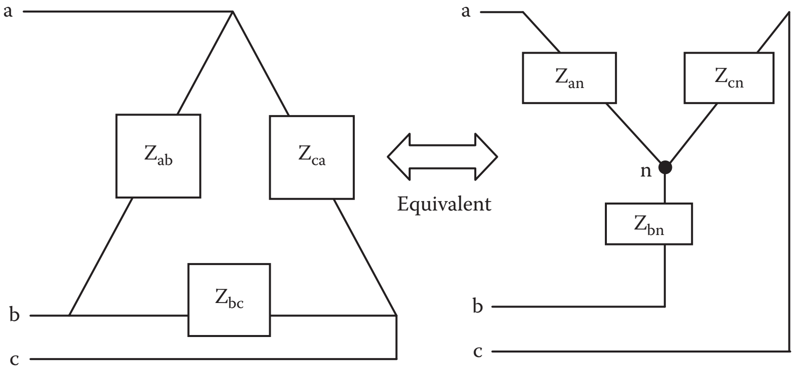


FIGURE 1.16 Δ-to-Y or Y to Δ equivalent impedance conversions.

Equations (1.31) and (1.32) hold true for 3-phase balanced or unbalanced load impedances. However, for balanced 3-phase Y or Δ loads, where load impedances in each phase are equal (i.e., both R and X in each phase are equal), they get simplified to

$$Z_{LN} = \tfrac{1}{3}\, Z_{LL} \text{ and } Z_{LL} = 3 \cdot Z_{\text{LN}} \tag{1.33}$$

where $Z_{LN} = Z_{an} = Z_{bn} = Z_{cn}$ and $Z_{LL} = Z_{ab} = Z_{bc} = Z_{ca}$.

Equation (1.33) can also be deduced as follows. Since ac power absorbed in a load impedance is given by $S = I^2\, Z = V^2/Z$, we have $Z = V^2/S$. Therefore, the equivalent Z in ohms for the same power (required for equivalency) must change as the V^2 ratio. For converting Δ into the equivalent Y, since V_{LN} in $Y = (1/\sqrt{3})\, V_{LL}$ in Δ, the equivalent Y value $Z_{LN} = (1/\sqrt{3})^2\, Z_{LL} =$ 1/3rd of Z_{LL} value in Δ. Similarly, for converting Y into equivalent Δ, the equivalent Δ value $Z_{LL} = (\sqrt{3})^2 = 3 \times Z_{LN}$ value in Y.

After transforming the entire system into an equivalent Y, and then combining series-parallel impedances into one equivalent total impedance per phase, we can easily determine the

$$\textit{Phase current} = \frac{\textit{Voltage per phase}}{\textit{Total impedance per phase}} \tag{1.34}$$

Example 1.12

A balanced Δ-connected Load-1 has $Z_{LL1} = 6 + j\,9\ \Omega$ in each phase of Δ. It is connected in parallel with a balanced Y-connected Load-2 with $Z_{LN2} = 2 + j\,3\ \Omega$ in each phase of *Y*. If these two parallel loads are powered by a 3-phase, Y-connected generator with 480 V line voltage, determine the line current at the generator terminals and the real power delivered by the generator.

SOLUTION

We first convert Δ-connected Z_{LL1} into an equivalent Y-connected Z_{LN1}, which is ⅓ Z_{LL1}, that is, $Z_{LN1} = \frac{1}{3}(6 + j\,9) = 2 + j\,3$ Ω/ph. The Y-connected Z_{LN2} is in parallel with Z_{LN1}. Therefore,

$$Z_{LN.Total} = \frac{1}{\dfrac{1}{Z_{LN1}} + \dfrac{1}{Z_{LN2}}} = \frac{Z_{LN1}\, Z_{LN2}}{Z_{LN1} + Z_{LN2}} = 1 + j1.5 = 1.8\angle 56.3°$$

Ω/ph (the detailed algebra is left as an exercise for the students).
The Y-connected generator phase voltage = 480 ÷ √3 = 277.1 V

∴ Generator line current = Generator phase current

$$= \frac{277.1\angle 0\,(\text{Reference})}{1.8\angle 56.3} = 153.9\angle -56.3 \text{ A}$$

Real power per phase, $P = V \times I \times \cos\theta = 277.1 \times 153.9 \times \cos 56.3° = 23{,}672$ W/phase
Alternatively, using total circuit resistance $R = 1$ Ω in $Z_{LN.Total}$, we get

$P = I^2 R = 153.9^2 \times 1 = 23{,}685$ W/phase (checks within rounding error).

PROBLEMS

Problem 1.1: The electrical potential of point 1 is 3000 V higher than that of point 2. If 200 C of charge per minute flows from point 1 to 2, determine the current and power flow from point 1 to 2, and the energy transferred in 15 min.

Problem 1.2: A sinusoidal voltage of 900 cos377t V applied across a circuit element draws 250 cos(377t – 30°) A. Determine the average power absorbed in the element.

Problem 1.3: For two phasors $\tilde{A} = 85\angle 40°$ and $\tilde{B} = 700\angle 50°$, find (i) $\tilde{A} + \tilde{B}$, (ii) $\tilde{A} - \tilde{B}$, (iii) $\tilde{A} \times \tilde{B}$, and (iv) $\tilde{A} / \tilde{B}$. Express each result in *j*-form and also in θ-form.

Problem 1.4: If phasor $\tilde{A} = \frac{8.5}{j6.5}$, express it in j-form, θ-form, and *e*-form.

Problem 1.5: A source voltage of 240 V_{rms} is applied to a 30∠30° ohms load impedance. Find the average current, rms current, peak current, and average power delivered to the load.

Problem 1.6: A coil having an inductance of 30 mH and resistance of 3 Ω is connected to a 120-V, 60-Hz source. Determine the kWh energy drawn by the coil in an 8-hour period.

Problem 1.7: Figure P1.7 is in the time domain. Express it in the phasor domain with impedances in ohms, and determine the current phasor and the average power.

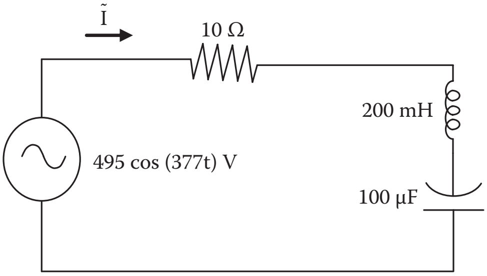


FIGURE P1.7

Problem 1.8: A 240 V, 50 Hz, 1-phase voltage is applied to the circuit shown in Figure P1.8. Determine the current, power factor, and power delivered by the source.

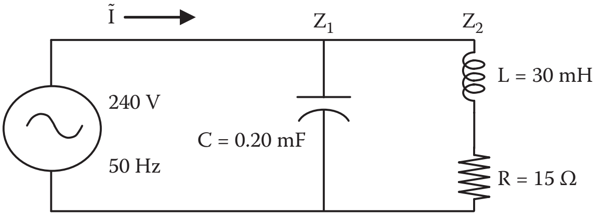


FIGURE P1.8

Problem 1.9: For Figure P1.9 circuit, determine the current and power and draw the phasor and time diagrams.

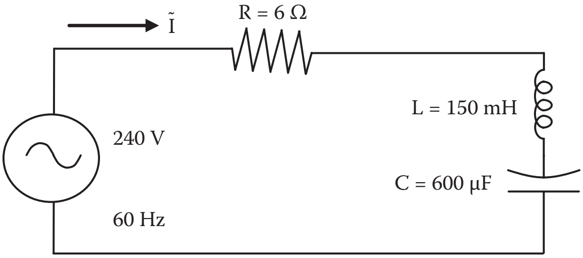


FIGURE P1.9

Problem 1.10: Three parallel loads draw power from a 480 V source. Load-1 is 25 kW and 15 kVAR lagging, Load-2 is 10 kW and 8 kVAR leading, and Load-3 is 18 kW and 10 kVAR lagging. Determine the combined power factor and the total kVA drawn from the source.

Problem 1.11: A Δ-connected transformer has the line-to-line voltage phasor $\tilde{V}_{ab} = 480\angle 0$. Determine the line-to-neutral voltage phasors $\tilde{V}_{an}$, $\tilde{V}_{bn}$, and $\tilde{V}_{cn}$ in magnitudes and phase angles with respect to $\tilde{V}_{ab}$.

Problem 1.12: A balanced Y-connected Load-1 has $Z_{LN1} = 4 + j\,1\ \Omega$ in each phase of Y. It is connected in parallel with a balanced Δ-connected Load-2 with $Z_{LL2} = 12 + j\,6\ \Omega$ in each phase of Δ. If these two parallel loads are powered by a 3-phase, Y-connected generator with 480 V line voltage, determine the line current at the generator terminals and the real power delivered by the generator.

Problem 1.13: Determine the average and rms values of a square wave current of + 10 A for $t = 0$ to 2.5 ms and –10 A for t = 2.5 to 5 ms.

Problem 1.14: Determine the average and rms values of a rectangular wave current of +5 A for 0 to 2 ms and –5 A for $t = 2$ to 6 ms.

Problem 1.15: Powered by a 120 *V* rms voltage source, three impedances are connected in parallel: (i) $10 + j\,20$ ohms, (ii) $5 + j\,20$ ohms, and (iii) $10 + j\,30$ ohms. Determine the real power drawn by each impedance. Without making numerical calculations, how would you have identified the impedance that will draw the highest power?

Problem 1.16: A 120 V source is connected to a load impedance of $31.8 + j\,42.4\ \Omega$. Determine (a) the load current, (b) complex power, (c) real power, (d) reactive power, and (e) apparent power. Be sure to include with each answer the phase angle and the unit as applicable.

Problem 1.17: In the circuit shown below, determine $\tilde{I}$, $\tilde{I}_1$, and $\tilde{I}_2$, and draw the phasor diagram taking 120 V as the reference phasor.

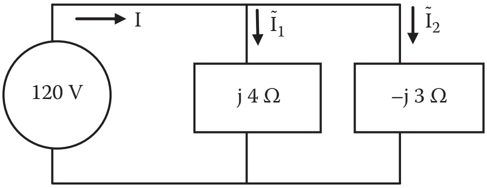


FIGURE P1.17

Problem 1.18: A 3-phase, 440 V, 60 Hz load draws 30 kW real power and 40 KVAR reactive power from the source: (i) determine the line current, complex power, and apparent power drawn from the source, (ii) draw the power triangle, and (iii) if the reactive power is reduced to 10 kVAR, keeping the same real power, determine the percentage change in the prime mover's fuel consumption rate.

Problem 1.19: A 440 V, 3-phase, Y-connected generator delivers power to a Δ-connected load with $6 + j\,9\ \Omega$ in each phase of Δ. Determine the line current and 3-phase power to the load.

Problem 1.20: A 3-phase Δ-connected load has $4.5 + j\,6.3\ \Omega$ impedance in each phase of Δ. For easy circuit calculations with other parallel loads connected in *Y*, the Δ-load needs to be converted into an equivalent *Y*. Determine the impedance *Z* in each phase of the equivalent *Y*.

Problem 1.21: Figure P1.21 shows a circuit in the time domain. First convert it into the phasor-domain showing the voltage and impedance as phasors, and then find the total impedance and current drawn from the source.

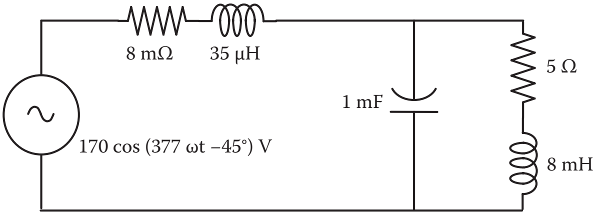


FIGURE P1.21

Problem 1.22: Prepare a Microsoft Excel spreadsheet as shown in the following table for one cycle of 60 Hz ac in increments of 15°. With $\theta = 0°$, plot the voltage, current, and power versus ωt using the CHART function in EXCEL. Find the average values of columns 2 through 5. Then, verify that (a) the square root of Avg4 is equal to the rms value $I_{\text{peak}}/\sqrt{2}$, and (b) average power Avg5 is equal to $V_{rms} \cdot I_{rms} \cdot \cos\theta$. Make sure to convert ωt degrees into radians for the Excel computations if necessary. Make and plot the second spreadsheet with $\theta = 30°$ and the third sheet with $\theta = 90°$. Examining your numerical answers and the charts, draw your conclusions as to the effect of lag angle θ on the average power.

ωt°	***v* = 100sinωt**	***i* = 150sin(ωt–θ)**	***i*²**	***v*i***
0				
15				
.				
360°				
Avg	Avg2	Avg3	Avg4	Avg5

QUESTIONS

Question 1.1 Why is ac universally adopted for the electrical power grid over dc?

Question 1.2 Why is the 3-phase power system universally adopted over the 1-phase ac power system?

Question 1.3 Explain the terms *phasor*, *phase difference*, and *operator j*.

Question 1.4 For the impedance boxes Z_1, Z_2, and Z_3 shown in Figure Q1.4, which of the R, L, and C elements and their relative magnitudes do you expect to see if you open the boxes? Answer for each box individually.

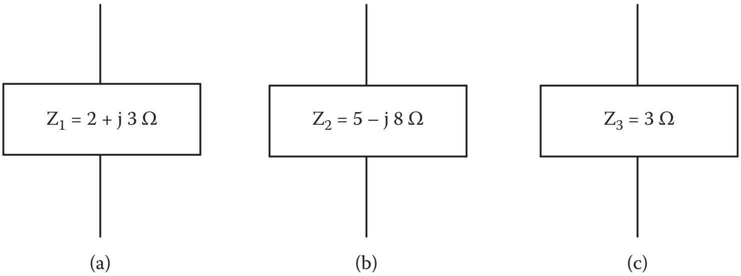


FIGURE Q1.4

Question 1.5 How do the basic circuit laws and theorems in dc and ac differ?

Question 1.6 If a 50 Hz impedance box of value $10\angle -30°$ ohms designed to work on a 240 V, 50 Hz source in Europe is connected to a 240 V, 60 Hz source in the United States, how would the current magnitude and phase angle change (will they increase or decrease?)

Question 1.7 What is the complex conjugate of current, and why is the conjugate (and not the straight) current used to determine the complex power in ac circuits?

Question 1.8 If the real power from the source to the load delivers real work (pumps water, heats space, etc.), what does the reactive power do?

Question 1.9 Why should the plant power engineer use power at or near unity power factor as much as possible?

Question 1.10 Draw a set of balanced 3-phase currents in a *Y*-connected system with neutral, and graphically using the tip-tail method, find their sum $\tilde{I}_a + \tilde{I}_b + \tilde{I}_c = \tilde{I}_\mathrm{n}$ returning via neutral wire.

Question 1.11 In a balanced 3-phase *Y*-connected power circuit, why does the circuit behave exactly the same with or without the neutral wire? What purpose does the neutral wire, where provided, serve in a 3-phase 4-wire system?

FURTHER READING

Alexander, C.K. and M.N.O. Sadiku. 2007. *Fundamentals of Electric Circuits*. Boston: McGraw Hill.

Dorf, R.C. and J.A. Svoboda. 1999. *Introduction to Electric Circuits.* New York: John Wiley & Sons.

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2 Shipboard Power System Architectures

Since the first electrical light was installed on the American ship *Columbia* in 1880 and the subsequent denial of insurance due to safety fears of electrical fire, the electrical power plants in modern ships have grown in size and complexity with the growing demands on ship size, speed, economy, safety, comfort, and convenience. From the original electrical power of a few kilowatts at 120 V, some of today's navy and cruise ships require an electrical load of over 100 MW at a voltage of 11 kV or higher.

2.1 TYPES OF SHIP DRIVES

The ship requires a large mechanical power for propulsion and a small electrical power for service loads. From the mechanical and electrical power systems point of view, ships are grouped as follows:

Mechanical-drive (conventional) ship: The prime mover directly drives the propeller via mechanical gears and a long shaft running through the center of the ship, and the electrical service loads are powered by the ship service generators. Most merchant cargo ships today fall in this category.

Electrical-drive ship: The propellers are driven by large electric motors powered by dedicated propulsion power generators, and the ship service power is produced by separate ship service generators. Most passenger cruise ships today fall in this category.

Integrated-electric ship: All required power for both the propulsion and the ship service loads is generated by the main generators with no separate ship service generators. The service load is provided via step-down transformer from the main bus. Integrated power systems are becoming more common in navy ships, where a high load demand during combat can be met by shedding nonessential service loads and diverting more power to the combat weapons. The navies of many countries are moving toward this category.

All-electric ship: On many ships today, some auxiliary equipment is either steam-powered (e.g., space heaters, laundry, and kitchen equipments), hydraulically powered (e.g., steering systems and submarine diving systems), or compressed-air-powered (e.g., valve actuators and surface-ship turbine engine starters). Converting these remaining nonelectrical equipments to electrical power would make the ship all-electric.

Almost all power systems in ships are 3-phase, 3-wire or 3-phase, 4-wire, grounded or ungrounded ac systems. The industry standards recommend (but do not require) that the power distribution system coming out of the generator be ungrounded for reliability, whereas the power distribution system for 120 V service loads be grounded for personnel safety.

2.2 ELECTRICAL DESIGN TASKS

The shipboard power system design generally requires the following tasks:

- Selecting the optimum power system configuration and voltage level best suited for the ship
- Load analysis to size the electrical generator kW and kVA ratings and the prime mover's kW or horsepower rating
- Power distribution routing for propulsion and service loads
- Sizing the feeder cables for required ampacity and for limiting the voltage drops per the applicable standard (usually 3%–5% of the rated voltage at steady state)
- Fault current analysis and protection device ratings at key locations
- Determining the sensor types and locations to monitor the system health

These tasks offer a great deal of design challenges and opportunities related to the power system analysis that meet all imposed or self-derived requirements in steady-state and transient conditions. More than a dozen power system modeling, simulation, and analysis tools are available in the market to help the design engineer perform these tasks. Examples of such commercial software available for shipboard or land-based power system design are:

EDSA™ (often used by navy, submarine, and marine engineers)
EASYPOWER™ (often used by large corporations such as GE and by refineries)
SKM™ (similar to ESDA, used by navy, submarine, and marine engineers)

Many recommended standards exist for designing shipboard power systems, such as the IEEE, USCG, and ABS in the United States. Other countries have their own standards, and IEC covers the international standards. These and other standards for shipboard electrical installations are covered in Chapter 13.

2.3 ELECTRICAL LOAD ANALYSIS

High power generation is required on ships for cruise lines, tugs, tankers, and dredgers. The generator rating depends to a large degree on a fairly close assessment of peak power and the relative timing of each load.

Various electrical loads in merchant ships generally fall in the following groups: (1) propulsion machinery, (2) auxiliary machinery, (3) cargo-handling machinery, (4) deck machinery, (5) HVAC equipment, (6) control electronics, (7) communication electronics, (8) shop load, and (9) hotel loads. Each individual load in these groups may draw different kW power during different phases of the ship operation.

2.3.1 LOAD FACTOR

Not all loads are turned on at all times, and many come one after another, each for a short duration of time in a set sequence. Therefore, the required generator capacity in kW is always less than sum total of kW ratings of all connected loads. This time diversity in their power demand is taken into account by assigning a load factor to each load equipment based on the heritage data on a similar ship built and operated earlier.

The *load factor (LF)*—also known by other names such as demand factor, diversity factor, utility factor, or duty factor—is defined as the average power over a period of time as a fraction of the peak (or rated) power of the equipment. Thus, the load factor indicates to what extent a specific load contributes to the total load on the generator, which aggregately powers all the connected loads. If the time-varying load power $p(t)$ is expressed in kW over any specific operating period T (e.g., at sea, in-port loading, etc.), then the load factor during that operating period is given by

$$Load\ factor = \frac{Average\ kW\ load}{Peak\ kW\ rating} = \frac{\frac{1}{T}\int_0^T p(t)\,dt}{Peak\ kW\ rating} \tag{2.1}$$

$$= \frac{Actual\ kWh\ used\ during\ T}{Peak\ rated\ kW \cdot T\ in\ hours}$$

Obviously, LF = 1.0 for continuously *on* load, LF < 1.0 for intermittent load, and LF = 0 for stand-by equipments.

For example, a 10 kW motor used at full load for 4 h and at one-half load for 2 h during a 12-h period on loading days at the port has a load factor of $(10 \times 4 + 5 \times 2) \div (10 \times 12) = 0.42$ during the daytime loading period. During other periods, it may have a different load factor, such as zero at night.

Example 2.1

One of the numerous pieces of electrical equipment in an industrial plant is turned on only during the daytime from 8 a.m. to 8 p.m. It draws 100% power for 2 h, 50% power for 3 h, and 30% power for 7 h. It remains off for the rest of the day. Determine its daily (24 h) and daytime (12 h) load factors.

SOLUTION

On the calendar day (24 h) basis,

$$\text{Daily Load factor} = \frac{100\times 2+50\times 3+30\times 7}{100\times 24} = 0.2333 \text{ or } 23.33\%.$$

However, when many such equipment with diverse use run only during the daytime, the 24 h load factor cannot be used for sizing the system kW rating.

Instead, the daytime load factor should be used for sizing the generator or incoming utility line to the plant, which is

$$\text{Daytime Load factor} = \frac{100\times 2+50\times 3+30\times 7}{100\times 12} = 0.4667 \text{ or } 46.67\%.$$

Caution: The LF approach gives good results only if numerous loads are connected, but only some of them work at the same time. It is not valid with very few loads, and must be used judiciously with an in-between number of loads. With few loads, or with some large loads and many small loads, the system kW rating is best derived by adding large loads on an hourly basis or shorter time intervals as needed.

An analogous factor used for electrical power plants—conventional or renewable—is the *capacity factor* of the entire power plant, which is defined as

$$\begin{aligned} \text{Capacity factor} &= \frac{\text{Average kW delivered}}{\text{Peak kW capacity}} \\ &= \frac{\frac{1}{T}\int_0^T p(t)\,dt}{\text{Peak kW capacity}} = \frac{\text{Actual kWh delivered during } T}{\text{Peak kW capacity} \cdot T \text{ in hours}} \end{aligned} \tag{2.2}$$

The power plant capacity factor may be calculated on a seasonal or annual basis. It measures the fraction of the plant's peak rated capacity in kW that is used over the period. A higher capacity factor of the power plant means the equipment is utilized for more hours, resulting in a lower capital cost per kWh delivered to the users.

2.3.2 LOAD TABLE COMPILATION

The first and foremost task for shipboard power system design is to compile a table of *all* connected electrical loads along with their load factors in all operating modes of the ships. The major operating modes during a cargo ship's port-to-port trip are

- In-port loading the cargo
- Maneuvering in and out of the port
- At sea cruising at full speed
- In-port unloading the cargo
- Anchor and standby
- Emergency operation

Since all connected load equipment does not draw full power continuously, we must use their load factors for deriving the combined contribution to the generator kW capacity requirement. That is, we must account for each wired load's peak kW rating and the load factor (time diversity in power demand).

The total load on generator in any particular operating mode is given by adding the (kW wired × LF) product for all equipments in that operating mode as illustrated

TABLE 2.1
Load Analysis Table for Ships (Suitable for EXCEL Spreadsheet)

Shipboard Load Group[a]	**Wired kW (Max. Possible Draw)**	**At Sea Cruising at Full Speed**		**Maneuvering In and Out of Port**		**In Port Loading**		**In Port Unloading**	
	kW_{wired}	LF	kW_{gen}	LF	kW_{gen}	LF	kW_{gen}	LF	kW_{gen}
	1	2	1×2	3	1×3	4	1×4	5	1×5
Propulsion machinery									
Auxiliary machinery									
Cargo equipment									
Deck machinery									
Shop loads									
Electronics									
Communication									
Hotel loads									
HVAC equipment									
$P = \Sigma\ \text{kW}_{\text{gen}}$ (sum of the column above)			Sum_1 $P_{\text{at-sea cruising}}$		Sum_2 $P_{\text{maneu-ver}}$		Sum_3 $P_{\text{inport-load}}$		Sum_4 $P_{\text{inport-unload}}$

* Load groups in the 1st column are subdivided into each for a detailed design.
Sum_k = Sum of the column above, where k = 1, 2, 3, 4, …. as many as required.

in Table 2.1. For example, the total generator power required at sea is given by the following sum of all connected equipment:

$$P_{\text{at-sea}} = \Sigma\ \text{kW}_{\text{wired}} \cdot \text{LF} \tag{2.3}$$

The generator kW rating is the maximum of all powers calculated by Equation (2.3) in all operating modes, that is,

$$P_{\text{gen}} = \text{Maximum of } \{P_{\text{at-sea}}, P_{\text{maneuver}}, P_{\text{inport-load}}, P_{\text{inport-unload}}\} \tag{2.4}$$

The generator kVA and the prime mover horsepower ratings are then

$$\text{Generator kVA} = \frac{P_{\text{gen}}\ in\ kW}{\text{Power factor at generator terminals}} \tag{2.5}$$

$$\text{Prime mover } HP = \frac{P_{\text{gen}}\ in\ kW}{0.746\, x\ Generator\ efficiency} \tag{2.6}$$

Allowing a 30% margin is common for growth and for uncertainties in loads during a preliminary power system design of a new line of ship. In heritage design, a 10% margin may be adequate.

Although each load has its own average contribution to the generator power rating, the wire ampere rating for each wired load must be based on its own peak kW rating with 30% margin as follows:

$$\textit{Ampere rating of each load wiring} = 1.3\frac{\textit{Peak watts each load can draw}}{\textit{Load voltage} \times \textit{Power factor}} \tag{2.7}$$

The cargo-handling machinery during in-port unloading generally consumes high power, making the $P_{\text{inport-unload}}$ the maximum load on a conventional ship. But, this has to be ascertained by actual load analysis. In an electrically propelled ship, the load during cruising at sea is generally the maximum.

Example 2.2

A load analysis for a typical midsize cargo ship finds the following electrical loads (not including 10,000 hp mechanical power for propulsion): 1250 kW at sea, 800 kW during maneuvering, 7000 kW for in-port cargo unloading, 700 kW for in-port cargo loading, and the emergency load power of 400 kW. (a) Determine the ship generator kVA and the diesel engine horsepower ratings. Assume a reasonable power factor and efficiency where needed. (b) If the shore power connection were available when required, how would the generator and engine ratings change?

SOLUTION

(a) First, we assume a generator efficiency 95% and a pf of 90% lagging. The ship's generator should be rated for Maximum {1250, 800, 7000, 700} = 7000 kW that will meet the maximum power demand during in-port cargo unloading. The generator kVA rating = 7000 ÷ 0.9 = 7778 kVA, and the generator input = 7000 ÷ 0.95 = 7368 kW, which is the diesel engine output rating. So, the engine horsepower rating = 7368 ÷ 0.747 = 9863 hp.

(b) For this ship in particular, use of shore power during in-port cargo unloading can significantly reduce the generator kW rating from 7000 to 1250 kW. This would reduce the generator kVA rating to 1250 ÷ 0.90 = 1389 kVA, and the diesel engine kW rating to 1250 ÷ 0.95 = 1316 kW or the horsepower rating to 1316 ÷ 0.746 = 1764 hp. These are significant reductions in both the generator and the diesel engine ratings, to about 1/5th. The subsequent capital cost reduction must be traded off with the high price of shore power compared with the self-generated power on the ship using cheap oil. However, many ships now may still favor expensive shore power due to recent emphasis on ship emissions at busy ports around the world.

Example 2.3

A small commercial office building takes a 208 V, 3-phase, Y-connected service from the utility company. Its daytime load analysis—assuming that the nighttime loads are very small—carried out based on the seasonal load variations gave the following results: winter 160 kW, spring 100 kW, summer 200 kW, and fall 90 kW. Determine the building incoming line ampere rating. Assume a reasonable power factor.

SOLUTION

We assume that there are numerous small loads, from lights to office equipments, air conditioners, heaters, etc. Therefore, the aggregate power factor of 0.90 lagging is a reasonable assumption.

Based on the load analysis, the building's incoming utility service line should be sized for Maximum {160, 100, 200, 90} = 200 kW, with the apparent power rating of 200 ÷ 0.9 = 222.2 kVA.

The line current is then derived from Equation (1.30) $I = \frac{222.2 \times 1000}{\sqrt{3} \times 208} = 617\ A$.

A margin of 30% is generally appropriate, requiring an 1.3 × 617 = 800 A incoming service line.

2.4 POWER SYSTEM CONFIGURATIONS

The selection of power system architecture for a ship primarily depends on the mission over the service life and the reliability desired. In discussing power system architecture, we often use the term *electrical bus*, which means the parallel conductor bars of a heavy cross section to which all generators and loads are connected. The generators feed the power to the bus, and the loads draw power from the bus. The bus bars are typically made of thin rectangular copper bars, solid or sometimes hollow to circulate cooling water in high-power systems. These bars are often called *bus bars,* or just the *bus*, or *lines*, or *rails* for their rail track look.

Modern power systems widely use power electronics converters, which inherently draw nonsinusoidal currents that cause high-frequency harmonics in the bus voltage. These harmonics requires harmonic filters. Some navy ships require very clean harmonic-free power for harmonic-sensitive loads. In that case, a separate clean bus is provided using a motor-generator set, which also provides electrical isolation for the loads.

2.4.1 Basic Conventional Ship

Figure 2.1 depicts the basic architecture shown by a single-line diagram for a conventional ship with mechanical propulsion. It shows two main 460 V generators and one 460 V emergency generator with a battery for start-up and to power essential loads that must be kept on all the time. It has a receptacle for connecting to the shore power when the ship is anchored at a port and generators are turned off. The loads are in two basic groups, heavy 450 V loads, and 115 V loads for small equipments and lighting. The refrigeration loads have their own control panel. The battery chargers draw power from the 120 V emergency bus.

2.4.2 Large Cargo Ship

A large cargo ship with conventional mechanical propulsion generally has a power system architecture shown in Figure 2.2. The main generator's power loads on the

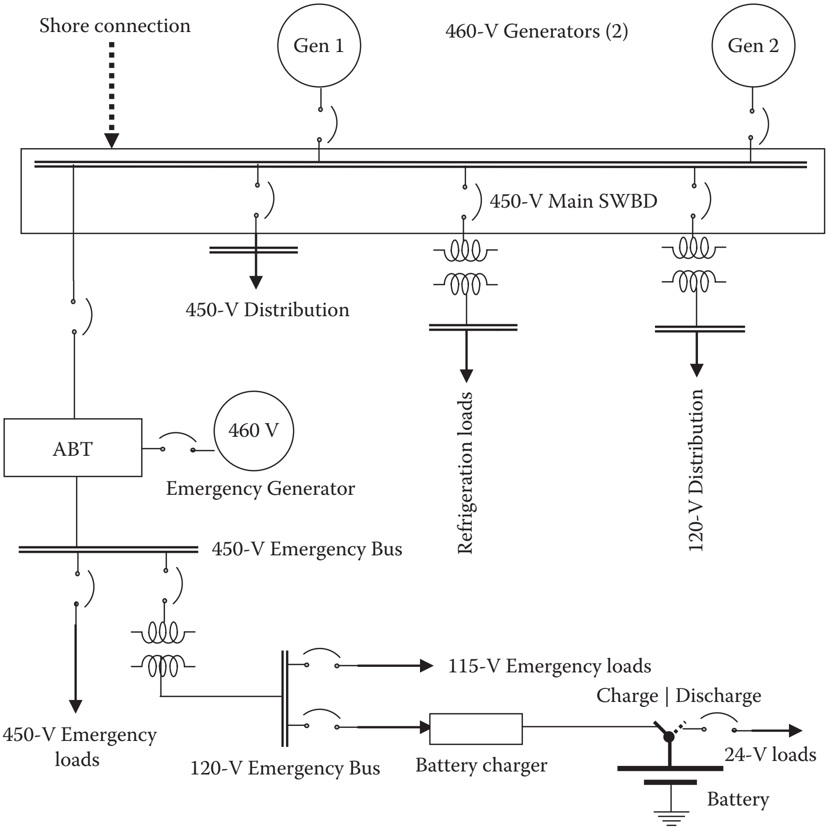


FIGURE 2.1 Basic power system for small cargo ship with emergency generator and battery backup.

main switchboard, and the emergency generator switchboard serves the essential loads, lighting, internal communication, and electronics circuits. The emergency switchboard is normally tied with the main bus, with an emergency power source from an emergency generator or a battery via an automatic bus transfer switch that has two interlocks with electrically operated power circuit breakers. With an automatically started emergency generator, the transfer occurs when the generator acquires a rated voltage at a rated frequency. With a manually started emergency generator, a temporary emergency power source—battery or uninterruptible power source (UPS)—is installed. All automatic bus transfer switches have two interlocks with electrically operated power circuit breakers.

ABS-approved UPS: The American Bureau of Shipping (ABS) recommends specific uninterruptible power sources for shipboard use. APC corporation makes several such UPS units (700, 1500, and 3000 VA), particularly for backing up the shipboard navigational electronics. Only units that are marked for shipboard use are acceptable as these are ungrounded and equipped with switching to break both lines L1 and L2.

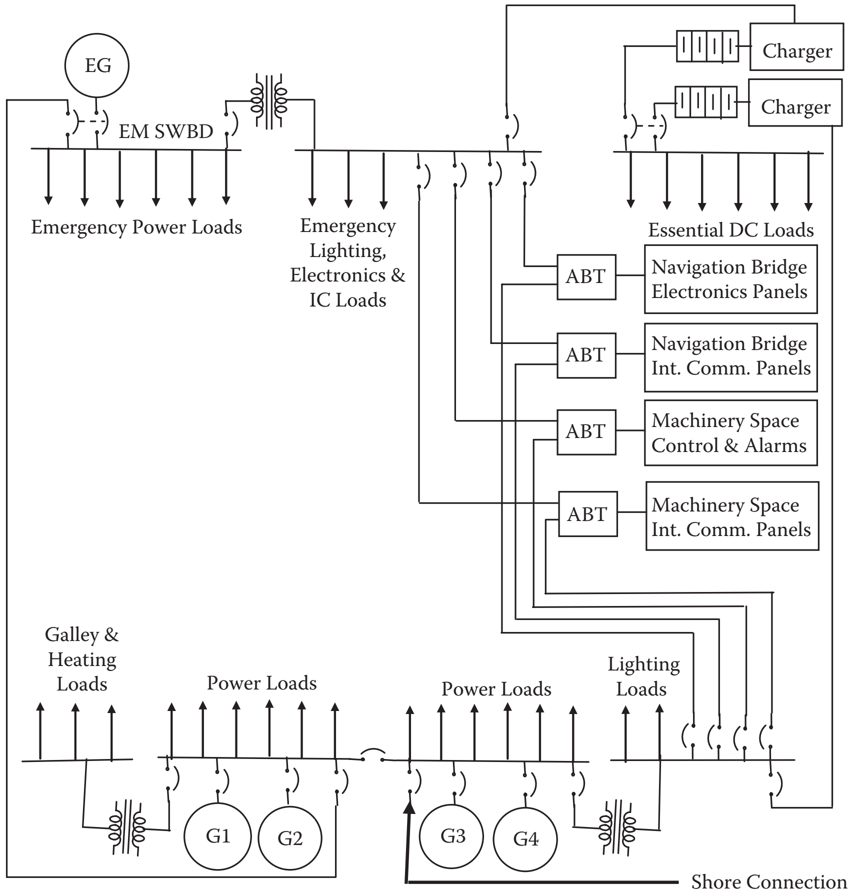


FIGURE 2.2 Large cargo ship power system with emergency generator and battery back-up (Adapted from IEEE-Std-45: Recommended Practice for Electrical Installations on Shipboard, 2002).

2.4.3 LARGE CRUISE SHIP

Figure 2.3 is a typical power system architecture for a large cruise ship with integrated electrical propulsion. The main generators power the propulsion motors and all service loads from one main bus. The industry standards require emergency power in all cruise ships. The emergency power distribution should have three buses as shown at the top. The temporary emergency power comes from the battery or UPS, which is then automatically transferred to the final emergency generator bus when it acquires the rated voltage at rated frequency.

The kW power and the main power reliability and availability requirements dictated by the industry standards require additional features in the shipboard power system. The following architecture incorporates these features often found in today's ships.

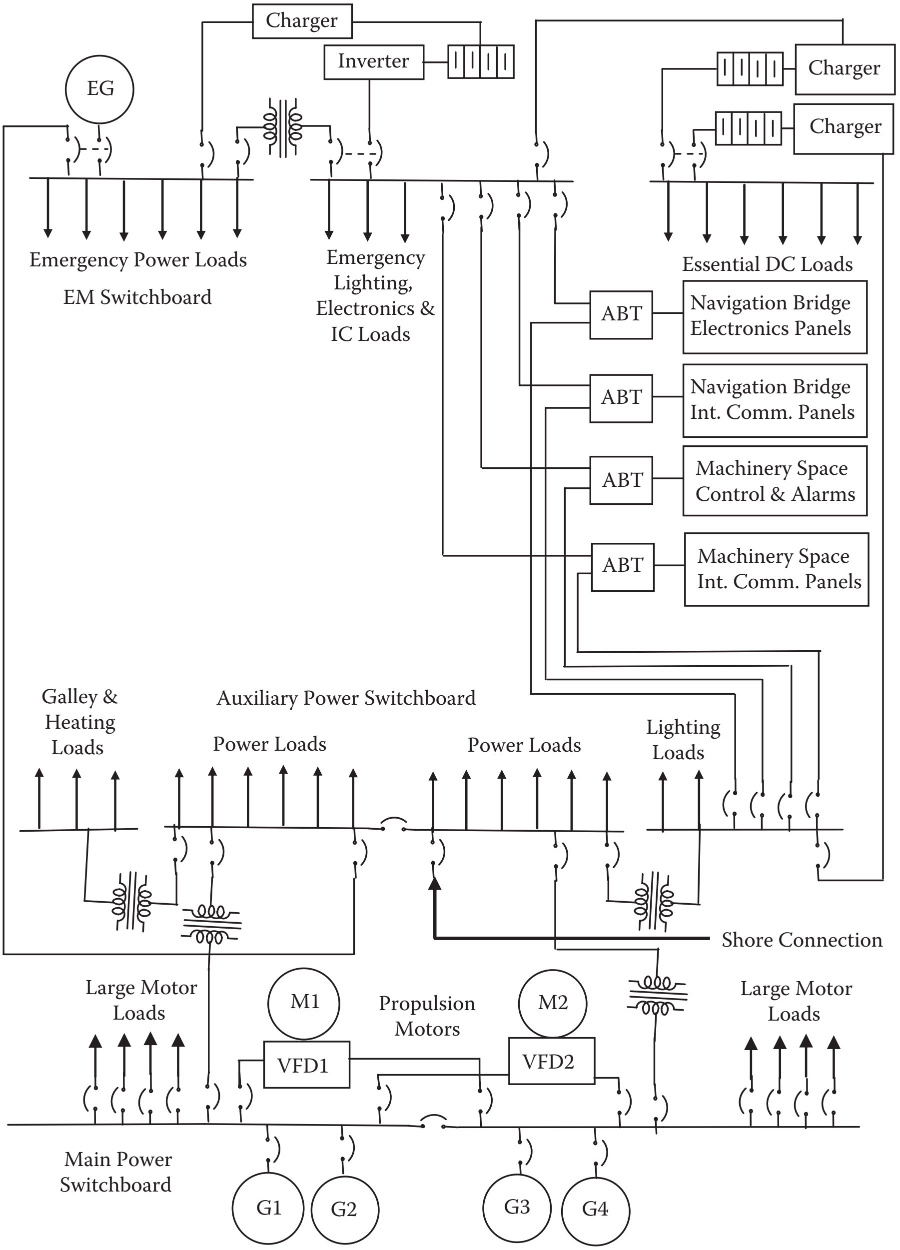


FIGURE 2.3 Integrated power system for large cruise ship with electric propulsion (Adapted from IEEE-Std-45: Recommended Practice for Electrical Installations on Shipboard, 2002).

2.4.4 RING BUS IN NAVY SHIP

A ring-bus architecture shown in Figure 2.4 is often used in high-power navy ships. It has four generators connected in parallel in a ring bus configuration and no emergency generator. Note that the four generator buses along with the four bus-ties form a square-shaped *ring*. Such an architecture without an emergency generator is possible when multiple generators are connected in parallel by a bus-tie in a way that any one can be out for service with the remaining generators supplying the total load. The physical locations of the generators and switchboards are generally in different fire zones, watertight compartments, and on different decks, providing the redundant power distribution system with separation.

2.4.5 ABS-R2 REDUNDANCY CLASS OF SHIP

In electrically propelled ships, ABS rules require a certain redundancy in the electrical power system. For example, ABS-R2 class redundancy requires the power system architecture shown in Figure 2.5, where two groups of generators with a bus-tie power two propulsion motors.

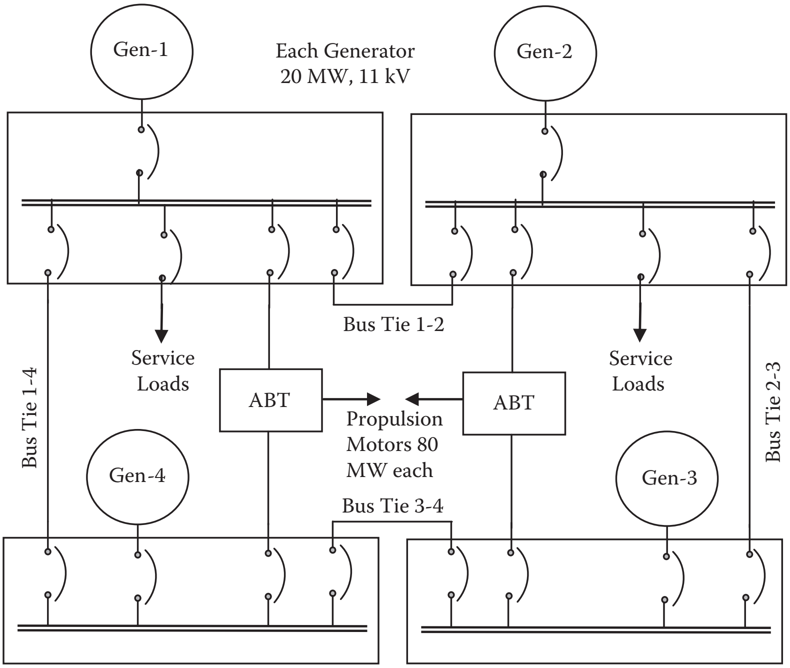


FIGURE 2.4 Ring bus typical in many navy ships with four main generators and no emergency generator.

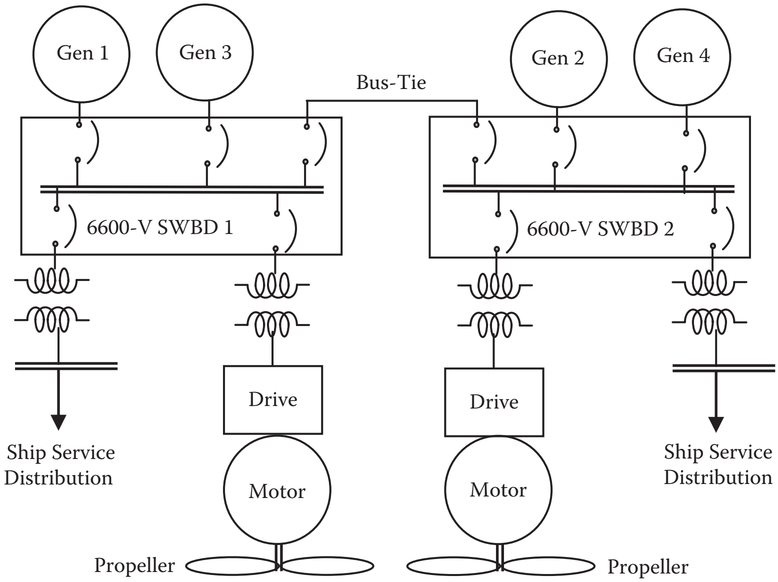


FIGURE 2.5 ABS-R2 redundancy class of ships with electric propulsion.

2.4.6 ABS-R2S Redundancy with Separation

In ships where a higher degree of safety and redundancy are required, the redundant power system ABS-R2 must also be physically separated with a watertight bulkhead separator as shown in Figure 2.6.

2.4.7 ABS-R2S+ with Two-Winding Propulsion Motors

For an even higher degree of redundancy on board, the redundant power system ABS-R2S is not only physically separated, but also has each propulsion motor built with two stator windings, each capable of delivering full power. This highly reliable architecture is shown in Figure 2.7, where two circles in the motor represent redundant stator windings.

In the foregoing ABS classifications, we note that R denotes redundancy, 2 denotes two alternate parallel paths of powering the loads, S denotes physical separation, and + denotes additional reliability features.

2.4.8 Clean Power Bus for Harmonic-Sensitive Loads

The widespread use of power electronics converters for pump and propulsion motor drives in ships has introduced power quality concerns. The power-frequency current and voltage chopping done in these converters generate high-frequency harmonics on the bus. Often, harmonic filters are installed on the bus, but they are inadequate for highly harmonic-sensitive loads—especially navigation electronics and some navy equipments. For such loads, harmonic-free power is generated using a

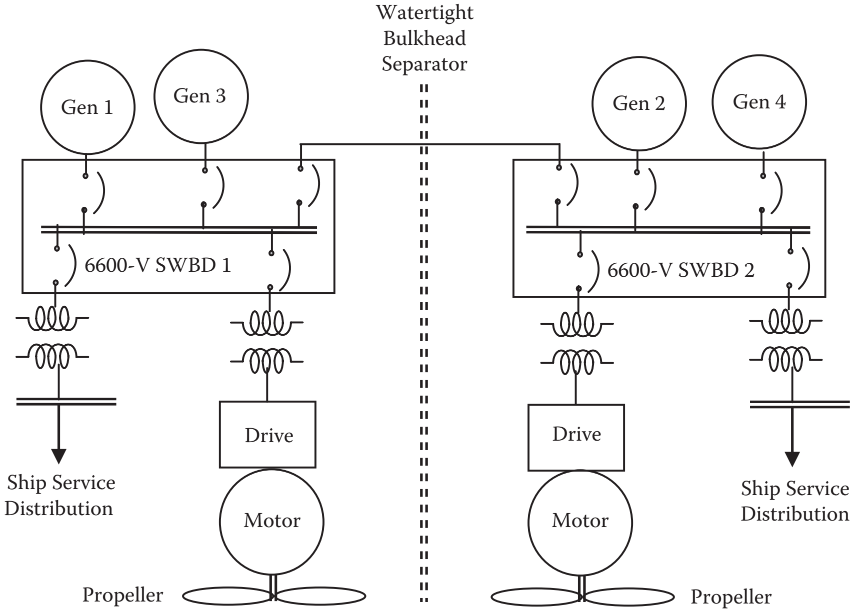


FIGURE 2.6 ABS-R2S redundancy with separation for ships with electric propulsion.

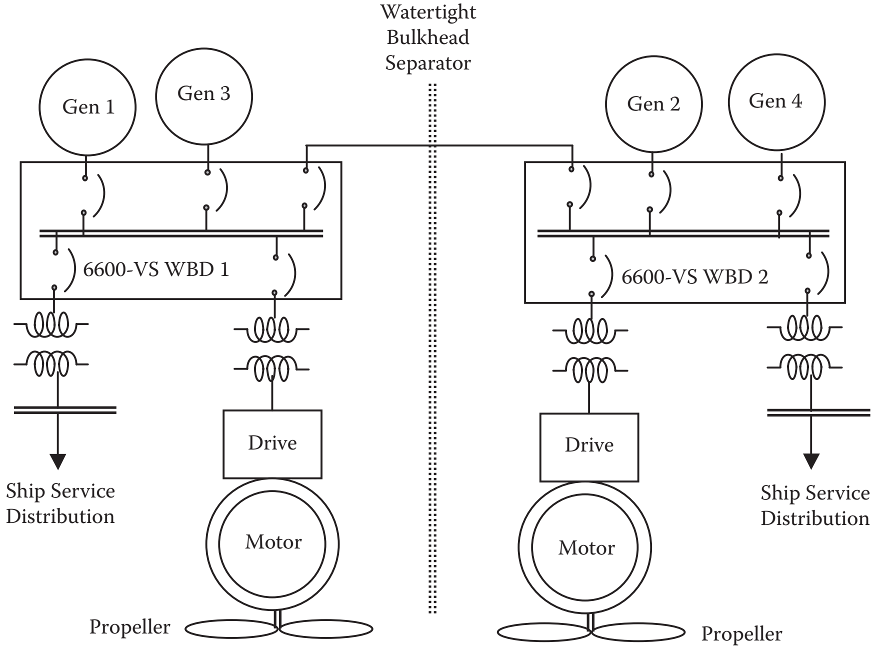


FIGURE 2.7 ABS-R2S+ redundancy with separation and two propulsion motors each with redundant stator windings.

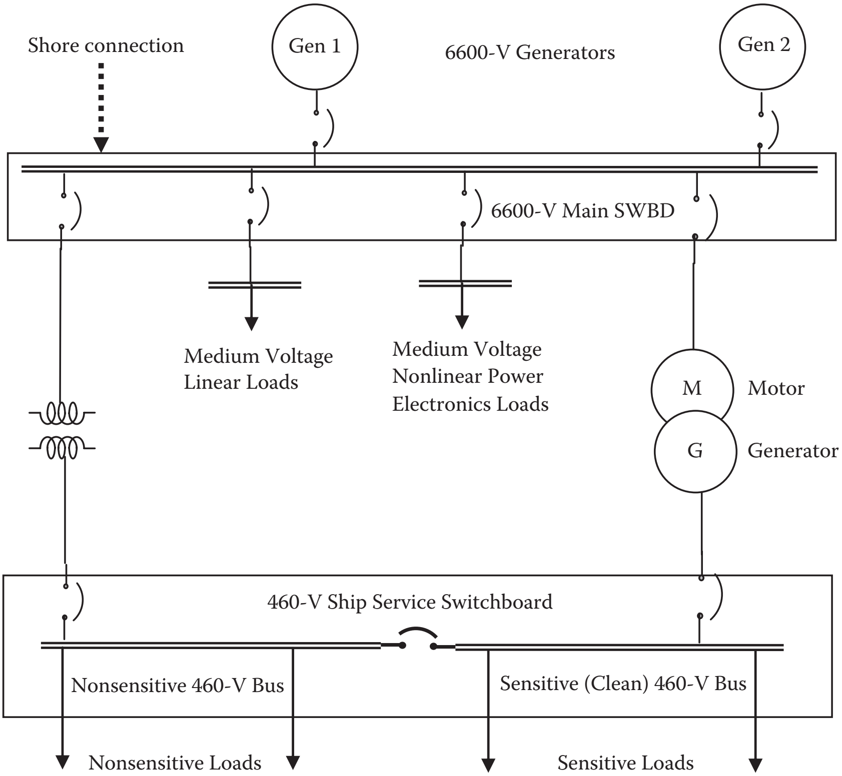


FIGURE 2.8 Clean power bus for harmonic-sensitive loads.

motor-generator set in an architecture shown in Figure 2.8. The voltage generated by this set is completely isolated from the main bus, as there is no electrical connection between the two.

2.4.9 EMERGENCY GENERATOR ENGINE STARTING SYSTEM

Figure 2.9 depicts the engine starting system for the emergency generator. It typically incorporates two electrical lines and one hydraulic line. It has redundant engine starting capability to start and take over the emergency loads within 45 sec after the failure of the main ship service power, which is detected by zero voltage at the main bus. The starting system is usually a combination of compressed air, hydraulic, and battery power (or an uninterruptible power supply).

2.5 COLD IRONING/SHORE POWER

Cold-ironing is the U.S. Navy's term for connecting the ship to the shore-side power source in-port with the ship's machinery shut down, thereby causing the

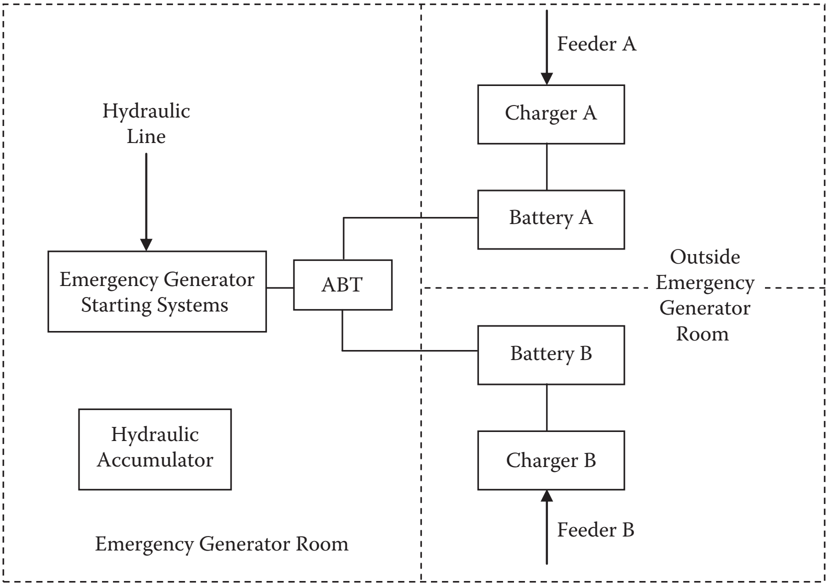


FIGURE 2.9 Engine starting system for emergency generator.

engine room and hull iron to turn cold. Availability of fast plug connections and seamless load transfer without blackout allow the full range of in-port functions to continue during the switchover. It is becoming an important part of the clean port regulations that major ports around the world have rigorously started to implement. For example, all major container, liquid bulk, and cruise ship terminals at the Port of Los Angeles will have shore-side electrical power within five years, and all container and one crude oil terminal at the Port of Long Beach will have it within five to ten years. This will allow shutting down dirty diesel-powered auxiliary engines when at port, and taking advantage of relatively cleaner—although more expensive—shore power for reducing ship emissions around the port.

2.6 EFFICIENCY AND RELIABILITY OF CHAIN

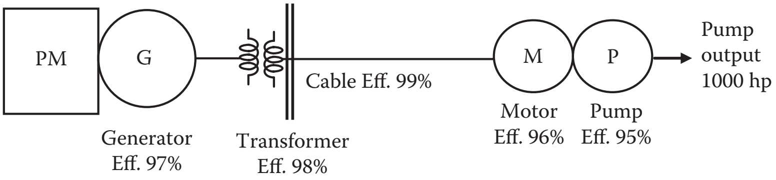
The power system is configured with equipment in series. In a long chain of equipment, system efficiency and reliability both suffer in the following way.

$$\begin{aligned} &\text{System efficiency} = \text{Multiplication of all individual equipment efficiencies} \\ &\text{System reliability} = \text{Multiplication of all individual equipment reliabilities} \qquad (2.8) \end{aligned}$$

For this reason, long chains of equipment in series should be avoided for good system design. The two examples that follow illustrate these two important concepts of chain efficiency and chain reliability.

Example 2.4

A water pumping line has a chain of equipment in series as shown below from the prime mover to the end pump that must deliver 1000 hp output. The motor is powered via cable and a step-down transformer connected to a dedicated 3-phase generator. Assume the following efficiencies: generator 97%, transformer 98%, cable 99%, motor 96%, and pump 95%. Determine the prime mover hp rating required for this pumping system.



SOLUTION

The efficiencies of series equipment multiply; hence, the total efficiency of the power delivered from the prime mover to the end pump is

$$\eta_{Total} = \eta_{pump} \times \eta_{motor} \times \eta_{cable} \times \eta_{trfr} \times \eta_{gen} = 0.95 \times 0.96 \times 0.99 \times 0.98 \times 0.97 = 0.8583$$

$\therefore$ Prime mover output = 1000 hp motor output ÷ 0.8583 overall efficiency = 1165 hp

Example 2.5

In the ABS-R2 redundancy class of ship depicted in Figure 2.5, if the probability of each side losing power due to failure or planned maintenance is 5% over one year, determine the probability of the ship going dark—having no power at all—over a one year period.

SOLUTION

The theory of probability is outside the scope of this book; hence, we will draw on your previous background and keep it simple so that we can understand even without previous exposure to the reliability calculations.

With two sides of an ABS-R2 power system in parallel, the ship would have no power if both sides have failed. For such a system, we multiply the unreliability (probability of failure) of each side to obtain the unreliability of the total system, that is,

Total system unreliability = 0.05 × 0.05 = 0.0025. That is, there is a 0.25% chance that the ship will have no power at all over a one-year period. Expressed positively for a management report, we say that this ship has a 99.75% reliable power system.

If two components were in series, each with the probability of failure 5%, such that any one failing will cause the entire chain system to fail, then we multiply the reliabilities (probability of working) of each component to obtain the reliability of the total system, which would in this case be 0.95 × 0.95 = 0.9025 or 90.25%. This is quite a reduction from 99.75% with parallel components. That is why modular systems with identical components in parallel to share the total load are always preferred and implemented in practice.

TABLE 2.2
Shipboard Electrical Circuit Designation Prefixes

Type of Circuit	Prefix	Type of Circuit	Prefix
Propulsion power	PP	Lighting	L
Ship service power	P	Emergency lighting	EL
Emergency power	EP	Electronics	R
Shore power	SP	Communications	C
Special frequency power	SFP	Degaussing	D
Controls	K	Cathodic protection	CPS

2.7 SHIPBOARD CIRCUIT DESIGNATION

All electrical circuits on ships are required to be identified in applicable documents and on equipment labels, such as nameplates and cable tags, with a designation prefix. Some selected designations prefixes commonly found in power circuits are listed in Table 2.2.

2.8 SHIP SIMULATOR

The ship simulator built for education and training purposes (Figure 2.10) can be a great tool in visualizing the overall ship systems—including the electrical power

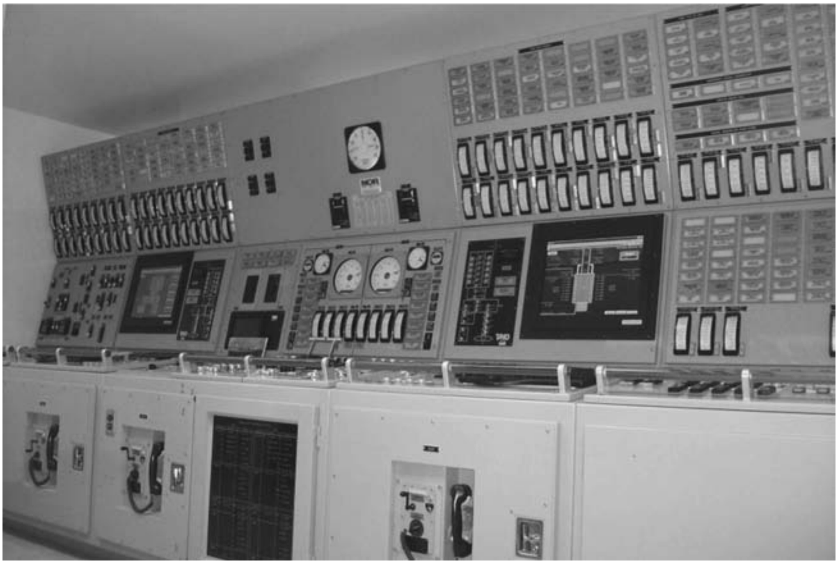


FIGURE 2.10 Ship simulator for a U.S. Navy oil tanker (from Captain Elwood Baumgart, U.S. Merchant Marine Academy).

system—in one place. It offers the opportunity of learning the effects of changes that can be made in operating modes on the ship performance, and can answer many *what if* questions. For example, the electrical load on generators increases with speed according to the exponential relation $\text{MW} = k \times \text{speed}^{\alpha}$, where α is a complex function of the ship surface submerged in water, propulsion resistance, age of the ship surface paint, and the ship speed, which determines the nature of the ocean water flow (streamline, turbulent, etc.). At low speed, $\alpha = 3$, which can approach 4 at high speed. Since it is extremely difficult for young engineers in training to determine the value of α analytically, the ship simulator with its built-in propulsion resistance computation can be a valuable tool in experimenting with a ship's operating speed. The simulator shown in Figure 2.9 is for the U.S. Navy's oil tanker USNS Henry J. Kaiser (T-AO187)—677 ft long × 97 ft wide, 2 × 24.3 MW diesel power—that can carry 180,000 barrels of fuel oil or aviation fuel at 20 knots speed.

2.9 SYSTEMS OF UNITS

The systems of units can pose a special problem in the shipping industry, since ships are built in one country and travel throughout the world, often with an international crew. The international system of units (meter-kilogram-second) is used in all industrially developed and developing countries. Countries still not using the System International (SI) units are the United States, Burma (Myanmar), and Liberia. In the United States, the British (English) system of units (foot-pound-second) is still in widespread, with no sign of change coming in the foreseeable future. Electrical engineers have been using SI units from the very beginning, but British units still prevail in the thermal and mechanical engineering fields.

The U.S. Metric Standard Act of 1968 to convert to SI units did not work in the United States, and since then no effort has been made to change. The conversion to SI has been on the back-burner of the U.S. government since then. Most specifications written by organizations such as ASME, ASTM, SAE, etc., include both British and SI units. By one rough estimate, educating U.S. students in both units cost about $10 billion annually. Even then, horror stories occasionally surface, such as a Mars rover spacecraft crash at a cost of hundreds of million dollars due to a unit conversion error (omission). The shipping industry, being global, has to endure the resulting confusion.

Both systems of units are used in this book depending on the data source. However, many numerical values are given in both units as far as possible. As a further help to the readers worldwide, an extensive table of unit conversion is given in the front pages of the book for use as and when needed.

PROBLEMS

Problem 2.1: Among numerous electrical equipment in an industrial plant, one is turned on during the daytime only from 8 a.m. to 8 p.m. It draws 100% power for 4 h, 50% power for 5 h, and 30% power for 3 h. It remains off for the rest of the day. Determine its daily (24 h) and daytime (12 h) load factors.

Problem 2.2: A midsize cargo ship generator needs to supply the following electrical ship service loads: 2500 kW at sea, 1800 kW during maneuvering, 9000 kW for in-port cargo unloading, 900 kW for in-port cargo loading, and emergency load power of 800 kW. Determine (a) the ship generator kVA and the diesel engine horsepower ratings, assuming a reasonable power factor and efficiency where needed, and (b) if the shore power were used when at port, how the generator and engine ratings would change.

Problem 2.3: A small factory has a 480 V, 3-phase, Y-connected service line from the utility company. Its daytime load analysis—assuming that the nighttime loads are very small—carried out based on the seasonal load variations gave the following results: Winter 460 kW, Spring 200 kW, Summer 400 kW, and Fall 190 kW. Determine the incoming power line ampere rating assuming a reasonable power factor.

Problem 2.4: A chain of equipment is in series in a pumping line from the prime mover to the end pump that delivers 5000 hp output. The motor is powered via cable and a step-down transformer connected to a dedicated 3-phase generator. Assume the following efficiencies: generator 96%, transformer 97%, cable 98%, motor 92%, and pump 94%. Determine the prime mover horsepower rating required for this pumping line.

Problem 2.5: ABS-R2 redundancy class of ship depicted in Figure 2.5 has the probability of each side losing power due to failure or planned maintenance equal to 3% over one year. Determine the probability of the ship going dark—having no power at all—over a one-year period.

Problem 2.6: Take a ship you are familiar with (or a hypothetical one) as an example, and compile the load table similar to Table 2.1 showing peak kW load and load factor for each load or load group in major operating modes. Then carry out the load analysis and derive the kVA rating of the ship generator and the horsepower rating of the prime mover. If applicable, you may borrow information from your previous work (such as a capstone project).

QUESTIONS

Question 2.1 In the ring bus shown in Figure 2.4, trace the *ring of power*. Identify a major advantage of the ring bus. How does it compare with the land-based power grid?

Question 2.2 Discus your experience in dealing with both U.S. and SI units. Do you think the United States should change to SI units soon, by compulsion if necessary? Considering that the 1968 act for changing the units in 10 years did not work, how can the U.S. government proceed now? What are the pros and cons of changing and not changing in the near future?

FURTHER READING

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3 Common Aspects of Power Equipment

All electrical machines have two sets of coils: one set (field coil in generator and motor, and primary coil in transformer) produces the working magnetic flux φ and the other set (armature coil in generator and motor, and secondary coil in transformer) reacts to flux φ and produces an electromotive force (voltage) in the generator and transformer, and a mechanical force (torque) in the motor. Both sets of coils are embedded in magnetic steel to produce the required flux with a minimum magnetizing (field excitation) current. The electromagnetic interaction between two sets of coils is governed by the following basic laws of physics.

3.1 FARADAY'S LAW AND COIL VOLTAGE EQUATION

A coil of N turns linking magnetic flux φ coming from an independent external source as shown in Figure 3.1(a) induces an internal voltage equal to $-\mathrm{d}\varphi/\mathrm{d}t$ in every turn of the coil. With N turns in series, it adds up to the terminal voltage

$$v = -N \times \mathrm{d}\varphi/\mathrm{d}t \tag{3.1}$$

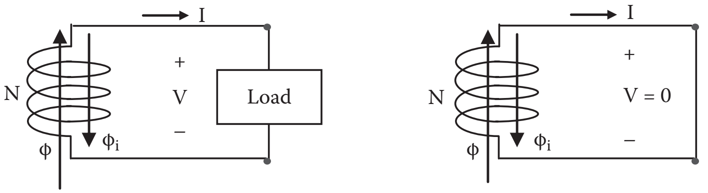
The − sign is due to Lenz, which indicates that the polarity of induced voltage and the resulting current in the coil (if in a closed loop) is such as to oppose $\mathrm{d}\varphi/\mathrm{d}t$, that is, to oppose any change (increase or decrease) in flux φ. Stated differently, the coil current direction will be such as to produce an internal reaction flux φ_i to counter the flux coming from the external source as shown in Figure 3.1(b).

Constant Flux Linkage Theorem: A corollary to Faraday's law when applied to a coil with shorted terminals as shown in Figure 3.1(b), where the terminal voltage $v = 0$ leads to $-N \times \mathrm{d}\varphi/\mathrm{d}t = 0$ or flux $\varphi = \text{constant}$, that is, the flux linking any shorted coil remains constant. To do this, the shorted coil produces an internally circulating current, and the resulting flux is just sufficient to cancel out the change in incoming flux such that the net flux through the coil remains constant (zero or any other value). Some examples of the working of this theorem are:

Losing radio or cell phone signals in a long road tunnel or a concrete building with a reinforced steel grid of numerous shorted metal rings (called Faraday rings) shown in Figure 3.1(c)

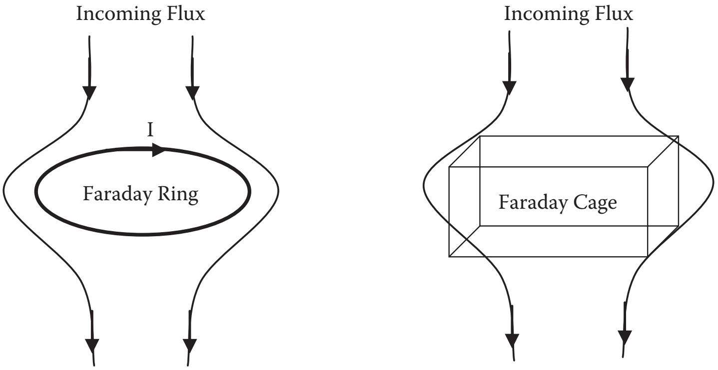
Faraday cage shown in Figure 3.1(d) that shields the inside from electromagnetic signals coming from the outside

Superconducting coil for energy storage where the coil with zero resistance retains its flux and the associated magnetic energy forever



(a) Flux φ from external source and ϕ_i due to load current I

(b) Net flux ($\phi - \phi_i$) remain constant (frozen) in shorted coil



(c) Shorted conductor ring keeps flux away (shield)

(d) Conductor cage keeps flux away (shield)

FIGURE 3.1 Faraday's law and constant flux theorem in a shorted coil.

Basic Voltage Equation: In any electrical machine, either rotating (generator and motor) or stationary (transformer), if a coil of N turns is subjected to an alternating flux $\varphi = \varphi_m \cos\omega t$, where φ_m = maximum value (peak amplitude) of the magnetic flux in weber, and $\omega = 2\pi f$ = angular frequency in rad/sec, then a voltage is induced in the coil that is given by

$$v = -N\frac{d\phi}{dt} = -N\frac{d\{\phi_m \cos\omega t\}}{dt} = \{N\,\varphi_m\,\omega\}\ \sin\omega t = \{V_m\}\sin\omega t$$

where the voltage amplitude $V_m = N\,\varphi_m\,\omega = 2\pi f\,N\,\varphi_m$

$$\therefore\ V_{rms} = V_m/\sqrt{2} = (2\,\pi/\sqrt{2})\,f\,N\,\varphi_m = 4.444\,f\,N\,\varphi_m \text{ volts} \tag{3.2}$$

Example 3.1

Determine the rms value of voltage induced in a 30-turn coil on a 20 cm × 20 cm cross section core with peak flux density of 1.65 T alternating at 50 Hz.

SOLUTION

Tesla is the unit of flux density B in weber/meter^2 (Wb/m^2), from which we get the peak flux in the core = 1.65 T × (0.20 m × 0.20 m) = 0.066 Wb. Then, using Equation (3.2), we have

$$V_{rms} = 4.444 \times 50 \times 30 \times 0.066 = 440 \text{ V}$$

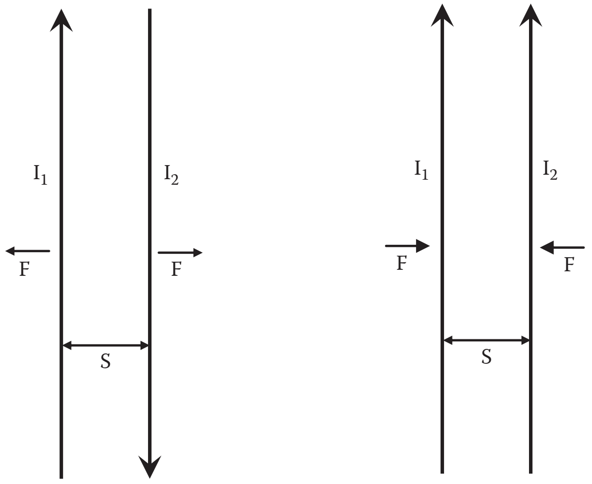
3.2 MECHANICAL FORCE AND TORQUE

Two parallel conductors with currents I_1 and I_2 in the *opposite* direction as shown in Figure 3.2 produce a *repulsive* force

$$F = \frac{5.4 K I_1 I_2}{S_{\text{inches}}} \cdot 10^{-7} \text{ lbf /foot length} \tag{3.3}$$

where K = conductor shape factor (K = 1 for round conductors and < 1 for rectangular bars), and S = center-to-center spacing between the conductors (in inches). In SI units with S in cm, it becomes

$$F = \frac{200\, K I_1 I_2}{S_{cm}} \cdot 10^{-7} \text{ N/meter length} \tag{3.4}$$



(a) Repulsive force between currents in opposite directions

(b) Attractive force between current in same directions

FIGURE 3.2 Mechanical force between two parallel bus bars.

If we use proper polarities of I_1 and I_2 in Equations (3.3) and (3.4), then a positive force would mean an attractive force and a negative force would mean a repulsive force. The conductors carrying currents I_1 and I_2 in the same direction produce an attractive force, and I_1 and I_2 in the opposite direction—as in 1-phase and dc bus bars—produce a repulsive force.

Example 3.2

In a 1-phase switchboard, two parallel rectangular bus bars 1.1 in. apart at center lines experience a 20,000 A_{rms} current when a load gets short-circuited. Determine the peak mechanical force per foot length of the bus bars. Assume a bus bar shape factor K of 0.8. State whether this force is attractive or repulsive.

SOLUTION

In 1-phase power circuits, the currents in two parallel bus bars are equal and opposite. Therefore, we must use peak currents $I_1 = -I_2 = \sqrt{2} \times 20{,}000$ in Equation 3.3, which gives the peak force

$$F = \frac{5.4 \times 0.8 \times (\sqrt{2}\,20{,}000) \times (-\sqrt{2}\,20{,}000)}{1.1} \times 10^{-7} = -314.2 \text{ lbs/ft}$$

The – sign indicates a repulsive force. The bars must be braced to avoid mechanical damage due to bending stresses or deflections between supports.

A coil with current I placed in a magnetic flux φ of another coil (Figure 3.3) produces a mechanical torque equal to

$$\text{Torque} = K\, I\, \varphi \sin\theta \tag{3.5}$$

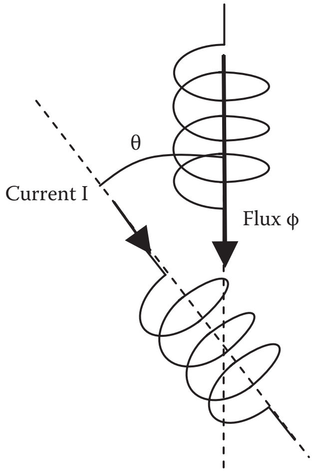


FIGURE 3.3 Mechanical torque between two coils.

where K = constant of the geometry, and θ = angle between the magnetic center lines of two coils.

Two coils with currents I_1 and I_2 and the maximum mutual inductance M_o, which occurs when $\theta = 0$, produce a torque with magnitude given by

$$\text{Torque} = M_o I_1 I_2 \sin\theta \tag{3.6}$$

3.3 ELECTRICAL EQUIVALENT OF NEWTON'S THIRD LAW

If the rotor of a motor is held fixed (blocked from rotating), and the stator is allowed to rotate freely, the "stator" would rotate at the same speed that the rotor would have in the opposite direction. This is due to the reaction torque on the stator. It can be viewed as the electrical equivalent of Newton's third law: *the action and reaction are equal and opposite*, both of which can be electrical or mechanical. If the generator stator coil is pushing electrical current outward, a back torque (reaction) is produced on the rotor coil, which prevents the rotor from turning forward. The prime mover must overcome this back torque by using fuel to maintain a constant speed. We feel such a back torque when turning a megger for testing the insulation resistance. In the motor operation, if the rotor coil is pushing a mechanical torque outward, a back voltage (reaction) is produced in the stator coil, which opposes the current from going inward. This back voltage must be overcome by the applied source voltage at the stator terminals in order to maintain constant speed.

The mechanical analogy of the back voltage in a motor and the back torque in a generator is the back pressure in water pipes and air compressors. The pump must overcome the back pressure in order to push the fluid forward in the desired direction.

Thus, there is a back torque in the rotor of an electrical generator, and there is a back voltage in the stator of a motor. Both the back torque and the back voltage are the reactions that must be overcome by the primary power source to keep the machine delivering steady power outward. Stated differently, there is always a motor reaction in a generator, and a generator reaction in a motor.

3.4 POWER LOSSES IN ELECTRICAL MACHINE

Three types of power losses occur in all electrical machines. They can be expressed as follows, where K is a proportionality constant (different in different equations):

Ohmic loss in conductor = I^2R or $K \cdot P^2$ if delivering power P at a constant voltage.

Magnetic loss in iron core = $K\,\phi_m^2$ or $K\,V^2$, since V and the magnetic flux ϕ_m are linearly related in the unsaturated region of the machine operation per Equation (3.2).

Mechanical loss in bearing friction and aerodynamic windage depending on the speed of rotation.

Regarding the magnetic loss, the alternating magnetic flux produces two types of power loss in the ferromagnetic iron core: (1) hysteresis loss $P_{hyst} = K_h f B_m^n$ due to hysteresis (magnetic friction), and (2) eddy current loss $P_{\text{eddy}} = K_e f^2 B_m^2 t^2$ due to internally circulating (eddy) currents in the core body. Here, K_e and K_h = proportionality constants, f = frequency, B_m = maximum (peak) flux density, t = thickness of the core laminations perpendicular to the flux flow, and n = Steinmetz constant (typically 1.6 to 1.8 for magnetic steel used in electrical machines).

Since P_{eddy} depends on the core lamination thickness squared, the magnetic core is made of thin sheets (laminations) to minimize the eddy current loss. Widely used laminations are 9, 11, 13, 19, and 23 mils thick (1 mil = 1/1000 inch = 25.4 μm). In a given machine, using 9 mil thick laminations would result in about 1/4th the eddy loss compared to that with 19-mil-thick laminations. The hysteresis loss, however, does not change with lamination thickness.

The total power loss in the magnetic core is then $P_{\text{core}} = P_{\text{hyst}} + P_{\text{eddy}}$, which leads to

$$P_{\text{core}} = K_h f B_m^n + K_e f^2 B_m^2 t^2 \text{ watts} \tag{3.7}$$

Vendors of the magnetic core laminations provide the curves of total power loss in watts per kilogram of core weight at various frequencies, flux densities, and lamination thickness for the grades of core steel they supply. The electrical machine design engineer uses such vendor-supplied curves.

3.5 MAXIMUM EFFICIENCY OPERATING POINT

The losses in any power equipment can also be classified broadly into three categories:

- Voltage-related loss, which varies approximately with the flux density squared or applied voltage squared. Since most power equipment operates at a constant voltage and constant frequency, this loss is a *fixed loss* at all load currents.
- Current-related loss in diode type elements, varying with the load current or power output.
- Current^2-related loss in the conductor resistance, varying with square of the load current or square of the power output in constant voltage equipment.

At a constant voltage operation, therefore, the total loss can be expressed as a function of the load power output P_o as follows:

$$\text{Total Loss} = \text{Fixed loss } K_o + \text{Linearly varying loss } K_1 \cdot P_o + \text{Square variable loss } K_2 \cdot P_o^2$$

And efficiency η is the ratio of output to input powers, that is,

$$\eta = \frac{Power\ Output}{Power\ Input} = \frac{Po}{Po + Total\ Loss} = \frac{Po}{Po + K_0 + K_1 P_o + K_2 Po^2} \tag{3.8}$$

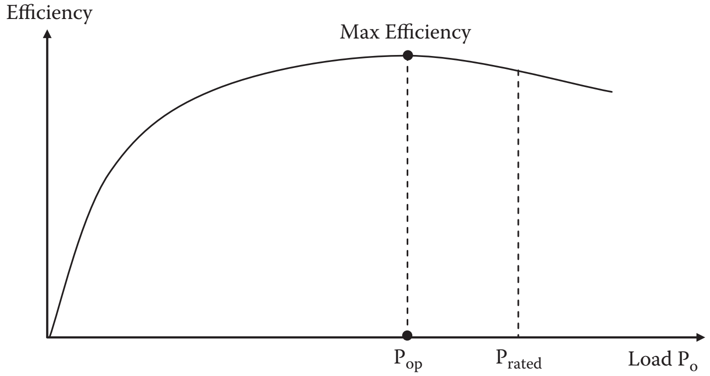


FIGURE 3.4 Efficiency versus load power output and maximum efficiency point.

The efficiency varies with load power per Equation (3.8), which is depicted in Figure 3.4. Due to the fixed loss, the efficiency is zero at zero load, and is low at light loads. The efficiency peaks at some load point, and then decreases due to high square-variable loss at high loads. We, therefore, deduce that there has to be a load point where the efficiency has the maximum value. To find that point, we find the derivative of η with respects to P_o and equate it to zero, that is, $d\eta/dP_o = 0$. Solving the resulting equation leads to $K_o = K_2 P_o^2$. That is, *the equipment efficiency is maximum* when *the fixed loss equals the square variable loss.*

It is noteworthy that the linearly varying loss in diode-type devices does not matter in determining the maximum efficiency point. It just degrades the efficiency in a fixed ratio, that is, $\eta = P_o/(P_o + K_1 \cdot P_o) = 1/(1 + K_1)$, which does not vary with load.

For equipment such as transformer, motor, and generator, the fixed loss comes from the magnetic loss in the core operating at constant voltage (constant flux amplitude), and the square-variable loss comes from the I^2R loss in winding conductor resistance. Thus, in electrical machines, an important conclusion is stated as follows:

At the maximum efficiency point:

$$\text{Fixed core loss} = \text{Square-variable conductor loss} \tag{3.9}$$

For saving energy, the equipment should be operated at the load P_{op} in Figure 3.4 at which the efficiency is maximum. This load point is usually at 75%–80% of the full rated load. Large power users often specify the maximum efficiency point at the power level at which the equipment is likely to operate most of the time, and specify the rated load equal to the maximum continuous load capability needed during the equipment service life.

Example 3.3

An electrical machine has a no-load loss of 10 kW and a full-load loss of 26 kW when delivering 500 kW. Determine its (i) full load efficiency and (ii) maximum efficiency with the corresponding load point in percentage of the full load and in kW.

SOLUTION

Full-load efficiency = 500 ÷ (500 + 26) = 0.9506

The no-load power loss of 10 kW remains constant as long as the machine remains connected to the lines, regardless of its level of loading. Therefore, the I^2R power loss in conductor at 100% load = 26 – 10 = 16 kW, which varies with load-squared. Per Equation (3.9), the efficiency is maximum when conductor loss is equal to the fixed loss, that is, when 10 = 16 × load^2. This gives load = $(10 \div 16)^{1/2}$ = 0.79 or 79% of the full load, or 500 × 0.79 = 395 kW.

Maximum efficiency at 79% load = 395 ÷ (395 + 10 + 10) = 0.9518

This is better than 0.9506 at full load. Although this is a small difference, it can add up to good energy savings over the year if the machine is large and works long hours every day of the year.

3.6 THEVENIN EQUIVALENT SOURCE MODEL

Any electrical power source (generator, transformer, battery, fuel cell, etc.), or for that matter, any complex electrical network, can be modeled by an internal source voltage V_s (also called Thevenin voltage V_{Th}) in series with an internal source impedance Z_s (also called Thevenin impedance Z_{Th}) as shown in Figure 3.5. If no load were connected, the terminal voltage would be the same as the source voltage. However, when the source is loaded as depicted in Figure 3.6(a), the terminal voltage at the load point is given by the source voltage less the voltage drop in Z_s under the load current I, that is,

$$\tilde{V}_T = \tilde{V}_s - \tilde{I} \times \tilde{Z}_s \tag{3.10}$$

The terminal voltage droop line with increasing load current is shown in Figure 3.6(b).

The Thevenin equivalent source parameters $\tilde{V}_s$ and $\tilde{Z}_s$ can be found by calculations or by test. Referring to Figure 3.6 again, if we measure the voltage at no-load and voltage drop $\tilde{I} \times Z_s$ at some load current, then it is easy to see that

$$\tilde{V}_s = \tilde{V}_T \text{ at no-load, and } \tilde{Z}_s = \tilde{V}_{drop} \div \tilde{I} = (\tilde{V}_{\text{T at no-load}} - \tilde{V}_{\text{T at load I}}) \div I \tag{3.11}$$

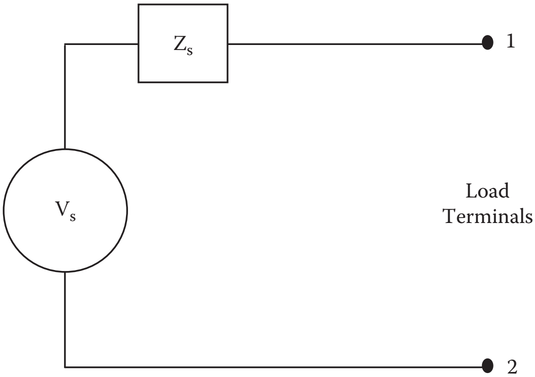
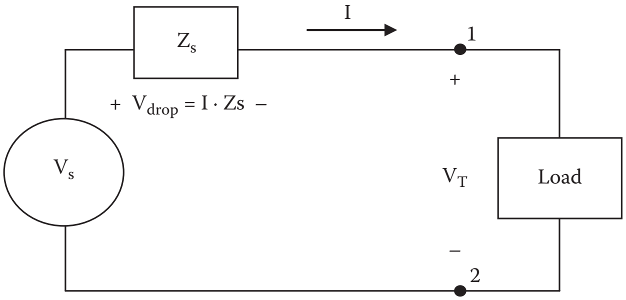
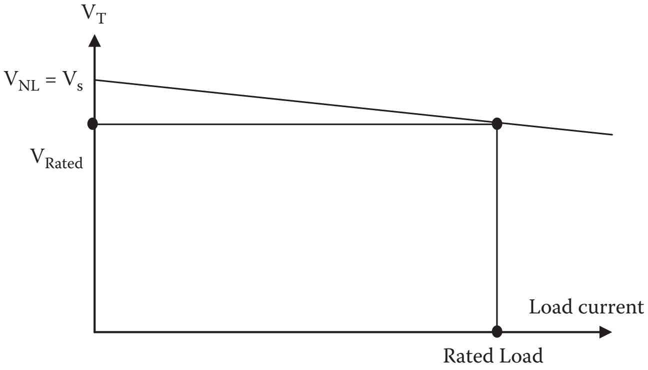


FIGURE 3.5 Thevenin equivalent source model of any electrical power source.



(a) Thevenin equivalent source with load current



(b) Terminal voltage droops with load

FIGURE 3.6 Drooping voltage with increasing load current at output terminals.

Since Equation (3.11) is a phasor relation in ac, mere voltmeter and ammeter readings are not enough to determine $\tilde{V}_s$ and $\tilde{Z}_s$. Some means of accounting for the phase angles between the voltage and current (e.g., oscilloscope) may be needed to determine $\tilde{V}_s$ and $\tilde{Z}_s$. In dc circuits, however, the voltmeter and ammeter alone can establish the Thevenin source parameters V_s and R_s.

Example 3.4

If a dc source with an open-circuit voltage of 240 V drops to 220 V under 25 A load, determine its Thevenin parameters, that is, the source voltage V_s and the source resistance R_s.

SOLUTION

The open-circuit voltage is the Thevenin source voltage, that is, $V_s = 240$ V.

Since the voltage drop of 240 − 220 = 20 V at 25 A load current must be in the internal source resistance, the Thevenin source resistance $R_s = 20\text{ V} \div 25\text{ A} = 0.8\ \Omega$.

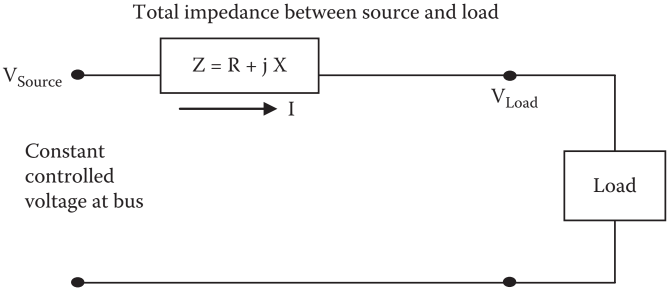
3.7 VOLTAGE DROP AND REGULATION

The rated voltage of an electrical power source is defined as the terminal voltage at full rated load current. The output terminal voltage rises with decreasing load current, rising to its maximum value at zero load (no-load) as indicated by Equation (3.10) and depicted in Figure 3.6(b). The percentage voltage regulation of a power source is defined as

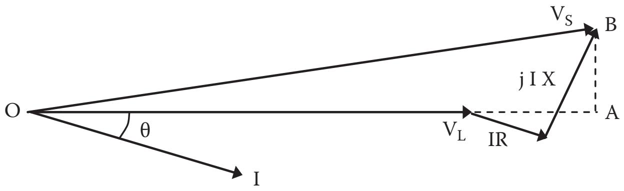
$$\% \text{ Voltage Regulation} = \frac{V_{\text{noload}} - V_{\text{ratedload}}}{V_{\text{ratedload}}} \times 100 \tag{3.12}$$

Voltage regulation is a common term used to describe the voltage change that a load connected to the transformer or generator terminals, or at the end of a long cable, would see. It indicates the voltage variation in the terminal voltage of the source from fully loaded to fully unloaded conditions. All loads should be designed to withstand this voltage variation. Alternatively, if a constant voltage is desired at the load terminals, the source voltage must be regulated by automatic feedback control in the percentage range equal to the percentage voltage regulation. Therefore, the *voltage regulation* defines the load voltage deviation without the voltage regulator, or the voltage range over which the regulator must control the source voltage.

For the most generic analysis that is repeatedly used in many chapters in this book, we can represent the total series impedance $Z = R + jX$ ohms between the source and the load as shown in Figure 3.7(a). The total Z can be due to one or more



(a) Load circuit



(b) Phasor diagram

FIGURE 3.7 Voltage drop in series impedance and voltage regulation.

impedances of the generator, transformer, and cable. The load draws current I at lagging pf cosθ at the load terminal voltage V_{Load}, which must be equal to the source voltage V_{source} minus the voltage drop $I \times Z$ in the series impedance. Therefore, the source voltage, which is maintained constant, must be equal to

$$\tilde{V}_{\text{Source}} = \tilde{V}_{\text{Load}} + \tilde{I} \times (R + jX) \tag{3.13}$$

The phasor diagram of Equation (3.13) is shown in Figure 3.7(b), the trigonometry of which gives the exact magnitude of the source voltage for any pf = cosθ,

$$V_S^2 = \text{OA}^2 + \text{AB}^2 = (V_L + I \times R\cos\theta + I \times X\sin\theta)^2 + (I \times X\cos\theta - I \times R\sin\theta)^2 \tag{3.14}$$

For a typical pf around 0.85, we can approximately write the source voltage $V_s = \text{OB} = \text{OA}$, and subsequently the voltage drop magnitude as

$$V_{drop} = V_S - V_L = \text{OA} - V_L = I \times R\cos\theta + I \times X\sin\theta = I \times (R\cos\theta + X\sin\theta) \tag{3.15}$$

We can take the following as the effective impedance between the source and the load:

$$Z_{\text{eff}} = R\cos\theta + X\sin\theta \tag{3.16}$$

which, when multiplied with current I gives the voltage drop by a simple application of Ohm's law:

$$V_{\text{drop}} = I \times Z_{\text{eff}} \tag{3.17}$$

All calculations are done using volts, amperes, and ohms per phase, and taking θ *positive for lagging pf* and negative for leading pf. Equation (3.16) is easy to remember: R (real part of Z) × cosθ which gives the real power + X (reactive part of Z) × sinθ which gives the reactive power. And, of course, the voltage drop is linearly proportional to the current and series impedance, as in Ohm's law. Therefore, in practical power circuits with a pf of around 0.85, we can explicitly write Equation (3.18) below in terms of the pf,

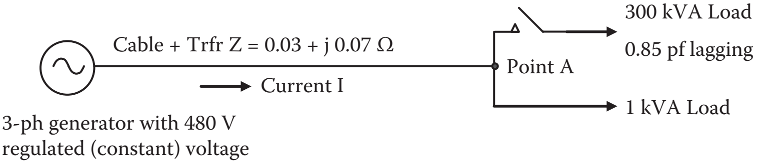
$$V_{drop} = I\{R \cdot pf + X \cdot \sqrt{1 - pf^2}\} \text{ for all practical power factors} \tag{3.18}$$

$$\text{Then, percentage voltage regulation} = (V_{\text{drop}}/V_L) \times 100\% \tag{3.19}$$

For $R \ll X$, Equation (3.18) is valid for a wide range of pf's ranging from 0 lagging to 0.9 leading, except that the algebraic sign before the reactance X is + for lagging pf and – for leading pf. Although this approximation is fairly accurate for most practical use, we must remember that the exact voltage drop is given by V_s from Equation (3.14) minus V_L.

Example 3.5

A 3-phase, 480 V, 5000 kVA generator connects to many feeders, one of which takes 1-phase power to two pieces of equipment at the other end via a cable and transformer with combined Z = 0.03 + j 0.07 Ω. The voltage at point A at the receiving end rises and falls as a large 1-phase, 300 kVA load varies over time. The small 1 kVA load remains online continuously, and must accommodate the voltage variations at point A with the 300 kVA load fully on or fully off. Determine the voltage range that the 1 kVA load will see, over which it must perform as intended.



SOLUTION

The 300 kVA load current I = 300 kVA × 1000 ÷ (√3 × 480) = 361 A/ph.

Assuming 0.85 pf lagging (i.e., cosθ = 0.85 and sinθ = 0.5268), Equation (3.15) gives the voltage drop in the cable

$$V_{\text{drop}} = 361\ (0.03 \times 0.85 + 0.07 \times 0.5268) = 22.5\ \text{V/ph} = 22.5\ \sqrt{3} = 39\ V_{LL}$$

$$\therefore \text{ Voltage regulation} = 39 \div 480 = 0.081 \text{ or } 8.1\% \text{ of } 480\ \text{V (a rather high percentage).}$$

$$\therefore \text{ Voltage at point A when the 300 kVA load is fully on} = 480 - 39 = 441\ V_{LL}$$

The voltage at point A when the 300 kVA load is fully off = 480 V_{LL} (the same as the generator voltage, since there is negligible current and hence negligible voltage drop in the cable).

In order to supply all loads, small and large, at a reasonable voltage range, the voltage variation at point A must be kept below 5% from full load to no load by proper design of the system.

3.8 LOAD SHARING AMONG SOURCES

Two or more electrical power sources are often connected in parallel to increase the load capability or reliability of the system. In the case of two sources, both sources must meet the following conditions before they can be connected in parallel to share the load:

- Voltage magnitudes must be equal within a few percents.
- Voltage polarities must be the same (+ connected to + and – connected to –).
- The phase sequence of 3-phase sources must be the same.
- Frequencies of ac sources must be equal (else voltages will get out of phase).

3.8.1 STATIC SOURCES IN PARALLEL

The load shared by two static sources—such as two transformers, batteries, fuel cells, etc.—is determined by their terminal voltage droop lines shown in Figure 3.8. With two static sources 1 and 2 sharing the load in parallel, their individual terminal voltages are given by

$$V_{T1} = V_{s1} - K_1 \cdot P_1 \text{ and } V_{T2} = V_{s2} - K_2 \cdot P_2 \tag{3.20}$$

where K_1 and K_2 = voltage droop rates = $\Delta V/\Delta P$ in volts/kW, and V_{s1} and V_{s2} are the internal voltages of source 1 and 2, respectively, that will be equal to their respective voltages at zero load.

By parallel connection at a common bus, we force both sources to share the load such that they have the same terminal voltage equal to the bus voltage, that is, $V_{T1} = V_{T2} = V_{\text{bus}}$

$$\therefore\ V_{s1} - K_1 \cdot P_1 = V_{s2} - K_2 \cdot P_2 = V_{\text{bus}} \tag{3.21}$$

$$\text{We must also have the total load, } P_{\text{Total}} = P_1 + P_2 \tag{3.22}$$

The parameters V_{s1}, K_1, V_{s2}, and K_2 are the source parameters given or known to us from the manufacturer's technical data sheets. Therefore, solving Equations (3.21) and (3.22) simultaneously gives P_1 and P_2 for a given P_{Total}.

Graphically, using the droop lines of source 1 and 2 shown in Figure 3.8, the load shared by two individual sources can be determined by trial and error. In that process, we find P_1 and P_2 at different values of V_{bus} until their sum $P_1 + P_2$ adds up to P_{Total}. As an example, for V_{bus} shown by dotted line in Figure 3.8, the load shared by source 1 is P_1 and that by source 2 is P_2. If P_1 and P_2 add up to P_{Total}, the dotted line is indeed the V_{bus} value. Otherwise, we try another value of V_{bus} until P_1 and P_2 add

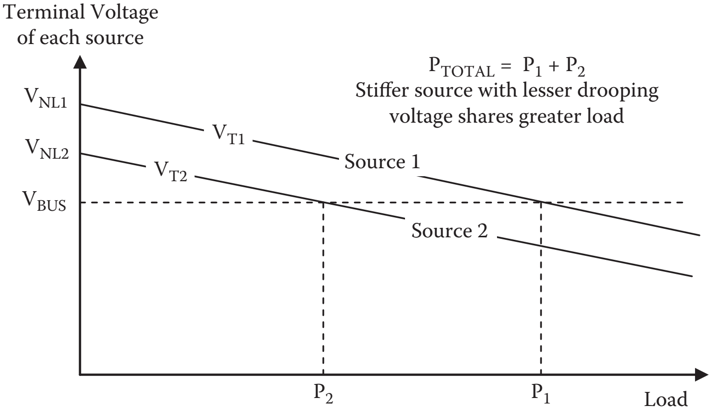


FIGURE 3.8 Voltage droop lines for two static power sources sharing load in parallel.

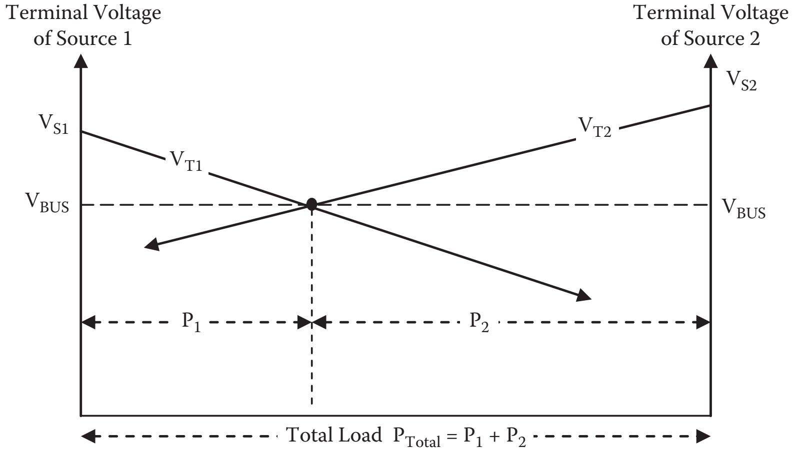
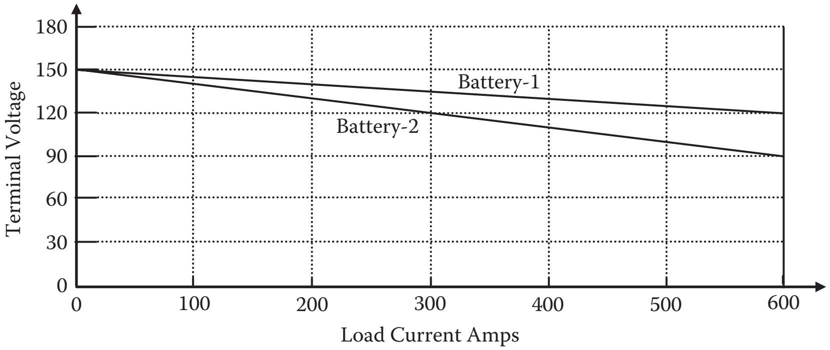


FIGURE 3.9 One-step method of determining load shared by two static sources in parallel.

up to P_{Total}. When the fun of trial and error fades, we can use the graphical method shown in Figure 3.9 to determine the load sharing in one step. In this method, the two voltage droop lines with vertical axes V_{s1} and V_{s2} are drawn apart by P_{Total} on the horizontal load axis, and the droop lines are drawn in the opposite directions. Then, the point of their intersection gives the individual loads P_1 and P_2 and also the bus voltage V_{bus} in one step.

Example 3.6

Battery-1 and Battery-2 have their terminal voltage versus load current droop lines as shown below. Determine analytically the load current shared by each battery and the battery bus voltage if they share a total load current of (i) 900 A and (ii) 600 A. In case of the 600 A total load, verify your answers by the one-step method with the Battery-2 line redrawn as needed.



SOLUTION

We first determine the droop line equations from Figure E3.6. Battery-1 voltage drops from 150 V at zero load to 120 V at 600 A, which gives

$$V_{T1} = 150 - \frac{150-120}{600} I_1 = 150 - 0.05 I_1$$

Similarly, we have $V_{T2} = 150 - \frac{150-90}{600} I_2 = 150 - 0.10 I_2$

(i) For a 900 A total load at a common bus voltage V_{bus}, we write

$$V_{T1} = 150 - 0.05\, I_1 = V_{T2} = 150 - 0.10\, I_2 = V_{bus} \qquad \text{(a)}$$

$$\text{And the total load current } I_1 + I_2 = 900 \text{ A} \qquad \text{(b)}$$

The simultaneous solution of Equations (a) and (b) gives

$$I_1 = 600 \text{ A},\ I_2 = 300 \text{ A, and } V_{bus} = 120 \text{ V}$$

(ii) For 600 A total current, we repeat the foregoing procedure, which leads to

$$I_1 = 400 \text{ A},\ I_2 = 200 \text{ A, and } V_{bus} = 130 \text{ V}$$

As the load current decreases from 900 to 600 A, we note that the battery bus voltage rises from 120 to 130 V, as expected.

These answers can also be derived graphically in one step by plotting the Battery-2 line backward starting from the right-hand side and drooping as we increase its load current from 0 to 600 A toward the left-hand side.

3.8.2 Load Adjustment

The load sharing can be adjusted by raising or lowering the voltage droop line of one or both sources working in parallel. This is generally done by changing the no-load voltage, which in turn raises or lowers the terminal voltage versus load line while maintaining the droop rate. In transformers, this is easily done by changing the taps on the primary or secondary side. In batteries and fuel cells, it is done by adding or subtracting one or more cells in one stack. For two sources in parallel to share an equal load, it is important to have their no-load voltages and the droop rates equal or as close as possible.

For two dc generators in parallel, the foregoing analysis for static sources also applies. For two motors in parallel sharing a torque load on a common shaft, the analysis presented here applies equally well after substituting the voltages in Equation (3.21) with individual motor speeds versus horsepower load. The horsepower sharing between two locomotive engines hauling a long train can also be determined by using their respective speed versus horsepower droop lines.

The load sharing among ac generators is complex, as both the real and the reactive powers are shared. This topic is covered in Chapter 4.

3.9 POWER RATING OF EQUIPMENT

The following applies to all equipment (electrical or mechanical) commonly found in electrical power plants.

The electrical power ratings of generators and transformers are stated in kVA or MVA (not in the real power kW or MW). These machines can deliver rated voltage and rated current, but the real power delivered by the machine depends on the power factor, which is primarily determined by the nature of the load connected. We often see the electrical power plant capacity expressed in MW, since it refers to the mechanical output rating of the prime mover, which is the dominant investment in the power plant. Moreover, the MW rating also determines the plant's fuel consumption rate, such as the number of coal cars coming daily in the thermal power plant. That is why the MW rating draws more attention than the MVA rating.

- Nameplate rating refers to the *full load output*, and input = output + internal losses.
- Efficiency = output power/input power, which varies with the output power.
- Cable rating is stated in terms of the voltage and the current-carrying capacity (ampacity).
- The mechanical power rating of motor or engine is stated in hp in British units or kW in SI units.
- Equipment rating is limited by the operating temperature limit on insulation in electrical equipment and the cooling method used in the design.

The service factor (SF), indicated on the equipment nameplate, is the equipment's ability to carry temporary overload without thermal or mechanical failure or measurable reduction in life. Industry standards require all electrical equipment to have an SF of at least 1.15 typically for 2 hours. It means the equipment can carry 15% overload for 2 h. However, the SF is often misinterpreted as the factor by which we can continuously overload the motor by 15%. If it is continuously overloaded by 15%, the equipment may run hotter than the design temperature by more than 10°C, reducing the service life to less than one-half, which is a significant penalty to pay.

3.9.1 TEMPERATURE RISE UNDER LOAD

The power rating of the electrical equipment is limited by the temperature rise above the ambient air that cools the equipment. The industry-standard ambient air temperature is 40°C. It can be higher, up to 65°C, in special places such as the boiler room uptake. The maximum permissible operating temperature depends on the class of insulation used in the equipment. For example, a temperature rise of 80°C is permitted in equipments with class B insulation. In 40°C standard ambient air, it would have an operating temperature of 80°C + 40°C = 120°C. For some insulations, such as Nomex^{TM}, the allowable operating temperature can be up to 200°C.

The temperature rise is determined by the cooling medium and the surface area available for dissipating the internal power loss that heats the equipment body.

Air-cooled equipment dissipates the internal heat mostly by convection and radiation, and negligibly by conduction. For electrical equipment normally operating around 100°C, the temperature rise above the ambient air is approximately given by

$$\Delta T_{^\circ C} = K \times (\text{watts})^{0.8} \tag{3.23}$$

where watts = power loss to be dissipated and K = constant for a given body.

3.9.2 SERVICE LIFE UNDER OVERLOAD

During an overload, the power loss in the conductor rises with the load squared, and the temperature also rises according to Equation (3.23). Overloading above the rated temperature limit reduces the equipment life to about ½ for every 10°C rise above the design temperature. Mathematically,

$$\mathit{Actual\ Life} = \frac{\mathit{Rated\ Life}}{2^{\left(\Delta T/10\right)}} \tag{3.24}$$

where $\Delta T = \Delta T_a - \Delta T_R$ = Temperature rise above the rated design temperature rise

ΔT_a = Actual temperature rise above ambient air at the operating load
ΔT_R = Design temperature rise above ambient air at rated load

This 10°C rule for half-life is based on typical properties of insulations normally used in electrical machines. Experience on other electrical components, such as power electronics devices, suggest a different rule, such as every 7°C rise for reducing the life to one-half.

Example 3.7

A cable is designed for a 30-year life with a 50°C rise in 40°C ambient air at rated load. If it is continuously overloaded by 15%, determine its new expected life.

SOLUTION

The power loss in cable is only due to resistance, and hence it varies as the load current squared. Denoting the rated and overload conditions by suffixes 1 and 2, respectively, the power loss at 15% overload is given by $W_2 = 1.15^2 \times W_1 = 1.3225\ W_1$. With the same heat dissipation area, Equation (3.23) in the ratio gives

$$\frac{\Delta T_2}{\Delta T_1} = \left(\frac{W_2}{W_1}\right)^{0.8} = 1.3225^{0.8} = 1.25$$

Since $\Delta T_1 = 50$°C, we have $\Delta T_2 = 1.25 \times 50 = 62.5$°C, and $\Delta T_2 - \Delta T_1 = 62.5 - 50 = 12.5$°C.

The cable, therefore, operates 12.5°C hotter than the design temperature. If this continues until the end, its life gets reduced by the 10°C rule in Equation (3.24), that is,

$$\textit{Life at overload} = \frac{30}{2^{\left(\frac{12.5}{10}\right)}} = 12.61 \text{ years.}$$

This is a significant reduction of life, which illustrates that continuous overload, even by 10%–15%, is not economical. That is why 15% overload on electrical equipment is limited to an hour or two, and that also only on occasion.

3.10 TEMPERATURE EFFECT ON RESISTANCE

The conductor resistance (usually in ohms) rises with the operating temperature. The conductor resistance R_2 at a new temperature T_2 can be derived from a known resistance R_1 at T_1 as follows:

$$\text{For the conductor, } R_2 = R_1 \times \{1 + \alpha\,(T_2 - T_1)\} \tag{3.25}$$

where α = temperature coefficient of conductor resistance = 0.0039 per °C for both copper and aluminum of electrical grade.

The insulation resistance (usually in megaohms) drops at a higher temperature. For the equipment to function at the rated voltage, the insulation must be sound, with a certain minimum resistance value measured by a megger applied between the coil conductor and ground. The industry standards require the insulation resistance, adjusted to 40°C if measured at a different temperature, to be greater than the minimum recommended value R_{min}. The IEEE Standard 43-2000 suggests the following R_{min} values at 40°C for different equipments, where kV_{LL}= line-to-line rated voltage of the equipment in kilovolts:

$R_{\text{min}} = \text{kV}_{LL} + 1\ \text{M}\Omega$ for rotating machines built before 1970
$R_{\text{min}} = 5\ \text{M}\Omega$ for new rotating machines rated below 1 kV_{LL}
$R_{\text{min}} = 100\ \text{M}\Omega$ for new rotating machines rated above 1 kV_{LL}
$R_{\text{min}} = 2\ \text{kV}_{LL}\ \text{M}\Omega$ for all transformer coils

If the insulation resistance were measured at temperature T, then its value adjusted at 40°C must be compared with the R_{min} value that is required at the standard ambient of 40°C. The adjusted value is given by

$$\text{R}_{40^\circ C} = \text{K}_T \times \text{R}_T \tag{3.26}$$

where R_T = insulation resistance measured at T°C, and K_T = temperature correction factor for insulation resistance. The industry data suggests that the factor K_T doubles for every 10°C rise in insulation temperature above the standard ambient of 40°C, that is,

$$K_T = 2^{\left(\frac{T-40}{10}\right)} \quad \text{where } T = \text{actual temperature of insulation in °C} \tag{3.27}$$

Example 3.8

The stator winding insulation resistance of a 100 hp, 460 V motor was measured at 10 MΩ when sitting idle at a normal room temperature of 20°C. Determine whether this motor can be connected to the line per the IEEE Standard.

SOLUTION

First, for this motor to be capable of being connected to the line voltage, the winding insulation must have $R_{\min} = 5$ MΩ at 40°C. The measured value of 10 MΩ at 20°C must be first adjusted to 40°C before comparing with $R_{\min}$ and making a judgment about its service worthiness. Using Equation (3.27), the temperature correction factor

$$K_T = 2^{\frac{20-40}{10}} = 2^{-2} = 0.25, \text{ which gives } R_{40^\circ\mathrm{C}} = 0.25 \times 10 = 2.5 \text{ M}\Omega.$$

Since this is less than the 5 MΩ minimum required in service, the motor winding insulation does not meet the industry standard. The stator winding must be rewound and varnish-impregnated before connecting to the lines.

PROBLEMS

Problem 3.1: Determine the rms value of voltage induced in an 80-turn coil on a 30 cm × 25 cm cross section core with peak flux density 1.7 T alternating at 60 Hz.

Problem 3.2: Two parallel 1-phase rectangular bus bars 2 cm apart at their center lines experience 30,000 A_{rms} current when a load gets short-circuited. Determine the peak mechanical force per meter length of the bus bars. Assume the bus bar shape factor $K = 0.85$.

Problem 3.3: A transformer has a no-load loss of 30 kW and full-load loss of 90 kW when delivering 3 MW power. Determine its (i) full load efficiency, and (ii) maximum efficiency with the corresponding loading level in percentage of the full load and in kW.

Problem 3.4: A dc source with open-circuit voltage of 120 V drops to 105 V under 15 A load. Determine its Thevenin parameters, that is, the source voltage V_s and the source resistance R_s.

Problem 3.5: From a 3-phase, 480 V generator switchboard, a feeder takes 3-phase power to two equipments at the other end via cable and transformer with the combined series impedance $Z = 0.04 + j\,0.09$ Ω per phase. The voltage at receiving end point A rises and falls as a large 3-phase, 500 kVA load at point A varies over time. The small 1 kVA load, also connected at point A, remains online continuously, and must accommodate the voltage variations at point A with the 500 kVA load fully on or fully off. Determine the voltage range that the 1 kVA load will see, over which it must perform within its specifications.

Problem 3.6: Two batteries' terminal voltage versus load current droop lines are shown in Figure P3.6. Determine analytically the load current shared by each battery and the battery bus voltage if they share a total load current of (i) 500 A and (ii) 800 A. In the case of 500 A total load, verify your answers by a one-step method with the Battery-2 line redrawn backward as needed.

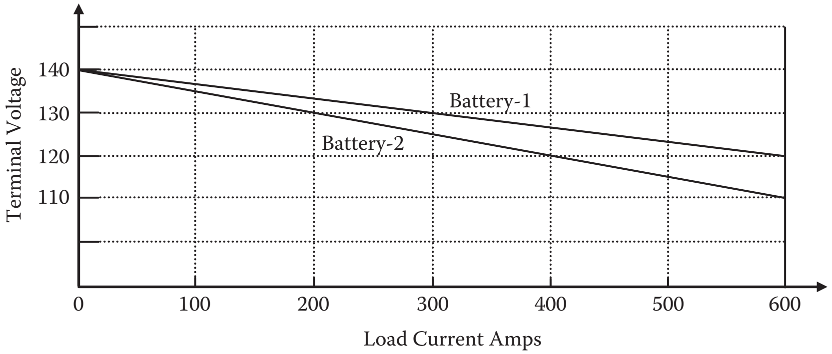


FIGURE P3.6

Problem 3.7: A piece of electrical equipment is designed for a 25-year life with a 70°C rise in 40°C ambient air at rated load. If it is continuously overloaded by 10%, determine its new expected life.

Problem 3.8: A 500 hp, 460 V motor winding insulation resistance was measured at 15 MΩ when sitting idle at a normal room temperature of 25°C. Determine whether this motor can continue in service per the IEEE standard.

Problem 3.9: A 75 kVA, 1-phase, 60 Hz transformer primary coil with 100 turns is connected to a 265 V source. Determine the peak flux in the core. If a magnetomotive force of 500 ampere-turns is needed to establish this flux, determine the core excitation current.

Problem 3.10: A feeder transformer delivers full load current at its output voltage of 460 V. When the load is removed, the output voltage rises to 480 V. Determine the voltage regulation of this transformer.

Problem 3.11: A generator winding delivering a rated load of 1000 kVA rises to 50˚C above 40˚C ambient air, making its operating temperature 40°C + 50°C = 90°C. If the generator is overloaded by 30%, determine its operating temperature. If always operated at 30% overload, determine the generator life if the rated design life is 25 years.

Problem 3.12: A 4160 V generator phase coil insulation resistance is 20 MΩ measured at 70°C soon after it was tripped. Compare this with the minimum required by the industry standard, and state whether this generator is good to continue in operation.

Problem 3.13: A 1-phase, 500 kVA, 440/120 V ship service transformer feeds service loads via bus duct with two bus bars each ½ x 3 in. in cross section and spaced ½ in. apart. An electrical system study has concluded that the worst-case fault current in the bus is 30,000 A at the first peak. Determine the peak mechanical force between the bus bars that will cause deflection and bending stress between the supports. Assume the bus bar shape factor *K* is 0.7.

Problem 3.14: A power electronics component designed to last 25 years under a rated load is always operated 5°C above the design temperature limit.

Determine its expected life if the 8°C rule for ½ life is found applicable to electronics components.

Problem 3.15: A 1000 kVA, 1-phase transformer operating at 85% power factor lagging at full rated load has a conductor power loss of 10 kW and core loss of 5 kW. Determine its maximum possible efficiency and the corresponding load level as a percentage of the rated load.

QUESTIONS

Question 3.1 Identify at least two electrical products (other than the transformer, motor, generator, and those identified in Section 3.1) that work on Faraday's law of electromagnetic induction.

Question 3.2 Using Faraday's law, explain why we lose radio and cell phone communications while going through a long tunnel made of steel-reinforced concrete structure.

Question 3.3 Using Faraday's law, explain the purpose of the front door screen in the microwave oven.

Question 3.4 How would the magnetic loss in a 60 Hz motor built in the United States change when operated in Europe at 50 Hz, and vice versa?

Question 3.5 State in one sentence when the electrical equipment operates at the maximum efficiency.

Questions 3.6 How would you determine the Thevenin equivalent source parameters V_s and Z_s of the entire power grid bringing power to the wall outlet at your workplace? Answer separately assuming that (a) the source reactance is negligible, that is, $Z_s = R_s$ (a pure resistance), and (b) the source reactance is not negligible, that is, $Z_s = R_s + j\,X_s$.

Question 3.7 You have two 120 V batteries working in parallel, one old and one new in age each with 60 cells in series. You wish the new battery to share a greater load. What would you do?

Question 3.8 In a long locomotive going uphill with two engines pulling, how would you determine their horsepower load-sharing to meet the total traction load?

Question 3.9 At what rate does the electrical equipment's service life degrade at higher operating temperature? Identify specific reasons for the degradation.

FURTHER READING

Hubert, C.I. 2002. *Electrical Machines.* Upper Saddle River: Prentice Hall.

Bosela, T.R. 1997. *Introduction to Electrical Power System Technology*. Upper Saddle River: Prentice Hall.

Wildi, T. 2002. *Electrical Machines, Drives, and Power Systems.* Upper Saddle River: Prentice Hall.

4 AC Generator

Nearly all electrical power in the world is generated by the 3-phase synchronous generator, which is also known as the ac generator or the alternator. It consists of three stationary coils (called the stator, armature, or phase coils), which are physically separated in space by 120° from each other, and a rotor with a dc coil that produces the dc magnetic field. Both the stator and rotor coils are individually embedded in ferromagnetic cores with an air gap that is consistent with the electrical and mechanical design requirements. Figure 4.1 is a simplified cross section of a 3-phase generator with three stator coils and salient poles.

A thermodynamic prime mover drives the rotor, which generates voltage in each phase of the three identical stator coils. The 3-phase voltages are equal in magnitude but 120° out of phase in time (or in ωt to be precise). The stator coils are usually connected in 3-phase *Y*. That way, the conductors in the stator slots need to have insulation to the ground for only $1/\sqrt{3} = 0.577$ or 57.7% of the line voltage, and hence, more conductors can be packed in the slots. In the conventional generator, the dc excitation field current comes from a small separate exciter via slip rings and carbon brushes.

One mechanical revolution of a 2-pole rotor generates one electrical cycle (360 electrical degrees) in the stator coil voltage. In a 4-pole rotor, one mechanical revolution generates 4/2 = 2 electrical cycles (2 × 360 electrical degrees). A rotor with P number of poles driven at n rpm generates $P/2 \times (n/60)$ electrical cycles per second. Therefore, the generator frequency is given by

$$f = \frac{P}{2} \cdot \frac{n}{60} = \frac{n \cdot p}{120} \text{ cycles/sec (Hz)} \qquad (4.1)$$

The prime mover must drive the generator at a constant speed to generate power at a constant frequency (60 Hz in the United States and 50 Hz in Europe). We also note here for later use that with a P-pole rotor,

$$\text{One mechanical degree} = P/2 \text{ electrical degrees} \qquad (4.2)$$

4.1 TERMINAL PERFORMANCE

The performance of a 3-phase generator at the terminals is based on the 3-phase power fundamentals covered in Chapter 1. The generator typically supplies power to more than one load, some of which may be connected in *Y* and some in Δ. In such cases, the Δ-connected load impedance is usually converted into its equivalent *Y*-connected

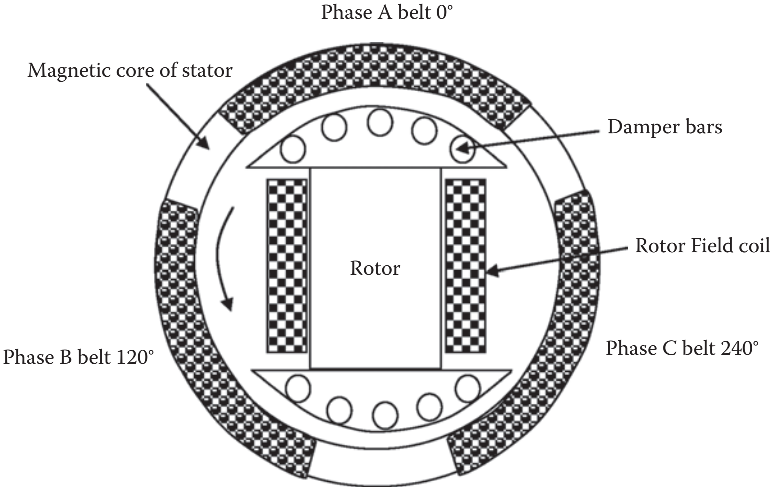


FIGURE 4.1 Three-phase synchronous generator cross section with salient pole.

impedance equal to one-third of the impedance value in Δ, and combined in parallel with other *Y*-connected loads. All calculations are then done on per-phase basis. This is illustrated in Examples 4.1 and 4.2.

Example 4.1

A 3-phase, 460 *V*, *Y*-connected generator is powering 500 kW balance 3-phase Δ-connected load at 0.85 pf lagging. Determine: (1) the line-to-line voltage and line current, (2) generator phase voltage and phase current, and (3) load phase voltage and phase current.

SOLUTION

As customarily implied, the generator voltage of 460 *V* is line-to-line. The line current is then derived from the 3-phase power Equation (1.29), that is, $P = 500 \times 1000 = \sqrt{3} \times 460 \times I_L \times 0.85$, which gives $I_L = 738.3$ A.

Referring to Figure 1.13, the *Y*-connected generator phase voltage = 460 $V \div \sqrt{3}$ = 265.6 *V*, and phase current = line current = 738.3 A.

Referring to Figure 1.14, the Δ-connected load phase voltage = line-to-line voltage = 460 *V*, and phase current = $738.3 \div \sqrt{3} = 426.3$ A.

Example 4.2

A balanced *Y*-connected load with $2 + j3$ ohms/ph (L-N) is connected in parallel with a balanced Δ-connected load with $9 + j12$ ohms/ph (L-L). Determine the combined total equivalent *Y*-connected impedance value per phase. If these loads were powered by *Y*-connected 480 V_{LL} generator, determine the current drawn from the generator lines.

SOLUTION

We first convert the balanced Δ-connected $9 + j12\ \Omega$ into an equivalent Y-connected load, which is ⅓ $(9 + j12) = 3 + j4\ \Omega$ per phase. This is in parallel with another Y-connected load with $2 + j3\ \Omega$ per phase. We now combine the two parallel Y-connected loads to obtain

$$Z_{\text{Total}} = \frac{1}{\dfrac{1}{3+j4}+\dfrac{1}{2+j3}} = \frac{(2+j3)(3+j4)}{(2+j3)+(3+j4)} = \frac{-6+j17}{5+j7}$$

$$= \frac{18.03\angle(180-70.56)}{8.60\angle 54.46} = 2.1\angle 55 = 1.2 + j1.72 \quad \Omega/\text{phase}$$

$$\therefore \text{ Generator phase current (also line current)} = \frac{480\big/\sqrt{3}}{2.1\angle 55^\circ} = 132\angle -55^\circ \text{ A.}$$

4.2 ELECTRICAL MODEL

The equivalent electrical model of the ac generator is shown in Figure 4.2, where

E_f = emf induced inside the armature coil due to rotor field
R_a = resistance of the armature coil (usually negligible)
X_s = synchronous reactance of the armature coil
V_T = voltage at the output terminals
I = load current (usually lagging the terminal voltage)

If n = rotor speed in rpm, φ_p = flux per pole, I_f = field current, and K_{m1} and K_{m2} are the machine constants, then in absence of the magnetic saturation

$$E_f = K_{m1}\, n\, \varphi_p = K_{m2}\, n I_f \tag{4.3}$$

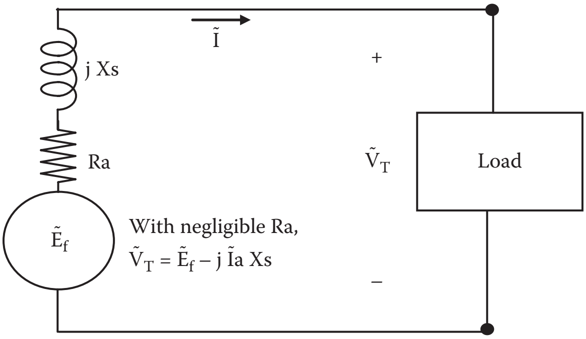


FIGURE 4.2 AC generator equivalent circuit model with load.

The terminal voltage is equal to E_f induced in the phase coil minus the voltage drop in R_a and X_s. In the phasor sense, it is given by

$$\tilde{V}_T = \tilde{E}_f - \tilde{I}(R_a + jX_s) = \tilde{E}_f - j\tilde{I}X_s \tag{4.4}$$

The value of R_a, being generally much smaller than X_s, is often ignored to simplify the analysis, as done on the right-hand side of Equation (4.4).

4.3 ELECTRICAL POWER OUTPUT

The performance analysis of a 3-phase machine is always conducted on a per-phase basis since three phases are identical except the 120° phase difference. If the generator is supplying a load current I lagging the terminal voltage V_T by phase angle θ, the phasor diagram of Equation (4.4), all with per phase values, which ignores the armature resistance R_a, is shown in Figure 4.3.

The power generated by each phase, P, is $V_T I \cos\theta$. Using the trigonometry of the phasor diagram, $IX_s\cos\theta = E_f\sin\delta$, or $I\cos\theta = E_f\sin\delta / X_s$, which leads to

$$P = V_T \cdot (E_f \sin\delta / X_s) \text{ or } P = \frac{E_f V_T}{X_s}\sin\delta = P_{\max}\sin\delta \tag{4.5}$$

$$\text{where } P_{\max} = \frac{E_f V_T}{X_s} \text{ watts/phase} \tag{4.6}$$

Since the power output depends on the angle δ between V_T and E_f, δ is known as the power angle (measured in electrical degrees).

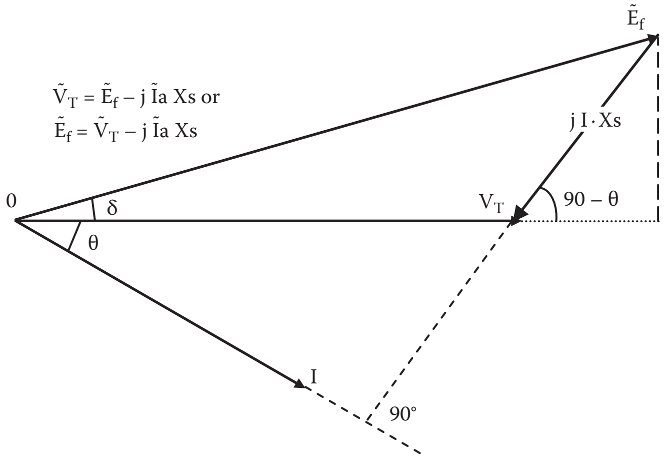


FIGURE 4.3 Phasor diagram of ac generator with negligible armature resistance.

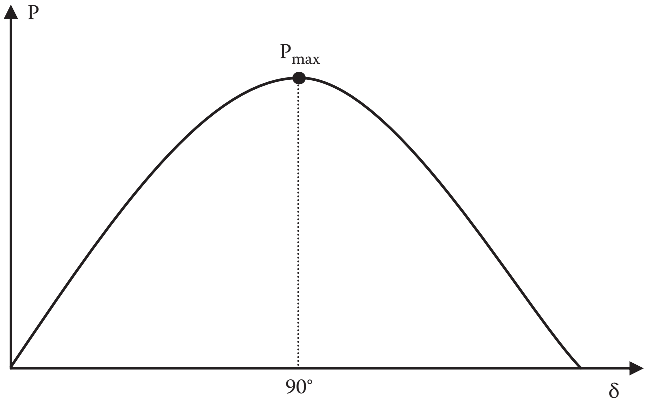


FIGURE 4.4 AC generator power output versus power angle δ.

The 3-phase stator currents produce a magnetic field that rotates exactly at the same speed as the rotor field (called the synchronous speed), and δ is the physical angle between the magnetic center lines of the rotor field and the stator field, the rotor field leading the stator field by angle δ.

The output power versus power angle relation as per Equation (4.5) is a half-sine curve shown in Figure 4.4. An increase in output power results in an increase in power angle only up to $P_{\max}$ at δ = 90°. Beyond this limit, the rotor and stator fields would no longer follow each other in a magnetic lockstep and would step out of the synchronous mode of operation, that is, the machine would become unstable and unable to produce steady power. Therefore, $P_{\max}$ is called the steady-state stability limit (or the pullout power) of the machine. It occurs at δ = 90°, and is equal to $P_{\max} = \frac{E_f V_T}{X_s}$.

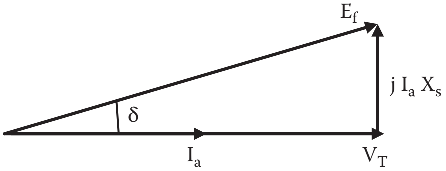
Example 4.3

A 30 MVA, 11 kV, 60 Hz, 3-phase, Y-connected synchronous generator has X_s = 2 Ω/ph and negligible R_a. Determine its power angle when delivering rated power at unity power factor.

SOLUTION

On a per-phase basis, V_T = 11 kV ÷ √3 = 6.351 kV, and ⅓ × 30,000 kVA = 6.351 kV × I_a.

∴ Armature current I_a = 1575 A. From the phasor diagram in the figure above at unity pf, we get



$$E_f = \sqrt{6351^2 + (1575\,x\,2)^2} = 7089\ V = 7.089\ \text{kV, and } P_{\max} = \frac{6.351 \times 7.089}{2} = 22.51\ \text{MW}$$

Power delivered by generator, $P = \frac{1}{3} \times 30$ MVA × 1.0 pf = 10 MW/phase

The power angle is then derived from Equation (4.5):

$P_{gen} = 10 = 22.51 \sin\delta$, which gives $\delta = 26.4°$

Example 4.4

A 3-phase, *Y*-connected synchronous generator is rated 10 MVA, 11 kV. Its resistance is negligible, and the synchronous reactance is 1.5 Ω per phase. Determine the generator voltage E_g when delivering rated MVA at 0.85 power factor lagging.

SOLUTION

E_g is an alternative symbol used in some books and national standards for the field excitation voltage E_f. With that noted, we make per-phase calculations as follows:

For *Y*-connection, $V_T = 11000 \div \sqrt{3} = 6351$ V/ph, which we take as the reference phasor.

Armature current at 0.85 power factor lagging, $\tilde{I}_a = 10 \times 10^6 \div (\sqrt{3} \times 11000) =$ 525 A at $\angle -\cos^{-1}0.85 = 31.8°$ lagging, that is, $I_a = 525\ \angle -31.8°$ A.

The generated voltage is then derived from

$$\tilde{E}_g = \tilde{V}_T + \tilde{I}_a \times jX_s = 6351\angle 0° + 525\angle -31.8° \times 1.5\angle 90°$$

$$= 6351\angle 0° + 787.5\angle 58.2° = 6766 + j\,669.3 = 6799\angle 5.65°\ V$$

So, the power angle δ is 5.65° at rated load and 0.85 pf lagging. The positive sign indicates that the generated voltage leads the terminals voltage, that is, the rotor magnetic field leads the stator magnetic field by angle δ. In the synchronous motor, the opposite is true. In rotating machines, the flux of the coil, which is the primary source of energy, leads the other coil's flux by mechanical angle equal to ($\delta_{\text{electrical}} \div$ number of pole pairs).

The electromechanical torque required from the prime mover to generate electrical power in all three phases is equal to (3 × power per phase) divided by the mechanical angular speed of rotor, that is,

$$T_{em} = \frac{3E_f V_T}{\omega_m Xs} \sin\delta\ \text{N-m} \tag{4.7}$$

where $\omega_m = 2\pi n/60$ mechanical rad/sec. The prime mover output kW or horsepower rating is then given by

$$\text{Prime mover } kW = \frac{T_{em(n.m)} \cdot \omega_{m(rad/\text{sec})}}{746 \cdot Generator\ efficiency} \quad \text{or} \quad \text{HP} = \frac{T_{em(Lb.ft)} \cdot Speed_{rpm}}{5252 \cdot Generator\ efficiency} \tag{4.8}$$

Example 4.5

A 5 MVA, 60 Hz, 6.6 kV, 4-pole, 95% efficient round rotor synchronous generator has the synchronous reactance of 7.0 Ω/phase and negligible armature resistance. When operating at unity power factor, determine (a) the maximum power it can deliver under steady state (no step load changes), (b) power angle (the rotor lead angle), and (c) back torque on the rotor in n-m and in lb-ft.

SOLUTION

(a) $V_{LN} = 6600 \div \sqrt{3} = 3810.5$ V, and $I_L = ⅓ \times 5 \times 10^6 \div 3810.5 = 437.4$ A in phase with voltage at unity pf. From the phasediagram at unity pf similar to Figure E4.3, we get

$$E_f = \{3810.5^2 + (437.4 \times 7.0)^2\}^{1/2} = 4888.2 \text{ V}$$

(b) $$P_{\max.3ph} = 3P_{\max.1ph} = 3x\frac{4888.2x3810.5}{7.0} = 7{,}982{,}785 \text{ watts} = 7.983 \text{ MW}$$

Using Equation (4.5) with power in 3-phase MW on both sides,
$5 = 7.983 \sin\delta$, which gives $\delta = 38.8°$

(c) Back torque T on rotor is derived from Power = $T \times \omega_{\text{mech}}$, where $\omega_{\text{mech}} = 2\pi \times 1800 \text{ rpm} \div 60 = 188.5$ mechanical rad/sec.

$$\therefore\ T = 5 \times 10^6 \text{ watts} \div 188.5 = 26{,}526 \text{ N-m.}$$

In British units, using Equation (4.8), HP = 5000 kW ÷ 0.746 = $T \times 1800 \div 5252$, which gives back torque $T = 19{,}556$ lb-ft.

Alternatively, we can use the unit conversion table in the front of the book, which relates as
$T_{n\text{-}m} = 1.3558 \times T_{lb\text{-}ft}$, or $T_{lb\text{-}ft} = 26{,}526 \div 1.3558 = 19{,}565$ lb-ft.

4.3.1 FIELD EXCITATION EFFECT

As per Equation (4.3), increasing the field current I_f increases E_f. One can visualize in Figure 4.3 that higher (longer) E_f in turn increases the terminal voltage V_T or the power factor angle θ, increasing the lagging reactive power output (kVAR) of the generator. The generator can supply more lagging kVARs to the load until I_f reaches its permissible limit. On the reverse, decreasing I_f decreases both E_f and the lagging kVAR output of the generator. The underexcitation can be adjusted such that $E_f = V_T$, when the machines would operate at unity power factor delivering the maximum real *kW* power to the load. However, this can exceed the prime mover capability, which is typically rated to drive the generator at a lagging power factor of 0.9. Further underexcitation can even make the generator operate at a leading power factor absorbing kVAR from the system. One downside of the underexcitation is lower E_f and lower steady-state power limit $P_{\max}$ as per Equation (4.5), and, in turn, a lower transient stability limit, which is discussed later.

If the load draws lagging kVAR, it must be supplied from the generator, or else the system cannot operate in a stable mode. The excitation of the machine in normal operation is adjusted such that it maintains the required terminal voltage at around 0.9 pf lagging to match with that of the load.

Example 4.6

One of six generators on a large cruise ship was repaired and brought back on line by synchronizing with the other five generators, which in a group can be assumed to make an infinite bus for the incoming generator. The incoming generator has the synchronous reactance $X_s = 1.5$ pu, $V_T = 1.0$ pu, and the field current $I_f = 1000$ A at the time of synchronization. After synchronization, while keeping the field current unchanged, the steam valves are adjusted such that it delivers 0.2 pu of its rated power.

(a) Determine the armature current in per unit value when delivering 20% power with rated $I_f = 1000$ A.
(b) If the field current I_f is now increased by 60% while keeping the steam input the same (i.e., the power output unchanged), determine the new armature current in per unit value.

SOLUTION

This example is posed in per unit values of power, voltage, current, and reactance. The per unit system is discussed in Chapter 6. Therefore, it is recommended that this example be read after reading Chapter 6 to appreciate the advantage of using the per unit system not only in the case of the transformer but also in overall power engineering studies. Until then we just say here that 1 *per unit* = 100 *percent*, that is, 1.0 pu = 100% of rated value.

(a) At synchronization, the incoming generator has no load, that is, no internal voltage drop, so it must have $E_f = V_T = 1.0$ pu $\angle 0°$ (say, reference phasor).

Then, $P = \frac{E_f V_T}{Xs}\sin\delta = \frac{1x1}{1.5}\sin\delta = 0.20$ pu, which gives $\delta = 17.46°$, and $E_f = 1\ \angle 17.46°$.

$$\therefore I_a = \frac{E_f - V_T}{jXs} = \frac{1\angle 17.46°\ - 1\angle 0°}{j1.5} = 0.2\angle 8.73°\ \text{pu.}$$

(b) The field current I_f is now increased by 60%, so the new E_f magnitude is now 1.6 × old value, which was 1.0.

$P = \frac{E_f V_T}{Xs} \sin\delta = \frac{1.6x1}{1.5} \sin\delta = 0.20$ pu, which gives $\delta = 10.8°$ and $E_f = 1.6 \angle 10.8°$ pu.

$$\therefore I_a = \frac{E_f - V_T}{jXs} = \frac{1.6\angle 10.8° \; - 1\angle 0°}{j1.5} = 0.43\angle -62.3° \text{ pu.}$$

We note here that, by increasing the field current while delivering the same output power, the power angle δ decreased, the armature current increased, and the power factor changed from 8.73° leading to 62.3° lagging with respect to V_T.

4.3.2 POWER CAPABILITY LIMITS

As described earlier, the excitation current not only affects the terminal voltage but also the operating power limit of the machine. Under normal excitation of the field coil, the permissible armature current limits the power output. Underexcitation of the field makes the machine operate near unity power factor, capable of delivering more real power, but that would overload the prime mover, as most prime movers are rated to drive the generator operating around 0.9 pf lagging. An overexcited machine can deliver more lagging kVARs until it reaches the heating limit of the field coil. All these limits jointly determine the generator's actual real power-generating capability as shown in Figure 4.5 with limiting boundaries coming from different performance limitations.

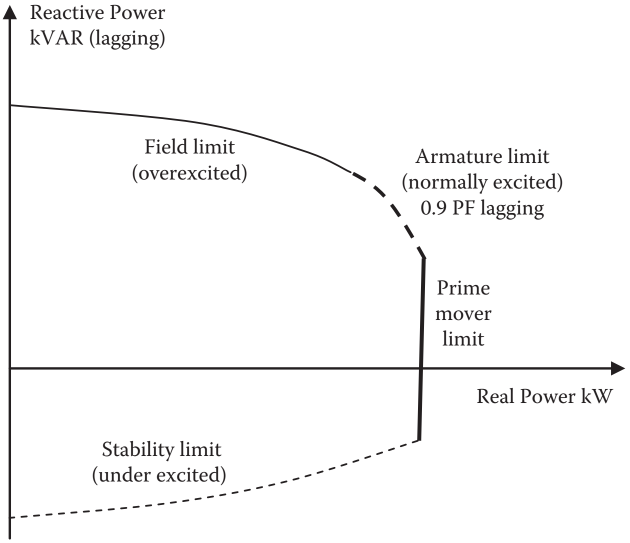
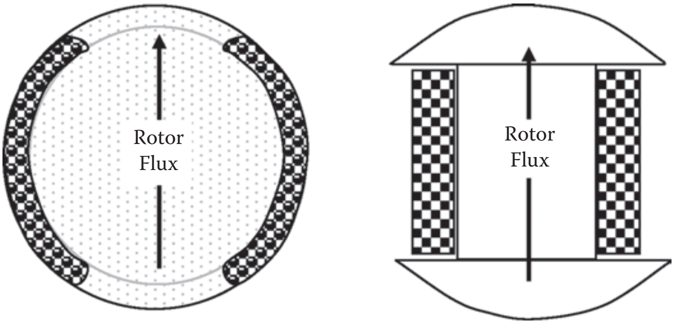


FIGURE 4.5 AC generator power capability curve with four limiting boundaries.



(a) Round rotor has field coil in slots on rotor periphery

(b) Salient pole rotor with separately wound field coil around poles

FIGURE 4.6 Round-rotor and salient-pole constructions.

4.3.3 ROUND AND SALIENT POLE ROTORS

Large ac generators are made of solid magnetic steel rotors of cylindrical shape, whereas small- and medium-size generators are made with laminated salient poles (protruding poles) as shown in Figure 4.6. The salient-pole construction provides more physical space for the rotor winding and also produces additional torque (called reluctance torque) that increases the machine power output at a small power angle δ. The flux in the main magnetic axis (called direct axis or *d*-axis) has a low-reluctance path (less air gap) than the flux in the quadrature axis (called *q*-axis). For this reason, the protruding salient poles have a natural tendency to align with the stator field axis under ferromagnetic attraction even in the absence of rotor current. This requires additional torque from the prime mover, twice in one electrical cycle, to drive the rotor away from said natural alignment. The total torque input and hence the power output of the generator has a sin(2δ) component superimposed on the sinδ component, as shown in Figure 4.7.

The power output analysis of the salient-pole generator requires employing the two-axis theory using Park transformation. It resolves the total machine magnetics in the d- and q-axis components, both rotating with the rotor but each with its own steady magnetic path. The d- and q-axis analysis is beyond the scope of this chapter, but the final result, which supports what we have reasoned, is as follows:

$$P_{\text{salient-pole}} = \frac{E_f V_T}{X_d}\sin\delta + V_T^2\frac{(X_d - X_q)}{2X_d X_q}\sin 2\delta \quad \text{watts/phase} \tag{4.9}$$

where X_d and X_q are the *d*-axis and *q*-axis synchronous reactance of the machine, respectively. For the salient-pole machines, the value of X_d is always greater than X_q. For a cylindrical rotor machine, the magnetic circuit in the *d*- and *q*-axis are identical, giving $X_d = X_q$. This makes the second term in Equation (4.9) vanish, leaving only the first term as in Equation (4.5).

Figure 4.7 shows that the salient-pole machine increases the total power generated at small power angle and has a higher $P_{\max}$ that occurs earlier in power angle δ.

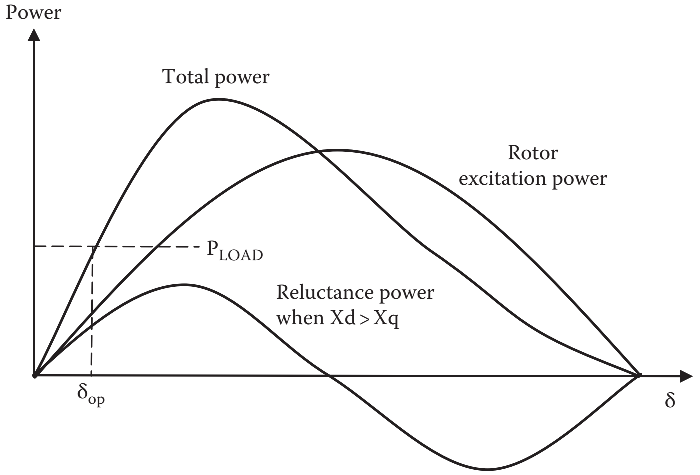


FIGURE 4.7 Salient pole power output versus power angle δ.

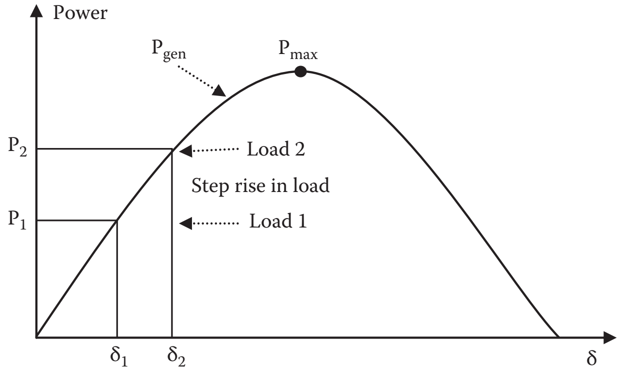
Therefore, for the same load power, the salient-pole machine would naturally run at lower δ. Further, having higher P_{max}, it can take a slightly greater step load change than the round rotor machine of comparable design before losing the transient stability limit discussed in the next section.

4.4 TRANSIENT STABILITY LIMIT

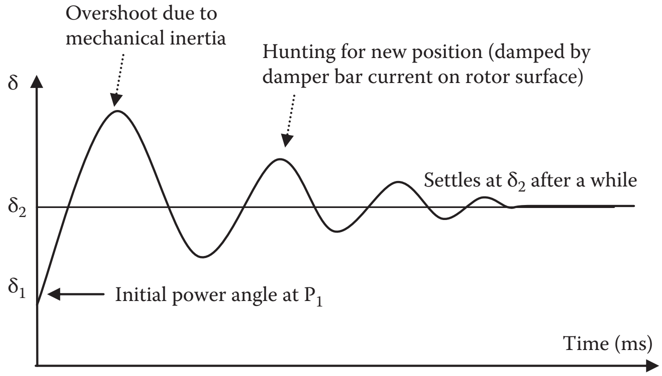
The maximum power limit P_{max} at $\delta = 90°$ in Figure 4.8(a) for the cylindrical rotor is called the steady-state stability limit. Any swing in δ beyond 90° may cause the rotor to lose synchronism and its power generation capability. Therefore, it is desirable to keep δ below 90° under all conditions, including any transient that may be encountered during normal and abnormal operations. For example, if the generator load were suddenly changed from P_1 to P_2 in one step, the rotor power angle would increase from δ_1 at old load P_1 to δ_2 at new load P_2. This takes some time due to the mechanical inertia of the rotor. No matter how short or long it takes, the rotor inertia and the electromagnetic restraining torque will set the rotor in mass-spring-damper type oscillations, swinging the rotor power angle beyond its new steady-state value of δ_2 as shown in Figure 4.8(b). If δ exceeds 90° any time during this swing, the machine stability and the power generation capability may be lost.

For this reason, the machine can be loaded only to the extent that, even under the worst-case load step, planned or accidental or during all possible faults, the power angle swing remains below 90° with sufficient margin. This limit on loading the machine is called the transient or dynamic stability limit, which is generally the power output for which δ is about 25° to 35° under normal steady-state operation.

For damping the transient oscillations of the rotor following a step load change, each pole face is provided with copper bars running along the length of the machine (Figure 4.9). All bars on each pole face are shorted at both ends, forming a partial squirrel cage of copper conductors on each pole surface. When the rotor oscillates



(a) P_{gen} vs. sinδ curve



(b) Transient δ after step load change

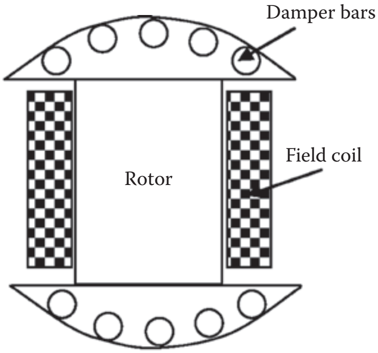
FIGURE 4.8 Transient oscillations of rotor power angle following step load rise.

around the synchronous speed, these bars see a relative slip with respect to the stator flux that runs exactly at the synchronous speed. This slip induces currents in the bars, as in the squirrel cage induction motor. The resulting I^2R power loss in the bars depletes the oscillation energy, cycle by cycle, until the oscillations are completely damped out and the induced currents subside.

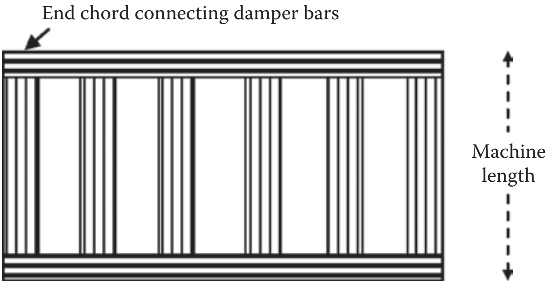
Thus, there is a small induction motor superimposed on the synchronous generator. It contributes damping only when the rotor oscillates around the constant synchronous speed. It is also used to start the machine as an induction motor to bring the generator near full speed before applying the dc excitation to the rotor and making it a synchronous machine then onward.

Equation 4.5 shows that the stability limit at a given voltage can be increased by designing the machine with low synchronous reactance X_s, which largely comes from the stator armature reaction component under steady-state (synchronous) operation.

Since many ac motors with direct online start are used on ships, such as for winches, the transient behavior of the generator needs close consideration. It may be necessary to select the shipboard generator with low reactance to improve the



(a) Cross section at mid-length of machine



(b) Top view of partial cage

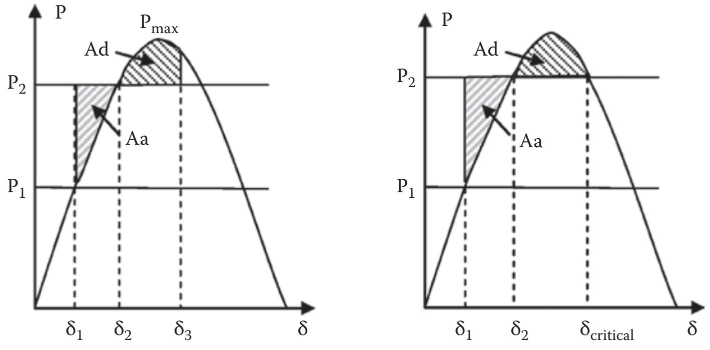
FIGURE 4.9 Damper bars near top surface of salient poles (used in round rotors also).

transient stability limit and to minimize the voltage dips, but that may increase short-circuit current levels.

Many shipboard loading events may constitute step load on the generator, such as turning on the bow and stern thruster, cargo and ballast pumps, cranes, main circulating pump on steam ships, high-power weapons on combat ships, and tripping a large load circuit breaker accidentally or for fault protection purposes. To maintain dynamic stability under such transients, sudden loading on the generator should be limited to no more than 25% to 30% of the rated power in one step, but the exact limit can be determined by the equal area criteria discussed in the following section.

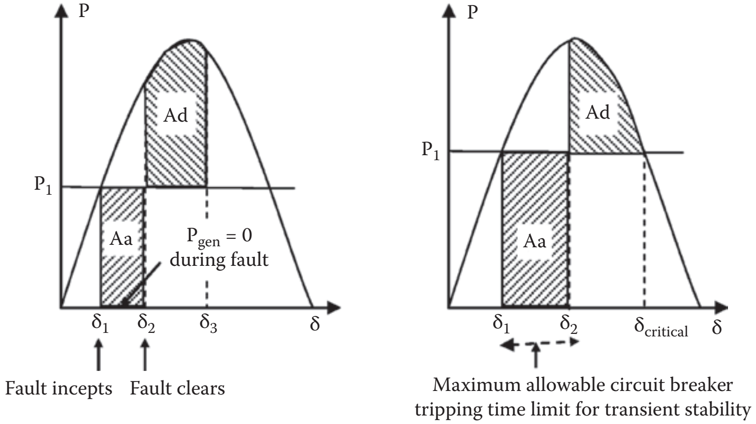
4.5 EQUAL AREA CRITERIA OF TRANSIENT STABILITY

If the generator load is suddenly increased in one step from P_1 to P_2, as shown in Figure 4.10(a), and the mechanical input from the prime mover is also changed from P_1 to P_2 to match with the load, the generator power angle (rotor lead angle) will start changing from δ_1 to δ_2 under the accelerating power $P_a = P_2 - P_1$. The rotor with large mechanical inertia will take a relatively long time to reach from δ_1 to δ_2, overshoot δ_2, and reach δ_3, which can be beyond the P_{max} point at $\delta = 90°$. Above δ_2, the generator supplies more power than the prime mover input P_2, and hence the



(a) Transient-stable swing

(b) Step-change limit for transient stability



(c) Transient-stable circuit breaker tripping after terminal fault

(d) Circuit breaker tripping time limit for transient stability

FIGURE 4.10 Equal area criteria of transient stability limit.

rotor decelerates back to δ_2 and undershoots below δ_2. The generator now has more mechanical input than electrical output, accelerating the rotor again. Such swings around δ_2 continue until damped out by the power loss in the damper bars. For simplicity, if we ignore the damper bar effect during the first swing, the machine will be stable if the accelerating energy equals the decelerating energy. In a torsional system, since the energy is equal to (torque × angle δ) and the torque equals (power ÷ angular speed), we can write the condition for the transient stability, that is, the accelerating energy = decelerating energy. In the integral form, it is

$$\int_{\delta 1}^{\delta 2} \frac{(P_2 - P)}{\omega} \cdot d\delta = \int_{\delta 2}^{\delta 3} \frac{(P - P_2)}{\omega} \cdot d\delta \tag{4.10}$$

where $P = P_{max} \sin\delta$ = actual electrical power generated at power angle δ.

The two integrals are areas under the generator's P-δ curve. The first one represents the accelerating area A_a, and the second the decelerating area A_d in Figure 4.10(a). The generator will be stable during the transient oscillations if the two areas are equal, and hence the name *equal area criteria of transient stability.*

Figure 4.9(b) depicts limiting step load change ($P_2 - P_1$) that will maintain the transient stability. It shows that the maximum value of $\delta_3 = \delta_{\text{critical}}$ can be the intersection of the generator P-δ curve and new load P_2. It can be determined by equating the two areas A_a and A_d. Knowing the initial load P_1, we can determine P_2, and the maximum permissible step load change ($P_2 - P_1$).

The generator can also lose stability if the circuit breaker trips under a short circuit, delivering no electrical power to the load, as shown in Figure 4.10(c). The rotor will accelerate under the prime mover input power, accelerating to increase the rotor angle to δ_2. If the fault is cleared in time and the circuit breaker is closed under the automatic reclosure scheme (common on land-based systems), the generator starts delivering electrical power in excess of the prime mover input and starts decelerating to its original power angle δ_1 after the inertial overshoot to δ_3. The transient swing will be such that the accelerating area A_a is equal to the decelerating angle A_d. The maximum allowable fault-clearing time for stability can be determined by applying the same equal criteria by equating A_a and A_d in the limiting case shown in Figure 4.10(d) and determining δ_{critical}. The length of time it takes for the machine to move from δ_1 to δ_{critical} comes from the equation of mechanical motion that equates the difference between the mechanical power input and the electrical power output. The difference goes in accelerating the rotor. Ignoring damping, we can write the following in terms of the torques:

$$T_{\text{mech}} - T_{\text{elec}} = J\frac{d\delta^2}{dt^2} \tag{4.11}$$

where J = polar moments of inertia of the rotor, $T_{\text{elec}} = T_{\max}\sin\delta$ = back torque on the generator shaft corresponding to the electric load, T_{mech} = mechanical torque input from the prime mover.

Equation (4.11) is called the swing equation. The solution of this differential equation, modified by power = torque × ω = torque × 2π*f*, leads to the critical fault-clearing time

$$t_{\text{critical}} = \sqrt{\frac{2H(\delta_{cr} - \delta_o)}{\pi f P_{\text{mech}}}}\ \text{sec} \tag{4.12}$$

where all δ's are in mechanical rad/sec, and H = rotor inertia in terms of kJ (kW-sec) kinetic energy stored at rated speed per kilowatt rating of the machine. The rotor inertial constant H is defined as the number of seconds the machine can deliver the rated power using its kinetic energy stored in the rotor inertia alone with no mechanical input from the prime mover. It includes the turbine rotor inertia as well.

Example 4.7

A 100 MVA, 4-pole, 60 Hz synchronous generator that operates at 0.90 power factor load has a cylindrical rotor 5 m long and 1.5 m in diameter. The average mass density of the rotor's copper and magnetic steel combined is 8 g/cm^3. Determine the kinetic energy of the generator rotor running at the rated speed. Assuming that the turbine rotor has inertia 1.5 × generator rotor inertia, determine the inertia constant H of the turbine-generator system.

SOLUTION

Rated speed of 4-pole, 60 Hz generator = 120 × 60 ÷ 4 = 1800 rpm.

$\therefore\ \omega_{\text{mech}} = 2\pi \times 1800 \div 60 = 188.5$ rad/sec.

Polar moment of inertia of cylindrical rotor, $J = \frac{1}{2}$ Mass × Radius^2, where Mass = $8 \times \pi \times (75\text{ cm})^2 \times 500\text{ cm} = 70.69 \times 10^6$ grams or 70,690 kg.

$\therefore\ J = \frac{1}{2}\ 70{,}690 \times 0.75^2 = 19{,}880$ kg.m^2.

Kinetic energy = $\frac{1}{2} J \omega^2 = \frac{1}{2} \times 19{,}880 \times 188.5^2 = 353.2 \times 10^6$ joules = 353.2 MJ.

Turbine rotor kinetic energy = 1.5 × 353.2 = 529.8 MJ, and the total kinetic energy of the turbine-generator rotor = 529.8 + 353.2 = 883 MJ.

Generator power output P = 100 MVA × 0.90 power factor = 90 MW, which can be supplied from the rotor kinetic energy for 883 ÷ 90 = 9.81 sec,

$\therefore$ By definition, $H = 9.81$ sec.

The swing equation also leads to the undamped mechanical natural frequency of rotor oscillations, which is given by

$$f_{mech} = \frac{1}{2\pi}\sqrt{\pi\, f_{elec}\,\frac{\left(\frac{dP_{elec}}{d\delta}\right)_o}{H}}\ \text{Hz} \tag{4.13}$$

Where suffix o represents the initial slope of the p–δ curve. This equation is similar to that for the mechanical mass-spring oscillations, namely,

$$f = \frac{1}{2\pi}\sqrt{\frac{spring\,cons\tan t\,K}{Mass\,M}}\ \text{Hz} \tag{4.14}$$

Comparing Equations (4.13) and (4.14), we see that dP_{elec}/dδ is like a spring constant, which is nonlinear, stiffest near $\delta = 0$, soft at operating δ's, and totally resilient at $\delta = 90°$. It provides no restraining stiffness at $\delta = 90°$, beyond which it is unable to operate stably and hence becomes unstable under steady-state loading. In determining $P_{\text{elec}} = P_{\text{max}}\sin\delta$ for the swing equation, the P_{max} must use the transient reactance X_d' of the generator (discussed later in Chapter 9).

Example 4.8

A 60-Hz round rotor generator has $E_f = 1.5$ pu, $V_{T(\text{bus})} = 1.0$ pu, $X_d = 1.3$ pu, and $H = 5$ sec. Determine the mechanical natural frequency of the turbine-generator oscillations under transient disturbance when operating at 50% load.

SOLUTION

Again, it is recommended that this example be read after reading about the per unit (pu) system in Chapter 6. Until then, we just say here that 1.0 pu = 100% of rated value, and work in the per unit system. The initial power angle δ_o is derived from Equation (4.5), that is, $P = 0.50 = \frac{1.5\times1.0}{1.3}\sin\delta_o = 1.154\sin\delta_o$, which gives $\delta_o = 25.68°$.

Synchronizing power coefficient (generator spring constant) that provides restraining force for transient swings in δ is given by

$$K = \mathrm{d}P/\mathrm{d}\delta = P_{\max}\cos\delta = 1.154\ \cos\delta_o = 1.04$$

The mechanical frequency of rotor oscillations following a step load change or any other transient disturbance is given by Equation (4.13), that is,

$$f_{mech} = \frac{1}{2\pi}\sqrt{\frac{\pi\,x\,60\,x\,1.04}{5}} = 1\,\mathrm{Hz}$$

The mechanical transients, which are much slower than electrical transients, occur in a synchronous generator supplied by pulsating power prime movers (e.g., diesel engine) or synchronous motor driving pulsating loads (e.g., compressor loads). If the transient oscillations are small around small δ, $\sin\delta \approx \delta$ in radians, and the torque becomes linear with δ, that is, $T = T_{\max} \times \delta$. The machine appears as a torsional spring with springs constant $K = T/\delta = T_{\max}$ n-m/rad. For large oscillations, the linearity no longer holds, and the solution must be obtained by a step-by-step numerical method on the computer. Moreover, as δ approaches 90°, the spring becomes softer and the machine may lose synchronism.

For a large turbine-generator, typical t_{critical} given by Equation (4.12) is less than one second for automatic reclosure of the circuit breaker after a fault for maintaining transient stability, and the mechanical oscillation frequency given by Equation (4.13) is generally less than 1 Hz.

4.6 SPEED AND FREQUENCY REGULATIONS

When the load torque rises, the automatic speed-regulating governor increases the fuel input to maintain the speed. However, it does not fully compensate for the load increase, and the prime mover speed drops slightly in approximately linear manner. This can also be explained in terms of the mechanical reaction torque (back torque) on the rotor, which is proportional to the stator current. As the load is increased, the rotor speed drops under increased back torque on the rotor. The generator speed regulator (governor) will allow more fuel to the prime mover to maintain the rotor speed constant. A practical speed governor must have a certain dead band (tolerance band), or else it would go through hunting oscillations in response to any load change. The dead band allows some decrease in speed without a response. Thus, as load increases on the generator, the steady-state speed of the rotor will decrease slightly due to the dead band in the prime mover speed governor. So, even with an automatic speed control, practical prime mover governors cannot maintain perfectly constant speed. As a result, the prime mover speed drops slightly with increasing load.

The speed regulation of a mechanical power source—steam or gas turbine, diesel engine, or motor—is defined in a manner similar to the voltage regulation of an electrical power source:

$$\textit{Speed regulation} = \frac{n_{\text{noload}} - n_{\text{ratedload}}}{n_{\text{ratedload}}} \tag{4.15}$$

Since the ac generator frequency is directly related with the prime mover speed, power engineers working with ac generators usually define the generator frequency regulation (GFR) in terms of the governor speed regulation (GSR):

$$GFR = GSR = \frac{f_{\text{noload}} - f_{\text{ratedload}}}{f_{\text{ratedlaod}}} \tag{4.16}$$

The frequency droop rate (FDR) or the governor droop rate (GDR) is defined as rate of frequency droop per kilowatt or megawatt of output load, that is,

$$FDR = GDR = \frac{\Delta f}{\Delta P}\ \text{Hz /kW or Hz /MW} \tag{4.17}$$

Example 4.9

A 60 Hz synchronous generator rated 100 MVA at 0.90 power factor has the no-load frequency of 61.5 Hz. Determine the prime mover speed regulation, the generator frequency regulation, and the generator frequency droop rate.

SOLUTION

We first note that, under rated operation, the generator output power would be 100 × 0.90 = 90 MW, and frequency 60 Hz (implied since the rated frequency of the generator is 60 Hz). Then,

$$\text{GFR or GSR} = (61.5 - 60.0) \div 60 = .025 \text{ pu or } 2.5\%.$$

Since the frequency drops from 61.5 Hz to 60 Hz from no-load to rated load of 90 MW, FDR = $\Delta f/\Delta P$ = (61.5 − 60.0) ÷ (90 − 0) = 1.5 Hz ÷ 90 MW = 0.01667 Hz/MW

4.7 LOAD SHARING AMONG AC GENERATORS

The ac generator voltage is controlled by the field current and the speed, but the frequency is controlled only by the speed. Since the generator power ultimately comes from the prime mover, the prime mover speed and the generators frequency both droop at the same rate with increasing load on the generator. With two ac generators sharing load in parallel, their terminal voltages and frequencies both must be equal, and the frequency droop lines of two ac generators determine their share of the total load. The frequencies of Generator 1 and Generator 2 at their own load powers P_1 and P_2 can be written in terms of their respective frequency droop rates as follows:

$$f_1 = f_{1\text{no-load}} - \text{FDR}_1 \cdot P_1 \quad \text{and} \quad f_2 = f_{2\text{no-load}} - \text{FDR}_2 \cdot P_2 \tag{4.18}$$

where f = operating frequency, $f_{\text{no-load}}$ = no-load frequency, and FDR = frequency droop rate. By parallel connection, we impose on the bus that $f_1 = f_2 = f_{\text{bus}}$, that is,

$$f_{1\text{no-load}} - \text{FDR}_1 \cdot P_1 = f_{2\text{no-load}} - \text{FDR}_2 \cdot P_2 = f_{\text{bus}} \tag{4.19}$$

$$\text{Total load} \qquad P_T = P_1 + P_2 \tag{4.20}$$

Solving Equations (4.19) and (4.20) simultaneously for P_1 and P_2 (with all other parameters known), we get the load shared by Generators 1 and 2. Figure 4.11 depicts the one-step method of determining the ac generator load sharing in a manner similar to that we discussed for the static power sources in Section 3.8.1.

Example 4.10

The no-load frequency of two AC generators is the same—60.5 Hz. The frequency droop rate of Generator 1 is 0.0006 Hz per kW, and that of Generator 2 is 0.0008 Hz per kW. If the two generators are in parallel supplying a total load of 1500 kW, determine the load shared by each generator and the operating frequency of the bus.

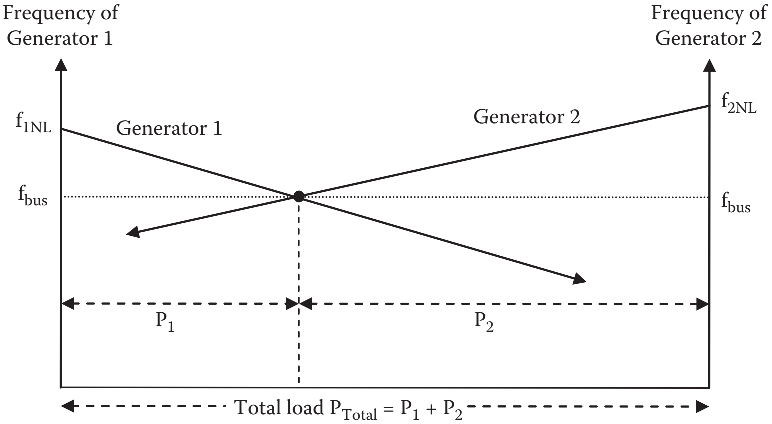


FIGURE 4.11 One-step method of determining two ac generators' load sharing.

SOLUTION

Denoting kW power P_1 and P_2 shared by Generator 1 and Generator 2, respectively, we write the two generator frequency droop line equations as

$f_1 = 60.5 - 0.0006\ P_1$, and $f_2 = 60.5 - 0.0008\ P_2$

In parallel operation, $f_1 = f_2 = f_{\text{bus}}$, and $P_1 + P_2 = 1500$ kW.

Therefore, we write $f_{\text{bus}} = 60.5 - 0.0006\ P_1 = 60.5 - 0.0008\ (1500 - P_1)$, which gives $P_1 = 857$ kW and $P_2 = 1500 - 857 = 643$ kW.

The bus frequency is then $f_{\text{bus}} = 60.5 - 0.0006 \times 857 = 59.986$ Hz.

Or, $f_{\text{bus}} = 60.5 - 0.0008 \times 643 = 59.986$ Hz, which is the same as the foregoing.

We note that Generator 1 is stiffer (droops less) than Generator 2, and hence it shares greater load, as expected.

It is noteworthy that the stiffer generator with low FDR (flatter line) shares the heavier load, and the weaker generator with high FDR (more drooping) shares the lighter load. This is analogous to load sharing between two mechanical springs in parallel. The stiffer spring, which droops less with load, shares greater load. Also, the machine with higher no-load speed (droop line shifted upward) would share greater load. When we change the governor setting, we essentially shift the fuel-input rate, and hence the speed line and the frequency line up and down, whereas the governor droop rate remains the same. In practice, load sharing is controlled manually or automatically by adjusting the prime mover's governor setting, which controls the input valve of the fuel (steam or diesel). The governor's automatic control system varies the fuel input rate and the speed directly proportional to the load. When the load increases, the fuel is increased, and vice versa.

With two generators working in parallel, if one generator momentarily slows down for any reason, it delivers less power and subsequently speeds up. The other generator takes the greater load and slows down. Such adjustment takes place until both generators run at the exact same speed to generate the exact same frequency and the exact same terminal voltage. We can say that parallel generators have a great team spirit in helping each other run at the same speed as determined collectively by the total load demand on the team.

Caution: In parallel operation, it is important that each machine shares the load (both in kW and in kVAR) within its own rated limit. The kW load can be balanced by adjusting the governor speed regulation, and the kVAR load can be balanced by the field excitation of each machine by adjusting the field rheostat in the voltage regulator. After such adjustments in both machines, the armature currents should be about the same percentage of their individual rated values and at about equal power factors.

4.8 ISOSYNCHRONOUS GENERATOR

If the frequency of Generator 1 were to be maintained constant regardless of its load as shown in Figure 4.12, that is, if Generator 1 were made infinitely stiff, its frequency droop under load is eliminated, and the bus frequency remains constant (flat) regardless of the total load. The constant frequency generator is known as the *isosynchronous generator*. Most governors have a set point adjustment that allows

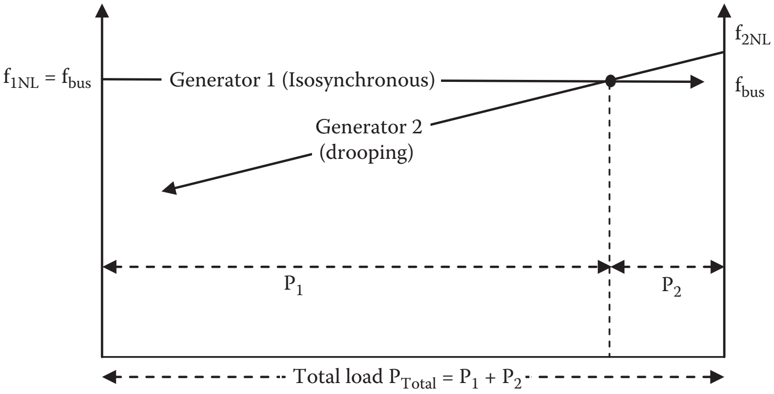


FIGURE 4.12 Load sharing between one isosynchronous generator and one drooping generator.

for varying the no-load speed of the prime mover. By adjusting this set point, one can adjust the frequency or the load shared by one generator with the other operating in parallel. In the isosynchronous generator, this adjustment is done by an automatic control system with precise compensation feedback. We note two points here when operating two generators in parallel:

1. Only one generator can be isosynchronous. Two isosynchronous generators cannot work in parallel since they would have no intersection that makes the common point of stable operation. In parallel, they would conflict with each other, could overload, and self-destroy in a continuous search of an intersection.
2. Beyond the initial load sharing, all additional load is taken by the isosynchronous generator, whereas the drooping generator load remains constant. This is analogous to two mechanical springs sharing a load, with one soft spring drooping with load and the other spring infinitely stiff like a solid metal block (we may call it isoshape spring) as shown in Figure 4.13. Regardless of the initial load sharing, any additional load will be taken by the solid metal block (isoshape spring) without additional drooping of either spring.

For this reason, the load shared by the isosynchronous generator is much greater than that by the drooping generator. Therefore, the prime mover governor for an isosynchronous generator is specifically designed for isochronous operation at any load from zero to 100% load. Such governor assures that the prime mover shares the load proportional to the generator kW rating by using direct measurement of kW. It provides rapid response to load changes, stable system operation, ability for paralleling dissimilar-sized engine-generator sets, and fine speed regulation under 0.25%.

Droop is inherent in all prime mover speed controls, but in an isochronous generator, it is recovered in a short time. It is a temporary droop of transient nature—more

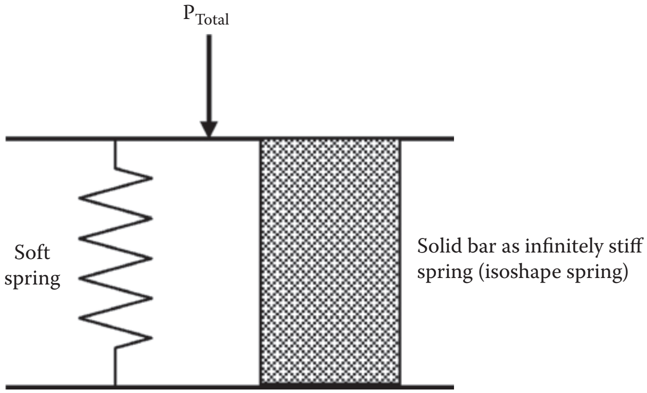


FIGURE 4.13 Load sharing between two springs in parallel, one soft and the other infinitely stiff.

commonly called the compensation. This gives bus frequency 60 Hz from no load to full load, as seen in Figure 4.14.

Example 4.11

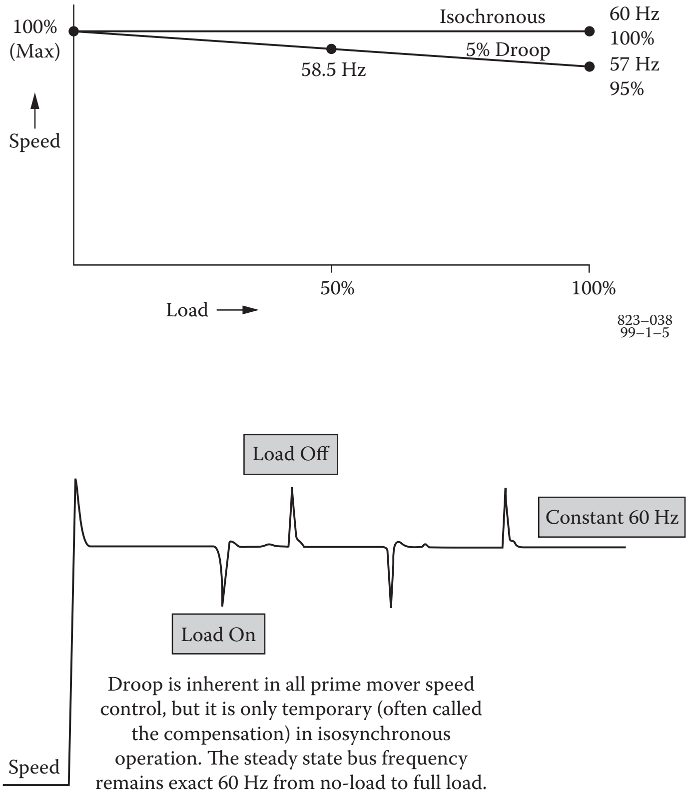
Two 60 Hz generators, one isosynchronous and one drooping, are sharing 15 MW each when supplying a total load of 30 MW with the bus frequency exactly at 60 Hz. If the total load is increased to 40 MW, determine the load on the isosynchronous generator and the bus frequency.

SOLUTION

Since the isosynchronous generator is infinitely stiff with no droop rate, it will take the entire additional load without a drop in frequency. Therefore, its new load will be 15 + (40 − 30) = 25 MW, and the new bus frequency will still be exactly 60 Hz.

4.9 EXCITATION METHODS

The synchronous generator excitation system is designed to produce the rotor magnetic field that can be varied to control the voltage and reactive power of the generator. In modern high-power machines, the synchronous reactance X_s is around 1.5 × base impedance of the machine. With such a high reactance, the phasor diagram in Figure 4.15 shows that E_f or the rotor field current required at the rated load at 0.9 lagging power factor can be more than twice that at no load with the same terminal voltage. A typical excitation system has the corresponding current and voltage ratings, with the capability of varying the voltage E_f over a wide range of 1 to 3, or even more, without undue saturation in the magnetic circuit. Most excitation systems operate at 200 to 1,000 V_{dc}. The excitation power to overcome the rotor winding I^2R loss ranges from ½% to 1% of the generator rating. For a large utility generator,



Droop is inherent in all prime mover speed control, but it is only temporary (often called the compensation) in isosynchronous operation. The steady state bus frequency remains exact 60 Hz from no-load to full load.

FIGURE 4.14 Top: Load versus frequency lines of isosynchronous and drooping generators, Bottom: Momentary frequency drop in isosynchronous generator (from Dana Walker, U.S. Merchant Marine Academy).

four types of excitation system—dc, ac, static, and brushless—are described in the following text.

DC exciter: A suitably designed dc generator supplies the main field winding excitation through conventional slip rings and brushes. Due to low reliability and a high maintenance requirement, the conventional dc exciter is seldom used in modern ac generators of large ratings.

AC exciter: It consists of a permanent magnet pilot exciter that excites the main exciter. The ac output of the pilot exciter is converted into dc by a floor-standing rectifier and supplied to the main exciter through slip rings. The main exciter's ac output is converted into dc by means of a phase-controlled rectifier whose firing angle is changed in response to the terminal voltage variations. After filtering the ripples, the dc is fed to the main generator field winding.

Static exciter: It has no moving parts, as opposed to the rotating exciters described. In the static exciter scheme, the controlled dc voltage is obtained

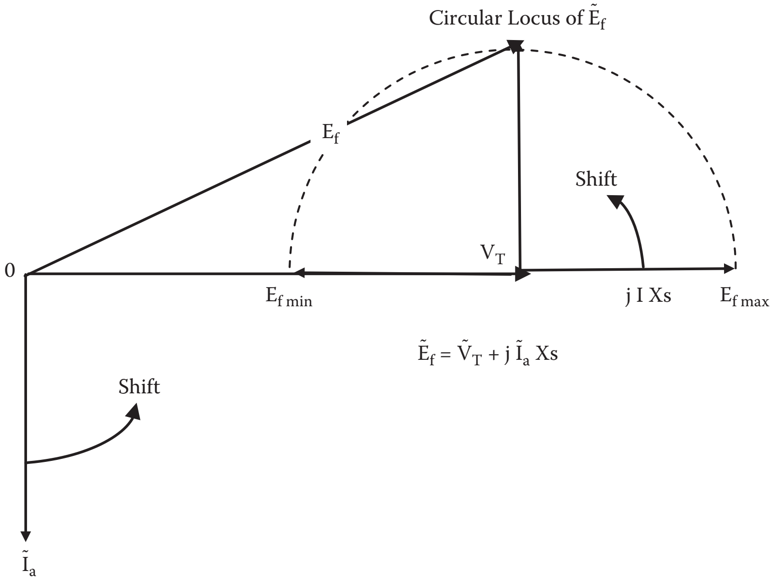


FIGURE 4.15 Variation in field excitation voltage E_f with load power factor varying from zero lagging to unity.

from a suitable stationary ac source rectified and filtered. The dc voltage is then fed to the main field winding through slip rings. This excitation scheme has a fast dynamic response and is more reliable because it has no rotating exciter with mechanical inertia.

Brushless exciter: Most modern synchronous generators of large ratings use the brushless scheme of excitation to eliminate the need for slip rings and brushes. The brushless exciter is placed on the same shaft as the main generator. The ac voltage induced in the exciter is rectified by rotating diodes on the rotor and filtered into pure dc. The dc is then fed directly into the rotor field coil.

The excitation control system modeling for analytical studies must be carefully done as it forms multiple feedback control loops that can become unstable. The IEEE has developed an industry standard for modeling the excitation systems. The model must account for any nonlinearity due to magnetic saturation that may be present in practical designs. The control system stability can be improved by supplementing the main control signal by auxiliary signals, such as speed and power, as required by the feedback control system stability.

4.10 SHORT CIRCUIT RATIO

The short circuit ratio (SCR) of the synchronous generator is defined as the ratio of the field current required to generate rated voltage at open circuit to the field current required to circulate rated armature current with short circuit. Thus, SCR is merely the reciprocal of the synchronous reactance X_s expressed in the per unit system (covered in Chapter 6), that is,

$$SCR = \frac{I_f \textit{ for rated voltage with open terminals}}{I_f \textit{ for rated armature current with shorted terminals}} = \frac{1}{X_{s(pu)}} \quad (4.21)$$

Thus, the short circuit ratio is a measure of the steady-state short circuit current in case of a terminal fault; it is the ratio of the fault current to the rated current of the generator, that is,

$$\text{Steady-state terminal short circuit current} = \text{SCR} \times \text{Generator rated current} \quad (4.22)$$

However, for the generator circuit breaker selection, the transient fault current using the generator's transient reactance—not the synchronous reactance—must be used, as covered in Chapter 9.

The SCR also indicates the machine's sensitivity to the change in rated load. A machine with higher SCR is larger in physical size and weight, and hence costs more. However, it has smaller X_s resulting in smaller internal voltage drop and smaller voltage regulation. The machine design engineer balances the SCR value for an optimum design.

4.11 AUTOMATIC VOLTAGE REGULATOR

Excitation control is required to (1) maintain the normal operating voltage, (2) vary kVAR generation to match with the load, and (3) increase the steady-state and dynamic stability. The manual control with field rheostat can be adequate for small generators, but the automatic voltage regulator (AVR) is common for large machines.

As discussed in Chapter 1, the complex power $\tilde{S}$ delivered by the generator must match the complex power drawn by the load. In $\tilde{S} = P + jQ$, the real power P is balanced by the prime mover fuel flow rate that is controlled by the prime mover governor. The reactive power Q, on the other hand, is balanced by the field current controlled by the AVR. The governor and the AVR are two independent controllers in an ac generator. The AVR is a part of the excitation system, which works as follows in brushless machines.

The AVR senses the voltage in the main generator winding and controls the excitation to maintain the generator output voltage within the specified limits, compensating for the load, speed, temperature, and power factor of the generator. Three-phase root mean square (rms) sensing is employed for finer voltage regulation. The excitation current is derived from a dedicated three-phase permanent magnet generator to isolate the AVR control circuits from the effects of nonlinear loads and to reduce radio frequency interference on the generator terminals. Protection of the exciter against sustained generator short circuit current is another feature of the permanent magnet rotor used in AVR. Additional features found in some AVRs are

- A frequency-measuring circuit continually monitors the shaft speed of the generator and provides underspeed protection of the excitation system by reducing the generator output voltage proportionally with speed below a presettable threshold.

- The maximum excitation is limited to a safe value by internal shutdown of the AVR output device. This condition remains latched until the generator has stopped.
- Provision is made for the connection of a remote voltage trimmer, allowing the user to finely control the generator output. The AVR has the facility to allow parallel running with other similarly equipped generators.
- Typical transient response times are: AVR itself in 10 ms, field current to 90% in 80 ms, and machine voltage to 97% in 300 ms. The AVR also includes a stability or damping circuit to provide good steady-state and transient performance of the generator.
- AVR includes a soft start or voltage ramp-up circuit to control the rate of voltage buildup when the generator runs up to speed. This is normally preset and sealed to give a voltage ramp-up time of approximately 3 sec. If required, this can be adjusted between the limits defined in the AVR specifications.

PROBLEMS

Problem 4.1: A 3-phase, Y-connected, 480 V generator is powering 1000 kW balance 3-phase Δ-connected load at 0.90 pf lagging. Determine (1) the line-to-neutral voltage and line current, (2) generator phase voltage and phase current, and (3) load phase voltage and phase current.

Problem 4.2: A balanced Y-connected load with 3 + j5 ohms/ph (L-N) is connected in parallel with a balanced Δ-connected load with 12 + j15 ohms/ph (L-L). Determine the combined total equivalent Y-connected impedance per phase. If these loads were powered by a Y-connected 2400 V_{LL} generator, determine the current drawn from the generator lines.

Problem 4.3: A 3-phase, 11 kV, 60 Hz, 10 MVA, Y-connected synchronous generator has $X_s = 1$ Ω/ph and negligible R_a. Determine its power angle δ when delivering rated power at unity power factor. (Hint: First calculate $\tilde{E}_f$ and $P_{\max}$.)

Problem 4.4: A 3-phase, Y-connected synchronous generator is rated 15 MVA, 13.8 kV. Its resistance is negligible, and the synchronous reactance is 2 Ω per phase. Determine the field excitation voltage E_f when delivering rated MVA at 0.90 power factor lagging.

Problem 4.5: A 3 MVA, 60 Hz, 6.6 kV, 4-pole, 96% efficient round rotor synchronous generator has the synchronous reactance of 4.0 Ω/phase and negligible armature resistance. When operating at unity power factor, determine (a) the maximum power it can deliver under steady state with no step loading, (b) rotor lead angle (power angle δ), and (c) diesel engine horsepower output.

Problem 4.6: A large cruise-ship generator was repaired and brought back on line by synchronizing with the other five generators, which collectively make an infinite bus for the incoming generator. The incoming generator has the synchronous reactance $X_s = 1.2$ pu, $V_T = 1.0$ pu, and the field current $I_f = 800$ A at the time of synchronization. After synchronization, while keeping the field current unchanged, the steam valves are adjusted such that the generator delivers 0.3 pu of its rated power. (1) Determine the armature current in per unit value when delivering 30% power with rated field current $I_f = 800$ A. (2) If the field current is now increased by 50% while keeping the power output the same, determine the new armature current in per unit value.

Problem 4.7: A 80 MVA, 4-pole, 60 Hz synchronous generator operates at 0.90 power factor lagging. Its cylindrical rotor of 5 m length and 1.5 m diameter has the average mass density of 8.3 g/cm^3. Assuming that the turbine rotor has inertia 1.5 × generator rotor inertia, determine the inertia constant H of this turbine-generator set.

Problem 4.8: A 50-Hz round rotor generator has $E_f = 1.4$ pu, $V_T = 1.0$ pu, $X_d = 1.5$ pu, and $H = 7$ sec. Determine the mechanical natural frequency of the turbine-generator oscillations under a transient disturbance when operating at 80% load.

Problem 4.9: A 60 Hz synchronous generator rated 30 MVA at 0.90 power factor has the no-load frequency of 61 Hz. Determine (1) the prime mover speed regulation, (2) generator frequency regulation, and (3) generator frequency droop rate.

Problem 4.10: The no-load frequency of two AC generators is the same—61 Hz. The frequency droop rate of Generator 1 is 0.001 Hz per kW, and that of Generator 2 is 0.0005 Hz per kW. If the two generators are in parallel supplying a total load of 1000 kW, determine the load shared by each generator and the operating frequency of the bus.

Problem 4.11: Two 40 MW, 60 Hz generators, one isosynchronous and one drooping, share equal load of 25 MW each when supplying a total load of 50 MW with the bus frequency exactly at 60 Hz. If the total load is increased to 70 MW, determine the load on the isosynchronous generator and the bus frequency.

Problem 4.12: A 4160 V, 60 Hz, 3-phase, Y-connected generator has the synchronous reactance of 0.9 pu and negligible armature resistance. At rated armature current, determine the field excitation current range for power factor changing from zero lag to unity to zero lead. Express the range in terms of the field current required when the generator is delivering the rated current at unity power factor.

Problem 4.13*:* A 3-phase, 500 kVA, 480 V, 60 Hz, Y-connected synchronous generator gave these test results at rated speed: (1) open circuit voltage at rated field current = 560 V_{LL}, and (2) short circuit current at rated field current = 305 A. When cold at 20°C, the average dc resistance of three armature phase coils measured by ohmmeter was 0.20 Ω. Determine the armature ac resistance and synchronous reactance per phase at operating temperature of 90°C.

QUESTIONS

Question 4.1 Explain the difference between electrical degree and mechanical degree in rotating electrical machines.

Question 4.2 List three reasons that may cause the generator to fail in building up the terminal voltage.

Question 4.3 Explain the counter torque developed in the generator when the load is connected to the stator terminals.

Question 4.4 Explain why the field excitation current of the generator alters the power factor as seen by the machine.

Question 4.5 In light of Equation (4.6), discuss the generator maximum power capability at lower frequency all the way up to zero frequency (dc). At very low frequency, what would limit the maximum power capability?

Question 4.6 Explain the construction and performance difference between the round-rotor and salient-pole synchronous machines.

Question 4.7 Identify two problems you may encounter in applying a large load in one step on a synchronous generator.

Question 4.8 What loading events in ships may cause large step load on the main generators?

Question 4.9 What causes damping in the rotor oscillations following a sudden step change in the generator load?

Question 4.10 In the synchronous machine, where the damper bars are located, what is their purpose and how do they work?

Question 4.11 A 1000 kVA, 460 V, 60 Hz, 3-phase, 4-pole generator has squirrel cage damper bars of 1 cm diameter on each of the rotor pole surface. What would be the damper bar current under the steady synchronous speed operation?

Question 4.12 Why cannot two generators at different voltage and frequency work in parallel?

Question 4.13 List the conditions that must be met before a synchronous generator can be paralleled with other generators or with the power grid.

Question 4.14 While transferring load from one generator to another, what changes really take place during this process in terms of the prime mover's governor speed regulation settings and the droop lines? Which way do they get adjusted, and how do these changes transfer the load?

Question 4.15 Clearly identify the benefits of using an isosynchronous generator. If two such generators are placed in parallel, what would you expect to happen?

Question 4.16 Two generators are operating in parallel, one with a droop, and the other isosynchronous. Explain their load sharing behavior when the total bus load increases and decreases.

Question 4.17 If the synchronous generator suddenly gets shorted at its terminals, how would the speed change before the turbine fuel supply is reduced or cut off?

FURTHER READING

Chapman, S.J. 1999. *Electric Machinery Fundamentals.* Boston: McGraw Hill.
Say, M.G. 1983. *Alternating Current Machines.* New York: John Wiley & Sons.

5 AC and DC Motors

Of the total electrical energy generated worldwide, about 58% is used by all motors combined, about 7% for lighting, and the remaining 35% for heating and other uses. Major types of motor are the synchronous motor, induction motor (also known as asynchronous motor), and dc motor. All have two sets of coils with different currents, say, I_1 and I_2. The electromagnetic interaction between two currents produces motor torque $T_m = K\, I_1\, I_2$. If the motor shaft has a load torque $T_{\text{Load}} < T_{\text{motor}}$, the motor would accelerate to a speed at which $T_{\text{Load}} = T_{\text{motor}}$, where it would stop accelerating and run at steady speed. The motor armature produces *back voltage,* or its equivalent, and draws current from the source, which is given by

$$Armature\ current = \frac{Applied\ voltage - Back\ voltage}{Effective\ Impedance\ of\ armature} \tag{5.1}$$

It is important to understand that the armature draws just enough current that is required to develop torque to meet the load torque at the steady running speed. A 100-hp-rated motor does not always deliver full 100 hp regardless of the shaft load. The motor delivers what is needed to drive the load and draws power from the source equal to what it delivers to the load plus the internal losses. Thus, the power drawn from the source may be less or more than the rated load, depending on the mechanically coupled load on the shaft. However, if continuously overloaded without added cooling, the motor would heat up and burn.

The shaft horsepower, torque, speed, and kW power delivered by the motor are related as follows:

$$HP = \frac{T_{lb.ft} \cdot n_{rpm}}{5252} \qquad or \qquad kW = \frac{T_{n.m} \cdot 2\pi n_{rpm}}{60 \times 1000} \tag{5.2}$$

Table 5.1 gives the breakdown of motor types and their energy usage in various horsepower ratings. It shows that about 98% of all motors are induction motors that use about 93% of the electrical energy used by all motors rated 5 hp and higher. Smaller motors do not use much energy because their use is intermittent, often less than an hour in a day. We now discuss the three most widely used motors in the industry.

5.1 INDUCTION MOTOR

The induction motor has been a reliable workhorse of the industry ever since it was invented by Nicola Tesla in 1888. It is the most widely used motor because of its simple, brushless, low-cost, and rugged construction. Most induction motors in use are 3-phase in large ratings or 1-phase in small ratings. The 3-phase induction motor

TABLE 5.1
Motor Types by Approximate Number and Electrical Energy Usage

Motor Category	Number in Category as Percentage of All Motors	Energy Usage as Percentage of Total Used by All Motors	Typical Usage Pattern and Applications
< 5 HP induction motors	88%	5%	Intermittent usage in small appliances
5–125 HP induction motors[a]	7%	50%	Heaviest energy usage motors
> 125 HP induction motors	3%	43%	Continuous usage in industry
Synchronous motors	1%	1%	Mostly in large sizes (> 5000 hp)
DC motors	1%	1%	Where easy speed control is needed

[a] Since induction motors in the 5–125 hp range consume about 50% of all the electrical energy used by all motors combined, the U.S. Department of Energy has focused on improving their design and efficiency.

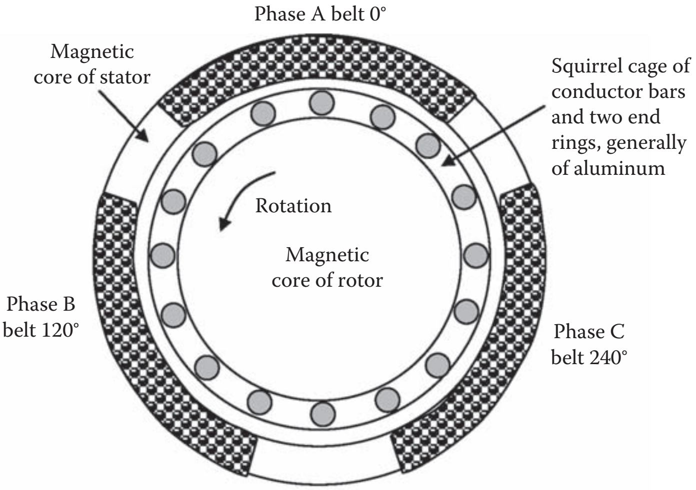
has three stator coils wound with wires, and the rotor in a squirrel-cage configuration as shown in Figure 5.1. The cage rotor is generally made of cast aluminum bars running along the machine length and two end rings shorting all the bars. The rotor cage in a high-efficiency motor is often made of copper, which has much better conductivity than aluminum. There is no electrical connection between the stator and the rotor. The 3-phase current in the three stator coils wound in P-poles configuration and powered at frequency f creates a magnetic flux that rotates at constant speed n_s (called the synchronous speed), which is given by

$$Synchronous\,rpm = \frac{120 \times frequency}{Number\ of\ poles} \tag{5.3}$$

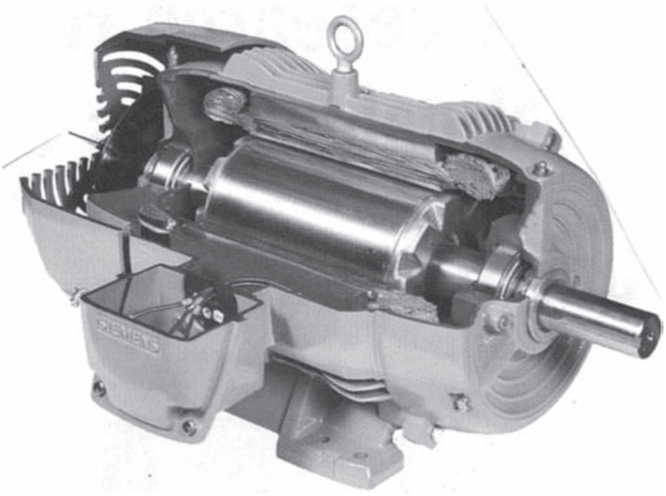
The number of magnetic poles created by the stator winding depends on the coil span, that is,

$$Number\ of\ poles\,P = \frac{\pi \times stator\,diameter}{stator\,coil\,span} \tag{5.4}$$

Injecting the rotor current from an outside source, as in the synchronous or dc motor, is not required in the induction motor. The sweeping (cutting) flux of the stator *induces* current in the rotor bars, which in turn produces the mechanical torque, and hence the name *induction* motor. This simplifies the construction to a great deal, as no brushes or slip rings are required. The power transfer from the stator to the rotor is brushless; it is done by magnetic flux as in the transformer. The cage rotor conductors with end rings constitute numerous shorted coils for the induced currents to circulate. From an analytical view point, the induction motor is essentially a transformer with a shorted secondary coil that can rotate.



(a) Radial cross section



(b) Axial cross section of high efficiency motor with copper cage rotor

FIGURE 5.1 Three-phase squirrel cage induction motor construction.

Most induction motors have a horizontal shaft, but vertical-shaft induction motors are also available for places where the footprint space is at a premium, such as on ships (Figure 5.2) or in city centers. Most induction motors have a squirrel-cage rotor, but some special 3-phase induction motors have a rotor wound with wires—just like the stator—with all three phase terminals brought out through slip rings. The rotor coil terminals are shorted outside the slip rings in normal operation, or with an external resistance to increase the starting torque or for speed control.

FIGURE 5.2 Vertical shaft motor saves footprints in ships. (From Raul Osigian, U.S. Merchant Marine Academy.)

For 50 and 60 Hz motors, the synchronous speed n_s of the stator flux is given in Table 5.2. The rotor speed n_r is always less than n_s, that is, the induction motor always runs at subsynchronous speed ($n_r < n_s$). The motor performance primarily depends on the rotor slip, defined as

$$\text{Slip speed} = n_s - n_r = \text{rotor slippage rpm relative to the synchronous speed} \tag{5.5}$$

$$\text{Slip } s = \frac{\mathrm{n_s - n_r}}{n_s} = \text{slip speed per unit of the synchronous speed} \tag{5.6}$$

Example 5.1

A 4-pole, 60 Hz, 3-phase, 1740 rpm induction motor runs at 1790 rpm at no load. Determine at rated load (1) the slip speed in rpm, (2) slip in per unit (pu) and percentage, and (3) speed regulation.

TABLE 5.2
Synchronous Speed of 50 Hz and 60 Hz AC Motors

No. of Poles	2	4	6	8	12
50 Hz motor	3000	1500	1000	750	500
60 Hz motor	3600	1800	1200	900	600

SOLUTION

Synchronous speed = 120 × 60 ÷ 4 = 1800 rpm

The 1740 rpm stated on the nameplate is the speed at rated load, and the no-load speed is given as 1790 rpm.

Using Equation (5.6), the slip speed at rated load = 1800 – 1740 = 60 rpm.

Slip at rated load = (1800 – 1740) ÷ 1800 = 0.0333 pu or 3.33%.

Using Equation (4.15), speed regulation = (1790 – 1740) ÷ 1740 = 0.0287 pu or 2.87%.

Under normal running operation, the slip is typically a small percentage of n_s. If all the load were removed, the motor speed will rise approximately by the slip percentage, although the exact speed regulation is determined by Equation (4.15), which applies to any source of mechanical power. Therefore, in the first approximation, *motor speed regulation = slip at rated load.* Furthermore, since the rotor sees the stator flux rotating at slip speed, the rotor current frequency is

$$f_r = s \times f \qquad \text{where } f = \text{frequency of the stator supply (main lines)} \tag{5.7}$$

The motor with load on the shaft, no matter how small, cannot run at the synchronous speed. At synchronous speed, $s = 0$, and there would be no rotor current induced and no torque produced for the motor to run. At a supersynchronous speed ($n_r > n_s$), the rotor sees the flux sweeping in the other direction, reversing the current direction, making the machine work as a generator, converting the shaft mechanical power into electrical power delivered out of the stator terminals to the power lines. Most wind power installations use induction machine as the generator driven at supersynchronous speed by the low-speed wind turbine with high gear ratio.

5.1.1 PERFORMANCE CHARACTERISTICS

The induction-motor-equivalent electrical model has a resistance R and leakage inductance L in both the stator and rotor circuits. There is a substantial amount of rotor leakage flux that cannot be ignored. It links only the rotor conductors and does not cross the air gap. The R and L are constant for the stator but vary in rotor with the rotor frequency due to skin effect in large rotor conductor bars. These variations are small and hence ignored here for simplifying the performance analysis without losing much accuracy. Denoting the constant rotor resistance R_r and leakage inductance L_r at stator frequency f, the rotor reactance $X_r = 2\pi f_r L_r$ varies directly with the slip frequency $f_r = s \times f$. The 3-phase stator voltages produce rotating magnetic flux of amplitude determined by the basic voltage Equation 3.2 in Chapter 3. The stator flux, less the leakage flux, flows in the magnetic circuit through the air gap and produces voltage in the rotor conductors equal to (Slip × V_{BR}), where V_{BR} = induced voltage in the rotor at blocked-rotor (i.e., speed = 0 or slip = 1.0). Since V_{BR} is linearly proportional to the stator voltage V by a constant factor K (equivalent to rotor-to-stator turn ratio), the rotor current can be expressed as the rotor voltage divided by the rotor impedance at rotor frequency $s \times f$, that is,

$$I_r = \frac{s V_{BR}}{\sqrt{R_r^2 + (2\pi s f L_r)^2}} = K \frac{s V}{\sqrt{R_r^2 + (2\pi s f L_r)^2}} \tag{5.8}$$

Since the torque is given by power divided by speed, the rotor torque = $I_r^2 R_r / \omega_r$. Using Equation (5.8) in this torque expression, we get the motor torque

$$T_m = K^2 \frac{s^2 V^2}{R_r^2 + (2\pi s f L_r)^2} \frac{R_r}{(1-s)} \tag{5.9}$$

Even with R_r and L_r constant (i.e., ignoring the skin effect), the motor torque varies with slip in a complex nonlinear manner. The complete torque–speed characteristic has a maximum torque point $T_{\max}$ in the middle (called the pullout or breakdown torque) as shown in Figure 5.3. Note that the speed scale has zero on the left-, and the slip scale has zero on the right-hand side. The torque increases linearly with the slip on the right-hand side of the hump, but inversely with the slip on the left-hand side of the hump, which leads to unstable operation. Therefore, the induction motor always runs on the right-hand side of the hump near the synchronous speed where the motor torque equals the load torque at the steady operating speed n_{op}. Figure 5.3 also depicts the line current versus speed. The motor draws high current at the start, which falls as the speed builds up to the steady-state operating speed.

At the start (or at standstill, or locked rotor, or blocked-rotor condition), speed = 0 and $s = 1$, R_r is negligible compared to high rotor reactance due to high rotor frequency. This simplifies Equation (5.9) to give the approximate starting torque

$$T_{\text{start}} = K_m \cdot \frac{V^2}{f^2} \tag{5.10}$$

Under normal running, with a typical low value of $s < 0.05$, the effective rotor reactance is negligible due to low rotor frequency. The running torque, therefore, is approximately given by

$$T_{\text{run}} = K_m \cdot s \cdot \frac{V^2}{f} \tag{5.11}$$

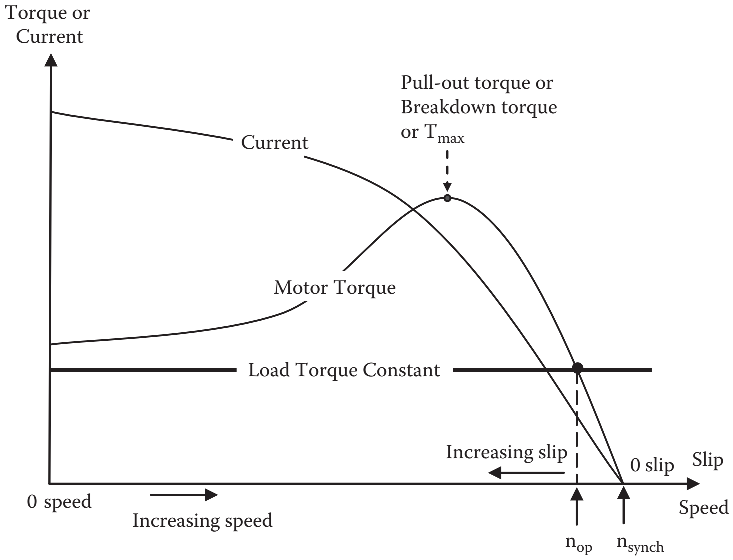


FIGURE 5.3 Torque and current versus speed and slip characteristics of induction motor.

In Equations (5.10) and (5.11), K_m = machine constant, s = rotor slip, V = applied line voltage, and f = supply line frequency. We note in Equation (5.11) that the motor running from a constant-voltage and constant-frequency source produces *torque linearly proportional to the slip* (emphasized *slip*, not the speed). There is no simple direct relation between the torque and the speed. Therefore, we must always derive the actual rotor speed from the slip found from the performance analysis.

Example 5.2

A 60 Hz, 3-phase, 4-pole induction motor delivers full load torque at 3% slip. If the load torque rises by 20% (overload condition), determine the new speed.

SOLUTION

The 60 Hz, 4-pole motor has the synchronous speed of 1800 rpm. Its full load speed at 3% slip would be (1 − 0.03) × 1800 = 1746 rpm.

Figure 5.3 and Equation (5.11) show that the motor torque is linearly proportional to the rotor *slip* in the normal running range of induction motor. The torque has no direct relation with speed; it must be derived only via the slip relation.

Therefore, with 20% rise in torque, the *slip* will also rise by 20% to new slip = 1.20 × 3 = 3.6%.

The new speed will then be (1 − 0.036) × 1800 = 1735.2 rpm. This is a drop of 1746 −1735.2 = 10.8 rpm, which is a 0.6% drop.

We note here that 20% overload causes the motor speed to drop by merely 0.6% (not by 20%). Obviously, the motor will deliver about 20% higher horsepower at 20% higher load torque at about the same speed.

Thus, the induction motor is not a constant horsepower motor; its horsepower output depends linearly on the torque loading.

Example 5.2 indicates that the induction motor speed drops by only a few percent from no-load to full load, that is, it runs essentially at a constant speed near the synchronous speed. Thus, the induction motor is not a constant horsepower motor; it delivers horsepower proportional to the torque load. Its speed can be changed only by changing the synchronous speed, which depends on the number of poles and the supply frequency. The motor speed in modern medium- and high-power installations is typically changed by the variable frequency drive (VFD) using power electronics converters. The power is supplied at a constant V/f ratio in order to create the rated flux to have rated torque under the rated armature current at all speeds starting from zero to full running speed. The need to maintain the constant V/f ratio to avoid magnetic saturation is evident from Equation (3.2).

Example 5.3

A 100 hp, 3-phase, 60 Hz, 460 V, 4-pole, 1750 rpm induction motor is used in an application that would load the motor to only 65% of the rated torque (this is obviously a higher-rated motor than we need). Determine (a) whether the motor at 65% load torque will run faster or slower than the rated speed of 1750 rpm, (b) the actual speed, and (c) the actual horsepower delivered to the load.

SOLUTION

(a) Since the induction motor slip increases with torque load, the speed decreases—although slightly—with increasing load, and vice versa. So, the motor would run faster at 65% load than at the full load rated speed of 1750 rpm.

(b) From Equation (5.2), we have

Rated torque = rated hp × 5252 ÷ rpm = 100 × 5252 ÷ 1750 = 300 lb-ft

For a 4-pole motor, synchronous speed = 120 × 60 ÷ 4 = 1800 rpm, so the rated slip = (1800 − 1750) ÷ 1800 = 0.0278 pu. Since the induction motor slip linearly varies with torque, at 65% load torque, the actual slip will be 0.65 × 0.0278 = 0.01806 pu, and the speed = 1800 (1 − 0.01806) = 1767.5 rpm.

(c) Actual horsepower delivered = (300 × 0.65) × 1767.5 ÷ 5252 = 65.55 hp, which is 65.55% of the rated 100 hp. Thus, we note that the induction motor hp output varies linearly with the load torque.

The induction motor efficiency depends on horsepower rating because large motors consuming high power are designed for better efficiency than small ones. As seen in Figure 5.4, large motors over 500 hp range have efficiency around 93% in the standard design, and around 96% in the high-efficiency design. The difference of 3% in efficiency improvement saves significant electrical energy in the motor running for 2000 to 3000 h or longer during the year.

The efficiency and power factor of a given induction motor depends on the loading level, as shown in Figure 5.5, which is for a standard 100 hp, 3-phase, 4-pole motor. The efficiency typically peaks at 75% to 80% of the rated power output.

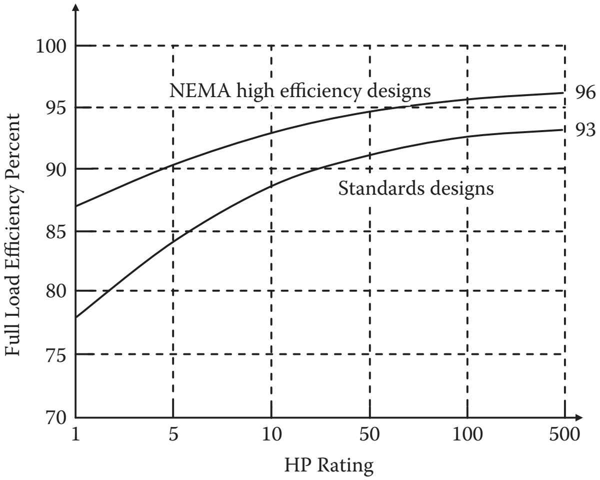


FIGURE 5.4 Induction motor efficiency at full load versus horsepower rating.

5.1.2 STARTING INRUSH kVA CODE

Like all motors, the induction motor starting directly at full line voltage draws high inrush current several times greater than the normal rated current as was seen in Figure 5.3. Such high current leads to the following ill effects:

- Unnecessary motor-fuse melting or circuit-breaker tripping (a nuisance since motor starting is not an abnormal fault condition)
- Momentary voltage drop in the cable that may disturb other sensitive components around the motor and may cause light flickers
- Excessive kVA drawn during starting causes heavy stress on the power lines and the source

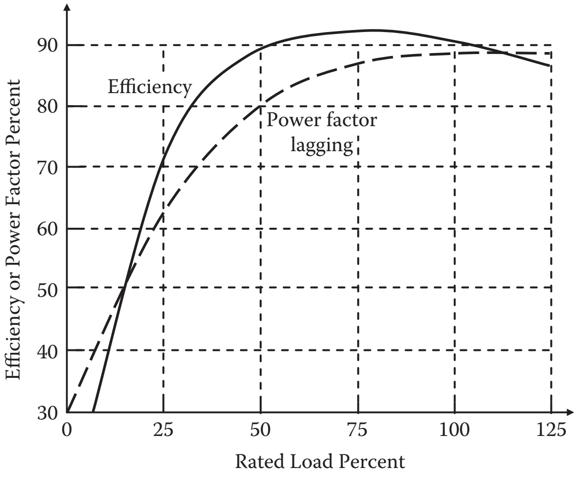


FIGURE 5.5 Standard 100 hp induction motor efficiency and power factor versus load.

TABLE 5.3
Motor Starting kVA Codes

Code Letter	Starting kVA per hp	Code Letter	Starting kVA per hp
A	0–3.14	L	9.0–9.99
B	3.15–3.54	M	10.0–11.19
C	3.55–3.99	N	11.2–12.49
D	4.0–4.49	P	12.5–22.99
E	4.5–4.99	R	14.0–15.99
F	5.0–5.59	S	16.0–18.99
G	5.6–6.29	T	18.0–19.99
H	6.3–7.19	U	20.0–22.39
J	7.2–7.99	V	> 22.4
K	8.0–8.99	—	—

Source: NEMA.

The source, cable, and circuit protection all must be sized for such inrush current, which is usually estimated from the code letter included in the motor nameplate. The code letter is expressed as the starting kVA per horsepower of the motor rating and is known as the locked-rotor or blocked-rotor kVA of the motor. It indicates the severity of the starting stress in terms of the kVA/hp it draws from the source on direct line starting. The range of codes found on motor nameplates is given in Table 5.3, which varies from low 3.14 to high 22.4 kVA/hp. A motor with higher code letter produces higher stress on the power system design.

Example 5.4

A 100 hp, 3-phase, 60 Hz, 460 V, 1710 rpm, 4-pole, letter code J, squirrel-cage induction motor with Δ-connected stator has efficiency of 90% and power factor 0.85 lagging when operating at rated load. Determine the input kW, kVA, and line current during normal operation, and the line current on direct start in Δ.

SOLUTION

Output power = 100 hp × 746 ÷ 1000 = 74.6 kW.
Input power = 74.6 ÷ 0.90 = 82.89 kW.
Apparent power = 82.89 ÷ 0.85 = 97.52 kVA.
Line current = 97.52 × 1000 ÷ (√3 × 460) = 122.4 A.

The starting current at rated voltage (i.e., direct on-line start in Δ, *without* using a Y-Δ or reduced voltage or reduced frequency starter) is given by the letter code found on the motor nameplate. For this letter code J motor, Table 5.3 gives 7.2 to 7.99 kVA/hp at starting, so we use the conservative number 7.99. The starting inrush kVA = 7.99 × 100 = 799 kVA.

$I_{\text{direct}\Delta\text{start}} = 799 \times 1000 \div (\sqrt{3} \times 460) = 1004$ A. This is 8.2× normal rated current of 122.4 A. However, 1004 A will gradually decay to 122.4 A as the motor gains speed.

5.1.3 TORQUE–SPEED CHARACTERISTIC MATCHING

Matching the torque versus speed characteristic of the induction motor with that of the load is an important consideration for system design. The motor connected to a high-torque load during start must have high starting torque, or else it will not start. Some motors have high starting torque but lower efficiency compared to others. The high starting torque requires rotor with high resistance, resulting in poor efficiency under running conditions. The National Electrical Manufacturers Association (NEMA) classifies motor designs according to the starting torque and the maximum breakdown (pullout) torque as depicted in Figure 5.6 and listed in Table 5.4. The motor design features depicted in Figure 5.5 are as follows:

Design B is a general-purpose motor used in centrifugal pumps, fans, blowers, and machine tools requiring low starting torque.

Design A is also for general purposes, but it has a higher pullout torque than the Design B motor.

Design C motor offers a good compromise between high starting torque and high running efficiency. It is suitable for plunger pumps and compressors.

Design D motor, having high rotor resistance, has the highest starting torque but has the lowest efficiency under full-load running conditions. It is primarily used with high inertia loads, such as flywheel-punch-presses, elevators, cranes, hoists, barge positioning, rail car pulling, ship and barge mooring, etc. Capstans and winches also use Design D motors for smooth starting under load and 300% starting torque.

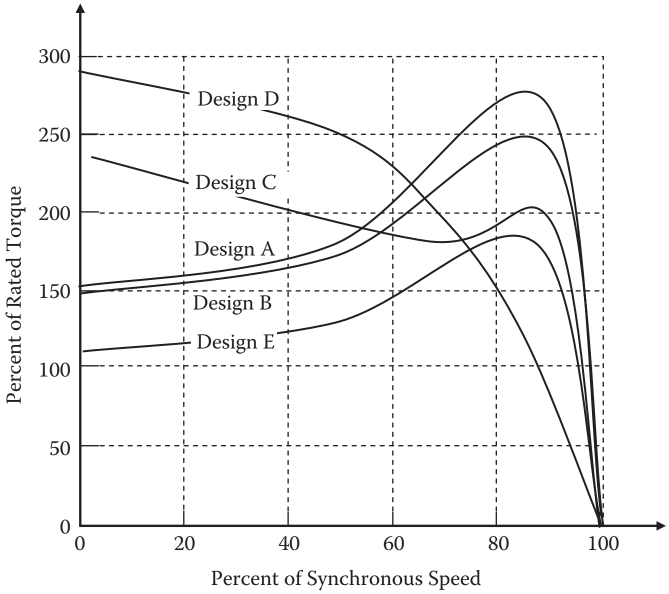


FIGURE 5.6 Induction motor torque versus speed characteristics by design classification.

TABLE 5.4
Starting Torque and Maximum Pullout Torque of Induction Motors in Selected Horsepower Ratings

HP Rating	Starting Torque (% of Rated Torque)			Pullout Torque (% of Rated Torque)		
Design A, B	**2-pole**	**4-pole**	**8-pole**	**2-pole**	**4-pole**	**8-pole**
1	—	275	135	250	300	215
5	150	185	130	215	225	205
10–200	135–100	165–100	125–120	200	200	200
250–500	70	80	100	175	175	175
Design C	2-pole	4-pole	8-pole	2-pole	4-pole	8-pole
5	—	255	225	—	200	200
25–200	—	200	200	—	190	190
Design E	2-pole	4-pole	8-pole	2-pole	4-pole	8-pole
1–20	180–140	190–150	150–120	200	299	180
50–100	120–100	130–110	120–100	180	180	170
300–500	80–75	90–75	90–75	160	160	160

Design E motor, having low rotor resistance, gives the lowest starting torque but highest running efficiency. It costs more but pays back in energy savings where the load runs for extended periods of time over the year. We see in Table 5.4 that Design E motors in 300–500 hp ratings have starting torque less than the rated torque. These motors, therefore, must be brought to speed under light load and then fully loaded after acquiring full speed. For this reason, they are primarily used for centrifugal pumps, fans, blowers, and machine tools, all requiring low starting torque.

Since the induction motor consumes 93% of all the electrical energy used by all motors combined, its efficiency reporting on the nameplate has become the industry standard. However, various international standards for testing and reporting the induction motor efficiency on the nameplate can be significantly different. For example, the following table compares the efficiency and power factor of a 70 hp induction motor derived from three international standards, namely, the American Standards for testing induction motor efficiency (IEEE-112), International Electrotechnical Commission Standards (IEC34-2), and the Japanese Electrotechnical Commission Standards (JEC-37):

	IEEE-112	**IEC34-2**	**JEC-37**
Efficiency	90.0	92.7	93.1
Power factor	86.2	86.2	86.3

Although the power factor test results are consistent, the efficiency tests leave much to be desired. The IEEE-112 standard results in the lowest efficiency. The difference comes from the different treatments of the stray-load loss (miscellaneous losses due to stray leakage flux not easy to quantify), which vary approximately with the load squared. The IEEE method derives it indirectly from tests, and the IEC34-2 assumes to be fixed 0.5% of the rated power, whereas the JEC-37 ignores it altogether, resulting in the highest efficiency. The point here is that a standard efficiency motor made in the United States cannot be compared with a high-efficiency motor made in Japan for energy-saving considerations.

5.1.4 MOTOR CONTROL CENTER

The motor control center design requires specifying the following:

1. The continuous and sustained short-circuit amperes that the bus must withstand without thermal and mechanical damage. Typical continuous current ratings of the motor control centers are 600 A, 800 A, and 1000 A, with typical short-circuit ampere rating of 40 kA, 65 kA, and 100 kA, respectively.
2. The motor starter size (usually Y-Δ starter) as specified by NEMA frame size below:

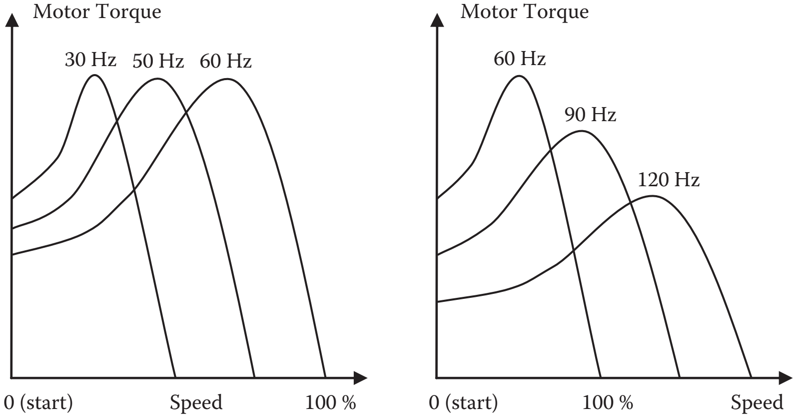
Motor HP Rating:	< 2	5	10	25	50	75	150	300
NEMA Frame Size:	00	0	1	2	3	4	5	6

   As an example, we read from the foregoing table that a 30 hp motor would need NEMA frame size 3 starter (rounded up to the nearest standard size).
3. Enclosure type, that is, ventilated dip proof or sealed watertight.
4. Protection type, that is, fuse or molded-case circuit breaker. The molded-case breaker trips all three phases together to eliminate single phasing and is usually of single element type without the thermal element. For this reason, the fuse protection, having a quick fusible link that melts quickly at higher current, has generally higher short-circuit current ratings and limits the damage.

5.1.5 PERFORMANCE AT DIFFERENT FREQUENCY AND VOLTAGE

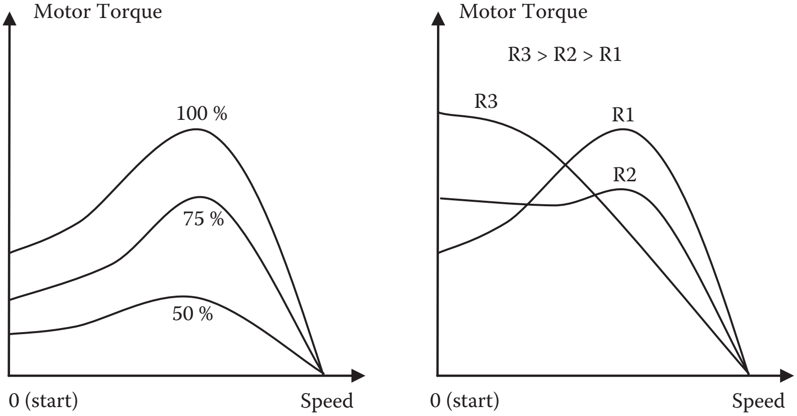
The induction motor is often used with variable frequency drives. At lower-than-rated frequencies, the drive also lowers the voltage to maintain the V/f ratio constant. This gives constant air-gap flux and constant torque at the rated armature current, as shown in Figure 5.7(a). At higher-than-rated frequencies, the motor is operated at rated voltage to lower the flux and torque as shown in Figure 5.7(b). This maintains constant horsepower to avoid high armature current and overheating.

At constant frequency and reduced voltage, the maximum torque point shrinks in voltage-squared proportions as shown in Figure 5.7(c). Such variable-voltage



(a) At lower frequency and constant air-gap flux (V/f ratio constant)

(b) At higher frequency and constant terminal voltage (reduced air-gap flux)



(c) At constant frequency and lower terminal voltage (reduced air-gap flux)

(d) At different rotor resistance (constant frequency and constant terminal voltage)

FIGURE 5.7 Induction motor torque versus speed curves at different frequencies and voltages.

operation is often used for speed control of small motors in fans and pumps where the load torque is proportional to speed squared. At constant frequency and constant voltage, the motor torque varies with rotor resistance as shown in Figure 5.7(d). High rotor resistance gives high starting torque, although with poor running efficiency. In the wound rotor induction motor, an external resistance is inserted in the rotor via slip rings and brushed to get high starting torque for heavy loads. Once the motor comes to full speed, the external resistance in the rotor circuit is reduced to zero by shorting the slip rings to gain better running efficiency.

5.2 SYNCHRONOUS MOTOR

The synchronous motor has the same construction as the synchronous generator, except that the current and energy conversion directions are reversed. The 3-phase stator currents produce the rotating magnetic field as in the induction motor, and dc current is injected in the rotor coil from outside through brushes and slip rings. Therefore, it costs more and requires more maintenance compared to the induction motor. The synchronous motor works on the magnetic attraction between the stator and the rotor magnetic fields, as opposed to the current induced in the rotor by the sweeping stator flux in the induction motor. With stator with *P* number of poles and supply frequency *f*, the synchronous motor runs exactly at synchronous speed at all loads, which is given by

$$Synchronous\, speed\ in\ rpm = \frac{120\,f}{P} \tag{5.12}$$

Since the synchronous motor is typically used in large size, it is usually driven by a variable frequency drive (VFD) for speed control.

The electrical model is the same as that for the synchronous generator we covered in Chapter 4, except that the armature current now reverses, and KVL gives $\tilde{V}_T = \tilde{E}_f + j\,\tilde{I}X_s$. The phasor diagram, therefore, changes accordingly as shown in Figure 5.8. The excitation current that determines E_f controls the motor power factor. Overexcitation results in leading power factor, and underexcitation results in lagging power factor. Some synchronous motors are designed to continuously operate in the overexcitation mode, providing leading kVARs to improve the system power factor, like the capacitors. For this reason, an overexcited synchronous motor with no mechanical load—running only to supply leading kVARs to the system—is called the synchronous condenser (capacitor).

Example 5.5

A 10,000 hp, 60 Hz, 1200 rpm, 4.1 kV, 3-phase, Y-connected synchronous motor draws the armature current of 1500 A at 0.80 leading power factor under rated load. It has synchronous reactance of 1.2 Ω/ph and negligible resistance. Determine the field excitation voltage (counter emf) E_f and the torque angle δ.

SOLUTION

We make per-phase calculations. For Y-connection, $V_T = 4100 \div \sqrt{3} = 2367\ V$, which we take as the reference phasor. The armature current at 0.8 power factor leading, $I_a = 1500\ \angle\cos^{-1}0.80 = 1500\ \angle 36.87°$ A.

Based on the phasor diagram in Figure 5.8, modified for leading power factor, the counter emf in the motor is given by

$$\tilde{E}_f = \tilde{V}_T - \tilde{I}_a \times jX_s = 2367\angle 0° - 1500\angle 36.87° \times 1.2\angle 90°$$

$$= 2367\angle 0° - 1800\angle 126.87° = 3447 - j\,1440 = 3735.7\angle{-22.67°}$$

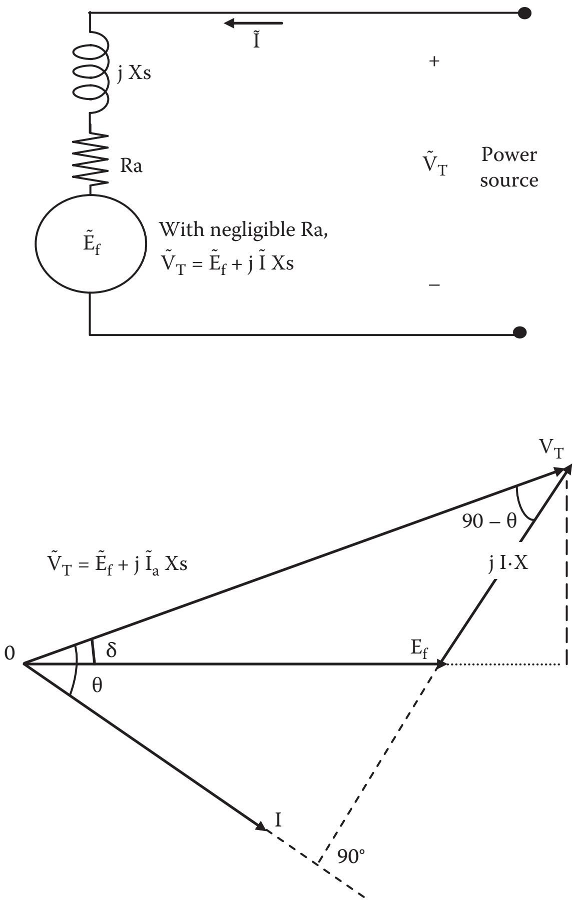


FIGURE 5.8 Synchronous motor electrical model and phasor diagram.

> Therefore, the torque angle δ = – 22.67° at rated load. The negative sign indicates that the counter emf lags the voltage applied at the terminals, that is, the rotor magnetic field lags (follows) the stator magnetic field. In the synchronous generator, the opposite is true.

The synchronous motor performance analysis under mechanical load is similar to that of the synchronous generator, resulting in the torque output at the motor shaft given by

$$T_{\text{motor}} = T_{\max} \sin\delta \tag{5.13}$$

where $T_{\max}$= maximum (pullout) torque the motor can develop, and δ = torque angle (electrical degrees). The T_{motor} versus δ plot shown in Figure 5.9 is similar to that of P_{gen} versus δ for the synchronous generator, and the same steady-state and transient stability issues with step load changes apply here as well. Note that

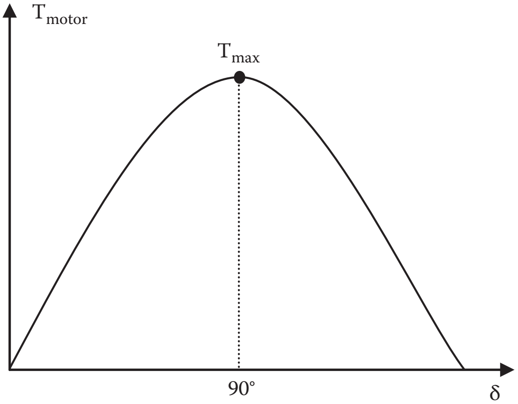


FIGURE 5.9 Synchronous motor torque versus torque angle δ.

δ is called the torque angle in the motor, as opposed to the power angle in the generator, but it is still the angle between the center lines of the stator and the rotor magnetic fields.

Equation (5.13) gives the maximum motor torque $T_{motor} = T_{max}$ at $\delta = 90°$. Beyond this point, the motor becomes unstable, falls out of synchronous operation, and stalls. The motor is normally designed to operate at δ equal to 25°–35° at rated load, and step load changes are kept below 30%–35% of the rated torque to assure transient stability. The damper bars here serve the same purpose as in the synchronous generator. The current induced in them during transient oscillations around the synchronous speed damps the electromechanical oscillations following a step load change. The partial cage formed by the damper bars and end chords is used for starting the synchronous motor as the induction motor under a reduced voltage and/or frequency before exciting the rotor field that eventually pulls the rotor in the synchronous mode. Then onward, the rotor runs at the synchronous speed in lockstep with the stator rotating the magnetic field. The torque is produced by the magnetic attraction between the two fields, with the rotor field trailing the stator field by angle δ. This is the reverse of the synchronous generator, where the rotor field leads the stator field by angle δ. One can think this way: the magnetic field of the primary source of power leads the other field by angle δ; the one that has the energy to supply leads the other, as found in most other places around us, in machines or in people.

Example 5.6

A synchronous motor is running at 70% rated load torque when the torque angle is 20°. Determine its pullout torque T_{max} (steady-state stability-limit torque) in percentage of the rated torque. If the load torque is increased to 100% rated torque in one step, determine the worst-case transient peak value of the torque angle. Ignore the damping bar contribution for a conservative estimate and assume the torque versus δ relation as approximately linear in the operating range.

SOLUTION

The synchronous motor torque is related to the torque angle δ by $T = T_{\max} \sin\delta$.

At 70% load operation, $0.70\ T_{\text{rated}} = T_{max} \sin 20° = 0.342\ T_{\max}$, which gives $T_{\max} = (0.70 \div 0.342)\ T_{\text{rated}} = 2.05\ T_{\text{rated}}$, or 205% of rated torque.

Under the step change in load torque from 70% to 100%, the new δ after the transient swing subsides will be $T = T_{\max} \sin\delta_{\text{new}}$, that is, $1.0 \times T_{\text{rated}} = 2.05 \times T_{\text{rated}} \sin\delta_{\text{new}}$.

$\therefore \sin\delta_{\text{new}} = 1.0 \div 2.05 = 0.488$, which gives $\delta_{\text{new}} = 29.2°$.

If we ignore damping in the first cycle and assume sinδ approximately linear, then the worst overshoot due to the rotor's mechanical inertia will be 100% of the change in δ, that is, $\delta_{\text{overshoot}} = 29.2° - 20° = 9.2°$, and $\delta_{\text{peak}} = 29.2 + 9.2 = 38.4°$.

The rotor angle will swing from 20° to 38.4° and will come back to 20° and then swing again upward. Such oscillations with 9.2° amplitude around the new steady-state value of $\delta = 29.2°$will continue until the swing energy is gradually dissipated by I^2R power loss in the rotor damper bars, cycle by cycle.

Since the worst-case swing peak of 38.4° is well below the 90° that is required for transient stability, the machine operation would be stable during and after the rotor oscillations following the step load change.

The rotor of the synchronous motor can have any one of the following configurations:

- Conventional rotor with dc excitation current injected from outside via slip rings and brushes
- Brushless rotor with exciter and diodes on the rotating shaft of the main motor
- Permanent magnet rotor (brushless, as no dc current is needed on the rotor)
- Reluctance rotor (brushless, as no dc current is needed; it works on self-induced magnetism in the ferromagnetic rotor)

The rotor with slip rings and brushes is the least desirable design because of its low reliability and high maintenance. Most large synchronous motors are of brushless design. The permanent magnet synchronous motor has been widely used in small- and medium-power applications but has recently drawn a good deal of interest in high-power motors for electric propulsion in navy and cruise ships. The neodymium–iron–boron permanent magnet is common in such designs since it offers high magnetic strength.

5.3 MOTOR HP AND LINE CURRENT

The motor delivers horsepower to drive load torque T at speed n. In the ac or dc motor, the three are related as

$$HP = \frac{T_{lb.ft}\, n_{rpm}}{5252} = \frac{T_{n.m}\, \omega_{mech\ rad/\text{sec}}}{746} \tag{5.14}$$

The nameplate fixed on the induction and synchronous motors includes the following key motor ratings: HP or kW output, 1-phase or 3-phase, frequency, line-to-line voltage, speed (rpm), temperature rise in degree Celsius, service factor (SF), power factor (PF), starting kVA code, efficiency, and design type, where

rpm = shaft speed at rated voltage and frequency when delivering rated horsepower

Temperature rise = average temperature rise of the conductor above standard 40°C ambient air under continuous rated load

SF = service factor = ratio of the maximum permissible continuous overload to the rated load for a short duration (usually <2 hours)

The full-load line current I_L drawn by the 3-phase induction or synchronous motor connected to line-to-line voltage V_{LL} can be determined from the following relation:

$$\sqrt{3} \cdot V_{LL} \cdot I_L \cdot PF = \frac{746 \cdot HP}{Efficiency} = \frac{1000 \cdot kW}{Efficiency} \tag{5.15}$$

The output rating of motor pumping fluid at a flow rate of Q m^3/h against the total pressure head H meters of fluid with specific gravity SG and pump efficiency η_{pump} is given by

$$Motor\ kW = \frac{Q \cdot H \cdot SG}{367 \cdot \eta_{\text{pump}}} \quad \text{and} \quad Motor\ HP = \frac{Motor\ kW}{0.746} \tag{5.16}$$

where the total pressure head H = static pressure + friction loss in pipes + loss in pump. The specific gravity of sea water is around 1.03, but that of crude oil varies from 0.76 to 0.85 depending on the type and temperature as listed in Table 5.5.

TABLE 5.5
Specific Gravity of Crude Oil at Various Temperatures

	Temperature °F	Temperature °C	Specific Gravity[a]
Crude oil 48°	60	15.6	0.79
API	130	54.4	0.76
Crude oil 40°	60	15.6	0.825
API	130	54.4	0.805
Crude oil 35.6°	60	15.6	0.847
API	130	54.4	0.824
Crude oil 32.6°	60	15.6	0.832
API	130	54.4	0.84

[a] Specific gravity = 1.00 for fresh water @ 60°F and 1.03 for sea water.

Example 5.7

A 100 hp, 3-phase, 460 V, 60 Hz, ac motor has a power factor of 0.90 lagging and efficiency of 94% on its nameplate. Determine the line current it will draw while delivering rated horsepower.

SOLUTION

Power drawn from the source = output ÷ efficiency = 100 hp × 746 W ÷ 0.94 efficiency = 79,362 W.

Using the general 3-phase power relation $P_{3\text{-}ph} = \sqrt{3}\ V_{LL}\ I_L$ pf,

Motor line current $I_L = 79{,}362 \div (\sqrt{3} \times 460 \times 0.90) = 110.68$ A

Example 5.8

A water-pumping system in a ship is designed to pump 90 m^3/h fresh water against 80 m of total pressure head. Determine the motor horsepower rating and the motor control center kVA rating, assuming 94% pump efficiency, 96% motor efficiency, and 90% pf lagging.

SOLUTION

The total head of 80 m may be partly due to elevation difference between the water entrance and discharge and partly in pipe friction; it does not matter to us how it is divided between the sources of pumping head.

Using Equation (5.16), $\text{Motor kW} = \frac{90 \times 80 \times 1.0}{367 \times 0.94} = 20.87$, and $\text{Motor HP} = \frac{20.87}{0.746} = 28$.

Motor input power in kW = 20.87 ÷ 0.96 = 21.74.

Motor control center kVA rating = 21.74 ÷ 0.90 = 24.16.

5.4 DUAL-USE MOTORS

Dual-frequency motors often used on ships are designed to operate either from 50 Hz or 60 Hz supply around the globe. The motor must be operated at the design frequency, or else its performance will severely degrade. For example, if a 60 Hz motor is connected to 50 Hz lines, it will run at 50/60 = 0.8333 pu or 83.33% of the 60 Hz rated speed, that is, 16.67% slower. If the same voltage is applied, its magnetic core would have 16.67% higher flux and would saturate, drawing significantly higher magnetizing current, leading to severe overheating. To avoid such overheating, 60 Hz motor voltage and horsepower ratings must also be reduced by 16.67% when operating at 50 Hz. Under such reduced load, however, the starting torque and the pullout torque remain essentially the same as in a 60 Hz operation.

In the reverse, a 50 Hz motor can be operated at 60 Hz with appropriate derating. However, its speed will be 60/50 = 1.2 × rated speed, or 20% over the design speed, and hence the torque load must be reduced by 20% to keep the horsepower load the same. Dual frequency motors are designed to operate at either 50 or 60 Hz safely but with appropriate derating.

Example 5.9

A 3-phase, 460 V, 60 Hz induction motor with a Δ-connected stator is designed to have the magnetic flux density in the core at the saturation limit of 1.6 T. If this machine is connected to 50 Hz lines, determine the voltage for limiting the flux density to the same 1.6 T saturation limit.

SOLUTION

Each phase of the Δ-connected stator sees the line voltage.

Using Equation (3.2) at 60 Hz, we have $V_{60\text{Hz}} = 460 = 4.444 \times 60 \times N \times \phi_m$.

At 50 Hz, we have $V_{50\text{Hz}} = 4.444 \times 50 \times N \times \phi_m$.

Taking the ratio of the two with the same number of turns N and to keep the same flux ϕ_m, we have $V_{50\text{Hz}}/460 = 50/60 = 0.8333$, or $V_{50\text{Hz}} = 0.8333 \times 460 = 383$ V. Thus, the voltage should be reduced in the same proportion as the frequency reduction.

The motor at 50 Hz will run at 50/60 = 0.8333 × 60 Hz rated speed. To avoid overheating, the current must be limited to the same 60 Hz rated value, which would give the same 60 Hz rated torque under constant air-gap flux.

Since horsepower = torque × speed, the horsepower loading at 50 Hz must be reduced to 50/60 = 0.833 pu or 83.33% of the 60 Hz horsepower rating.

Dual-speed motors are designed with multiple groups of stator coils that can be reconnected in two configurations in which the number of poles can be changed by a factor of two. Therefore, dual-speed motors are also known as pole-changing motors. They are used for house air-blowers, fluid transfer pumps, ballast pumps, and saltwater pumps.

Dual-voltage motors are designed with multiple groups of stator coils that can be connected in one of the two alternative—series or parallel—configurations such that the stator terminal voltage can be changed by a factor of two. The motor is normally connected to a high voltage to result in low current and low I^2R power loss. For example, a 230–460 V dual-voltage motor is normally connected for 460 V operations. In small sizes, they are used in 1-phase portable fan motors in the pump room.

5.5 UNBALANCED VOLTAGE EFFECT

The single-phasing (losing one line voltage completely) is an extreme example of voltage unbalance. Small unbalance in 3-phase voltages occurs when 1-phase loads are not uniformly distributed on three phases. The unbalance in 3-phase voltage is defined as the maximum deviation in the line-to-line voltages from the average value, that is,

$$\text{Unbalance voltage } UBV = \frac{\text{Max voltage deviation}}{\text{Average line voltage}} = \frac{V_{\text{max deviation}}}{(V_{ab} + V_{bc} + V_{ca})/3} \qquad (5.17)$$

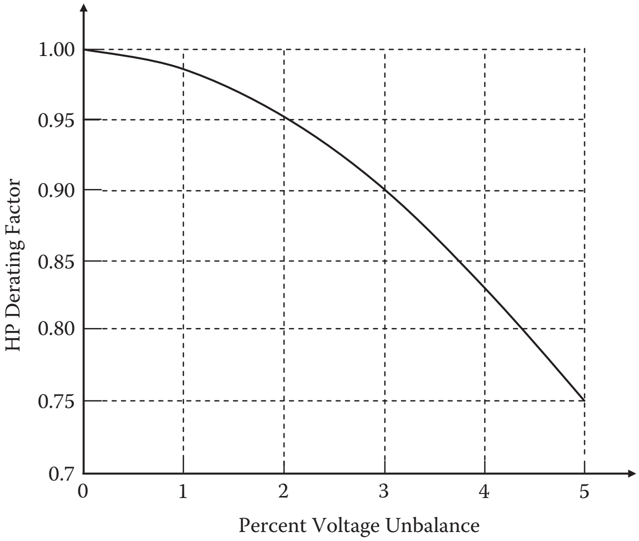


FIGURE 5.10 Three-phase motor derating factor versus voltage unbalance.

Experience indicates that the additional temperature rise under unbalanced voltage is given by

$$\%\Delta T = 2\ (\%UBV) \text{ and } \Delta T = (\%\Delta T/100) \times T_{\text{rated}} \tag{5.18}$$

Even a few percent unbalance in 3-phase voltages can degrade the motor efficiency and torque and increase the heating. The resulting overheating can be avoided by reducing the motor horsepower load, that is, the motor must be derated, as per Figure 5.10. Otherwise, the motor life will get shorter.

Example 5.10

A 150 hp, 3-phase, 460 *V* induction motor has the design life of 20 years at 110°C rise in conductor temperature above the ambient air. If it is continuously operated under unbalanced line voltages of 460, 440, and 420 *V*, determine (a) percentage voltage unbalance, (b) additional temperature rise when operating at rated load, (c) expected life at rated load, and (d) the required horsepower derating for the motor to last the design life of 20 years.

SOLUTION

(a) $V_{avg} = \frac{1}{3}(460 + 440 + 420) = 440\ V$.

The voltage deviations are $|460 - 440| = 20$, $|440 - 440| = 0$, and $|420 - 440| = 20$.

∴ Using Equation (5.17), imbalance in voltage, UBV = 20 ÷ 440 = 0.0455 pu or 4.55%.

Additional temperature rise, $\%\Delta T = 2\ (\%UBV) = 2\ (4.55\%) = 9.1\%$.

$\therefore \Delta T_{UBV} = (0.091)\ 110°C = 10°C$, that is, the motor would run 10°C hotter than the design temperature rise of 110°C.

Using Equation (3.24), $Expected\ Life = \dfrac{20}{2^{(10/10)}} = 10\ years$

The horsepower derating required for maintaining its design life of 20 years is derived from Figure 5.10, which shows that, at 4.55% UBV, the motor horsepower should be derated to 0.78 of the normal rating, that is, 150 × 0.78 = 117 hp.

The adverse effects of single-phasing or unbalance voltages can be analyzed in terms of symmetrical components presented in Appendix A. The analysis briefly runs as follows. The unbalanced voltages are resolved into symmetrical component voltages in the positive sequence (rotating counterclockwise), negative sequence (rotating clockwise), and zero sequence (not rotating at all). These sequence components are derived by the following equations set, where the operator a = 120° phase shift (similar to operator j = 90° phase shift that we routinely use to analyze ac power circuits).

Positive sequence voltage of phase A, $\tilde{V}a_1 = 1/3\ (\tilde{V}a + a \times \tilde{V}b + a^2 \times \tilde{V}c)$.

Negative sequence voltage of phase A, $\tilde{V}a_2 = 1/3\ (\tilde{V}a + a^2 \times \tilde{V}b + a \times \tilde{V}c)$.

Zero sequence voltage of phase A, $\tilde{V}a_0 = 1/3\ (\tilde{V}a + \tilde{V}b + \tilde{V}c)$ (5.19)

Note that operator a = 120° phase shift and a^2 = 240° phase shift. Once we calculate the sequence components in phase A, the positive sequence component voltages in phase B will be shifted by 120°, and that in phase C will be shifted by 240°, that is, $\tilde{V}_{b1} = a \times \tilde{V}_{a1}$, and $\tilde{V}_{c1} = a^2 \times \tilde{V}_{a1}$. The negative sequence components are $\tilde{V}_{b2} = a^2\ \tilde{V}_{a1}$ and $\tilde{V}_{c2} = a \times V_{a2}$. The zero sequence components are $V_{bo} = V_{a0}$ and $V_{c0} = V_{a0}$. All these voltages are per phase (not line-to-line).

We see here that (1, a, a^2) represents a balance symmetrical 3-phase phasors of unit magnitudes, and V_{a1} is phase A component of the unbalance voltages (V_a, V_b, V_c) on the balance positive sequence frame (1, a, a^2); V_{a2} is phase A component of the unbalance (V_a, V_b, V_c) on the balance negative sequence frame (1, a^2, a); and V_{a0} is phase A component of the unbalance voltages (V_a, V_b, V_c) on the zero sequence frame (1, 1, 1). For balance 3-phase voltages, the phasor sum ($\tilde{V}_a + \tilde{V}_b + \tilde{V}_c$) add up to zero, giving $V_{a0} = 0$, which we know to be true. For balance 3-phase voltages, V_{a2} will also reduce to zero, leaving only Va_1 to have nonzero value, which will be the actual value of $\tilde{V}_a$.

The sequence voltages applied to the motor will produce sequence currents and corresponding sequence fluxes. However, since most motors are connected in Δ, or ungrounded Y on the stator side, the three line current must add up to zero in the absence of any return path. Therefore, no zero sequence current can flow into the motor, and need not be considered any further. The performance of the motor with both positive and negative sequence voltages can be determined by using the superposition theorem, that is, by calculating the motor performance under the positive and negative sequence voltage separately and then superimposing the results. It can also be realized by the following reasoning.

The positive sequence voltage produces the magnetic flux rotating at synchronous speed in the positive direction and torque in that direction. The induction motor runs in the positive direction of rotating flux with a small slip. If a 60 Hz motor is running at 0.05 per unit slip, the rotor current frequency is 0.05 × 60 = 3 Hz, which will produce rated rotor current and I^2R loss. However, the negative sequence voltage produces the flux rotating backward at synchronous speed. For the rotor running in the forward direction at 95% of the synch speed, the slip from the negative sequence flux is 1.95. For such high slip, Equation (5.8) gives much higher rotor current, producing correspondingly high I^2R loss and negative torque. Both of these effects result in wasted energy and overheating, costing more in energy bills and reducing motor life.

Example 5.11

A 3-phase induction motor in 690 V distribution system under an abnormal condition is working on highly unbalanced 3-phase voltages $\tilde{V}_a = 450\angle 0^\circ$, $\tilde{V}_b = 450\angle -90^\circ$, and $V_c = 900\angle -225^\circ$. Determine its symmetrical component voltages V_a^+, V_a^-, and V_a^0, and comment on approximate motor current and power losses in the rotor and stator.

SOLUTION

The symmetrical components of the unsymmetrical voltages are derived from an equation similar to Equation (A.3) in Appendix-A, with all angles in degrees:

$$\begin{pmatrix} Va^0 \\ Va^+ \\ Va^- \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}\begin{pmatrix} Va \\ Vb \\ Vc \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1\angle 120 & 1\angle 240 \\ 1 & 1\angle 240 & 1\angle 120 \end{pmatrix}\begin{pmatrix} 450\angle 0 \\ 450\angle -90 \\ 900\angle -225 \end{pmatrix}$$

This matrix equation leads to

$Va^0 = \frac{1}{3}\{450\angle 0 + 450\angle -90 + 900\angle -225\} = 87.75\angle -225^\circ$
$Va^+ = \frac{1}{3}\{450\angle 0 + 450\angle 30 + 900\angle 15\} = 590\angle 15^\circ$
$Va^- = \frac{1}{3}\{450\angle 0 + 450\angle 150 + 900\angle -105\} = 222.3\angle -105^\circ$

The positive sequence voltage is 590 ÷ 690 = 0.855 pu of the rated voltage, which will produce 85.5% flux rotating in the positive direction.

The negative sequence voltage of 222.3 ÷ 690 = 0.322 pu will produce 32.2% of rated flux rotating in the reverse direction.

The zero sequence voltage of 87.75 ÷ 690 = 0.127 will produce 12.7% of rated flux not rotating at all (alternating but stationary in space).

If the rotor were running under such unbalanced voltage in the positive (forward) direction at 0.05 pu slip, then the rotor slip will be 2 – 0.05 = 1.95 pu under the negative sequence flux, and 0.95 pu under the zero sequence flux. Such high slips of 1.95 pu and 0.95 pu will produce excessive currents and I^2R power losses in both the rotor and the stator, burning the motor soon if the situation is not corrected.

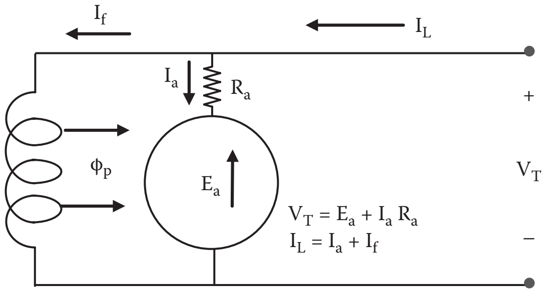


FIGURE 5.11 DC shunt motor electrical model.

5.6 DC MOTOR

The placement of the armature and the field coils in the dc machine is reversed from that in the ac machine. The dc motor has the field coil on the stator and the armature coil on the rotor. The dc is injected in the rotor (armature) via the commutator and brushes. Such construction features result in high capital and maintenance costs.

The electrical model of the most widely used dc motor—the shunt motor—is shown in Figure 5.11. The rotating armature in the dc flux produces voltage E_a that works as the back voltage. The applied terminal voltage V_T must overcome E_a and the armature voltage drop I_aR_a to push the current in. The motor speed would settle when the following KVL equation is satisfied, that is,

$$V_T = E_a + I_aR_a \qquad \text{where } E_a = K_m\ \varphi_p\ n \tag{5.20}$$

and I_a = armature current, R_a = armature resistance, φ_p = flux per pole that depends on the field current I_f, n= rotor speed, and K_m = machine constant. Equation (5.20) can be rearranged in terms of the armature current or the speed as follows:

$$I_a = \frac{V_T - K_m \quad {}_p n}{R_a} \qquad \text{or} \qquad speed\ n = \frac{V_T - I_a \cdot R_a}{K_m \quad {}_p} \tag{5.21}$$

Thus, the speed can be controlled by varying the applied voltage or by changing the flux per pole φ_p, which depends on the field current I_f in a linear nonsaturated region. Since the dc motor has relatively simple speed–torque–current relations, its speed control and the associated power electronics are simpler and lighter than those for the induction and synchronous motors. This was the main reason for the dc motor's wide use in the past where the speed control was required.

The starting current in the armature derived from Equation (5.21) with $n = 0$ is given by $I_{start} = V_T/R_a$, which is high since R_a is small. This high inrush current decays to the rated value (Figure 5.12) as the motor builds up the speed and the back voltage. To avoid overheating due to high starting current, large motors with high inertia needs reduced-voltage starting until the speed builds up and the current approaches the normal rated value.

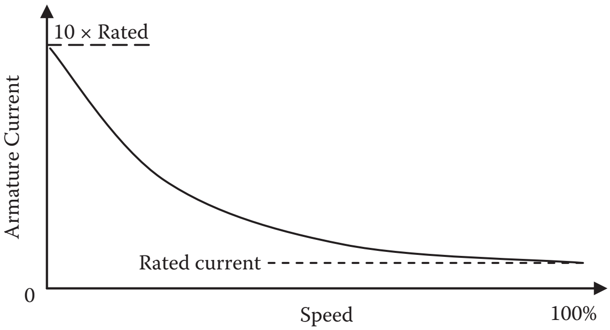


FIGURE 5.12 DC shunt motor current from start to full speed.

The steady-state full-load line current I_L drawn by the dc motor from line voltage V_T is given by the following equation:

$$V_T \cdot I_L = \frac{746 \cdot HP}{Motor\ Efficiency} \tag{5.22}$$

Example 5.12

A 50 hp, 240 V dc motor runs at 2350 *rpm* at rated conditions. If its field current is decreased by 20% while keeping the load current the same, determine its approximate and exact speed.

SOLUTION

The motor is assumed to be the shunt type, as it is the most widely used dc motor. Its speed changes inversely with the field current under the same loading. Therefore, a 20% decrease in the field current will increase the motor speed approximately by 20% to 2350 × 1.2 = 2820 *rpm*.

For exact speed, new field current = 0.80 × rated field current.

$$\therefore Exact\ new\ speed = \frac{Rated\ speed}{0.80} = 2937.5\ rpm.$$

The difference between the approximate and exact answers is 2937.5 – 2820 = 117.5 *rpm*, which is significant. The difference would be small if the change were small by a few percentage points but can be significant for a large change in the field current, as in this case. In general, using exact method—which is simple enough—is recommended.

The typical efficiency of dc machines are as follows:

HP Rating	5	10	50	100	500	1000	5000
% Efficiency	75	82	88	91	93	95	97

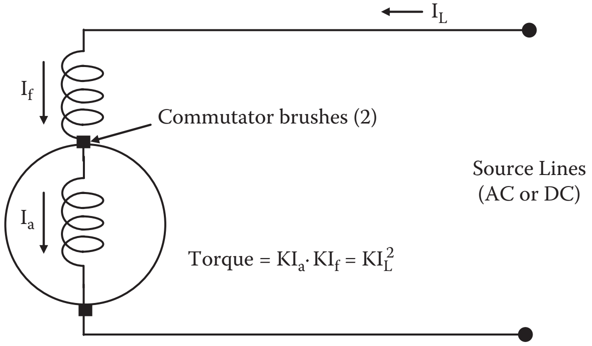


FIGURE 5.13 Universal series motor for ac or dc operation.

5.7 UNIVERSAL (SERIES) MOTOR AC OR DC

If the stator and rotor are connected in series (Figure 5.13), both carry the same current, positive or negative, the motor torque is always positive, as seen by Equation 3.6, regardless of ac or dc supply. The series connection between the stator and the rotor coils must necessarily use commutator and brushes, limiting the practical horsepower rating of this motor to small in order to avoid sparking at the brushes. Small-appliance size series motors in fractional horsepower ratings are called universal motors, whereas large motors in several horsepower ratings are called series motors. Since the torque is proportional to the current squared, and since the starting current is high (as in all motors), the series motor gives very high starting torque. For this reason, it is often used for moving large inertia load from rest, such as in trains, cranes, hoists, elevators, etc. However, the series motor has low flux at light load current, which speeds up the rotor as per Equation (5.21) and maintains constant horsepower output. The high speed at light load can self-destroy the rotor under centrifugal force, which is proportional to speed squared. For this reason, the series motor must not be used at very light load, and must be protected from accidental overspeed by centrifugal switch in the supply lines. The dc shunt motor, on the other hand, works at essentially constant flux and constant speed, and produces horsepower that is proportion to the load torque.

5.8 SPECIAL MOTORS FOR SHIP PROPULSION

In addition to the motors covered in this chapter, new advanced motors of large ratings in tens of megawatts have been and are being developed specially for electrical propulsion of ships. They are (1) superconducting synchronous motor, (2) permanent magnet synchronous motor, (3) homopolar dc motor, (4) axial flux motor, (5) transverse flux motor, etc.

5.9 TORQUE VERSUS SPEED COMPARISON

Figure 5.14 displays the speed versus torque load characteristics of four major types of motors. It shows that the synchronous motor maintains its speed constant regardless

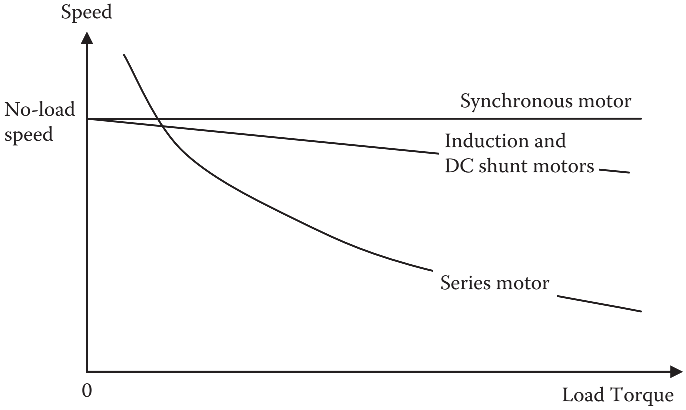


FIGURE 5.14 Speed droop versus torque load in ac and dc motors.

of the load, while the series (dc or ac) motor speed drops inversely with the load. The induction motor and dc shunt motor speeds, on the other hand, droop slightly down with increasing load.

PROBLEMS

Problem 5.1: A 2-pole, 60 Hz, 3-phase, 3460 rpm induction motor runs at 3580 rpm at no load. Determine at rated load (1) the slip speed in rpm, (2) slip in per unit, and percentage, and (3) speed regulation.

Problem 5.2: A 75 hp, 3-phase, 6-pole, 60 Hz induction motor delivers full load torque at 5% slip. If the load torque is increased by 15% as allowed by the service factor on the nameplate, determine the new speed and horsepower delivered.

Problem 5.3: A 150 hp, 3-phase, 60 Hz, 460 V, 4-pole, 1750 rpm induction motor is used in an application that loads the motor to only 70% of the rated torque. Determine (a) the actual speed, and (b) actual horsepower delivered to the load.

Problem 5.4: A 125 hp, 3-phase, 60 Hz, 460 V, 4-pole, 1720 rpm, letter code M, squirrel-cage induction motor with Δ-connected stator has efficiency 92% and power factor 0.90 lagging when operating at rated load. Determine the input kW, kVA, and line current during normal operation and the line current on direct start in Δ.

Problem 5.5: A 5000 hp, 60 Hz, 1200 rpm, 4.1 kV, 3-phase, Y-connected overexcited synchronous motor draws the armature current of 800 A at 0.85 leading power factor under rated load. It has the synchronous reactance of 1.5 Ω/ph and negligible resistance. Determine (1) the counter emf $\tilde{E}_f$, (2) torque angle δ, and (3) mechanical angle between the center lines of the rotor and stator magnetic fields.

Problem 5.6: A 1000 hp synchronous motor running at 80% rated load has the torque angle of 25°. Determine its pull-out torque $T_{\max}$ in percentage of the rated torque. If the motor load is increased to 100% in one step, determine the worst-case transient peak value of torque angle δ. Assume the torque versus δ relation approximately liner in the operating range and ignore the damping bar contribution.

Problem 5.7: A 250 hp, 3-phase, 460 V, 60 Hz, ac motor has power factor of 0.90 lagging and efficiency of 92%. Determine the line current it will draw under 10% overload and percentage change in stator conductor temperature rise above ambient air.

Problem 5.8: A water-pumping system in a ship is designed to pump 250 m^3/h fresh water against 70 m of total pressure head. Determine the motor horsepower rating and the motor control center kVA rating, assuming 95% pump efficiency, 92% motor efficiency, and 85% pf lagging.

Problem 5.9: A 3-phase, 125-hp, 460 V, 50 Hz induction motor with Δ-connected stator is designed to have the magnetic flux density in the core at the saturation limit of 1.65 T. If this machine is connected to 60 Hz lines, determine the voltage for limiting the flux density to the same 1.65 T saturation limit and horsepower at 60 Hz.

Problem 5.10: A 200 hp, 3-phase, 460 V induction motor has the design life of 25 years at 110°C rise in conductor temperature above the ambient air. If it is continuously operated under unbalanced line voltages of 430, 450, and 480 V, determine the (a) percentage of voltage unbalance, (b) additional temperature rise when operating at rated load, (c) expected life at rated load, and (d) the required horsepower derating for the motor to last the design life of 25 years.

Problem 5.11: A 3-phase induction motor in 440 V distribution system under an abnormal condition receives highly unbalanced 3-phase voltages $\tilde{V}_a = 360\ \angle 0°$, $\tilde{V}_b = 360 \angle{-90°}$, and $\tilde{V}_c = 720 \angle{-225°}$. Determine the symmetrical component voltages V_a^+, V_a^-, and V_a^0, and comment on approximate motor current and power losses in the rotor and stator.

Problem 5.12: A 100 hp, 240 V dc shunt motor runs at 3450 rpm at rated conditions. If its field current is decreased by 15% while keeping the same load current, determine its new speed.

Problem 5.13: A 3-phase induction motor running from 460 V supply lines is drawing 60 A rated current at 0.90 power factor lagging. The power losses estimated from the manufacturer's data sheet at this load are 3000 W in conductor, 1024 W in magnetic core, and 1000 W in friction and aerodynamic windage. Determine (1) output power and efficiency at rated load, and (2) maximum efficiency and the corresponding percentage of load.

Problem 5.14: A 3-phase, 50 hp, 460 V, Letter Code-G induction motor is connected to 480 V_{LL} lines via a transformer with $0.034 + j\ 0.097\ \Omega$/ph impedance and cable with $0.006 + j\ 0.003\ \Omega$/ph in series. Assuming the locked rotor power factor 0.20, determine the percentage of voltage drop on start with and without Y-Δ starter.

Problem 5.15: Determine the kVA load on a ship generator for powering a crude oil pump if (1) the pumping rate at discharge manifold is 5000 m^3/h against 100 m pressure head, (2) pump pressure head to manifold pipeline is 20 m, and (3) pump's internal pressure drop is equivalent to 10 m head of the fluid. Assume pump efficiency 88%, motor efficiency 95%, motor power factor 90%, and oil specific gravity 0.85.

Problem 5.16: Determine the running speed of a 100 hp, 440 V, 4-pole, 60 Hz synchronous motor at full load. If the voltage and frequency both were reduced to 80% and the load is reduced to 64%, determine its new speed.

Problem 5.17: For a 3-phase, 60 Hz induction motor with phase values of stator voltage $V = 120$ V, $R_{\text{rotor}} = 1\ \Omega$, $L_{\text{rotor}} = 1$ mH, compute and plot using EXCEL the rotor current and motor torque for slip varying over a wide range from $s = -1$ to $+ 2$ pu, that is, from backward rotating to supersynchronous speed. Then, in reference to the chart, discuss in each speed range the torque, current, and power variations, and what they mean in terms of using the induction machine as a motor, generator, or brake.

Problem 5.18: For the motor of Problem 5.17, compute and plot using EXCEL the rotor current and motor torque from slip $s = 1$ (starting) to 0.05 (full load) with rotor resistance varying from 0.5 to 2 Ω. Then, in reference to the chart, discuss the starting torque, full load speed, and full load current as the rotor resistance varies over the range. Also discuss what those variations mean in terms of the induction motor design for various starting torque and running efficiency requirements.

QUESTIONS

Question 5.1 Discuss the principal difference in construction and operation of the induction motor and the synchronous motor.

Question 5.2 Which two factors jointly produce a rotating magnetic field in 3-phase induction and synchronous motors?

Question 5.3 Why is the slip necessary for the operation of the induction motor?

Question 5.4 At what speed would an ideal induction motor on frictionless bearings run in a vacuum?

Question 5.5 Identify the advantage and disadvantage of using the Design D motor where high starting torque is required.

Question 5.6 Identify the adverse effects of motor current during starting.

Question 5.7 Explain the difference between the electrical degrees and mechanical degrees in 4-pole ac machines.

Question 5.8 What would happen if a 460 V, 60 Hz motor is connected to 460 V dc supply lines?

Question 5.9 What would happen if a 240 V dc shunt motor is connected to 240 V, 50 Hz supply lines?

Question 5.10 Explain the difference between dual-frequency and dual-voltage motors in construction and operation.

Question 5.11 Identify the effect of unbalanced voltage on motor operation and the remedy to maintain its normal service life.

Question 5.12 Identify the major advantage of the dc motor.

Question 5.13 Why have dc motors fallen out of favor now?

Question 5.14 What is required to change the direction of rotation of the induction motor and the dc shunt motor?

Question 5.15 List a few candidate applications where high-power series motors can be beneficial and the reason thereof.

FURTHER READING

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6 Transformer

The transformer changes the system voltage from one level to another using two mutually coupled coils on a magnetic core as shown in Figure 6.1. The two coils are electrically isolated—not connected by wires. The power transfer from one coil to the other takes place via the alternating magnetic flux in the core that links both coils. The core is made of magnetic steel that has high permeability (low reluctances) to the magnetic flux. The magnetic steel is also known as electrical steel for its use in electrical machines. It is basically mild steel with a few percent alloys that greatly improves the magnetic permeability. The magnetic core is made of thin laminations to keep eddy current loss low. It takes the flux created by the source side coil (usually HV, called the primary side, denoted by suffix 1) to the other coil (usually LV, called the secondary side, denoted by suffix 2). Almost all flux created by the primary coil reaches the secondary coil via the magnetic core, except a few percent of leakage in air between the coils.

The polarities of the voltage and current on two sides are opposite as per Lenz's law mentioned in Section 3.1. This means that the current is going into the positive polarity in the primary coil (bringing the power inward from the source), whereas it is coming out of the positive polarity in the secondary coil (delivering power out to the load).

The fundamental voltage Equation (3.2) for two coils with N_1 and N_2 turns and negligible resistance and leakage flux gives the following rms voltages V_1 and V_2 on two sides of the transformer (ignoring the negative sign),

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \tag{6.1}$$

Ignoring small power loss in core and coils, the conservations of energy requires the volt-ampere (power) balance on two sides, that is, $V_1 \cdot I_1 = V_2 \cdot I_2$, and also the ampere-turns balance, that is, $I_1 \cdot N_1 = I_2 \cdot N_2$, leading to

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{N_2}{N_1} \tag{6.2}$$

Although these relations are for the ideal transformer, they give fairly accurate results in practical transformers as well. The above transformer relations in rms values are emphasized in words below:

- Voltage ratio = turn ratio of the two coils.
- Current ratio = reciprocal (flip) of the turn ratio of the two coils.

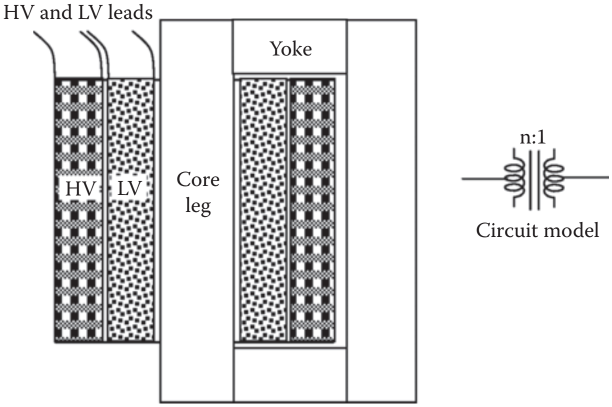


FIGURE 6.1 Single-phase transformer cross section with concentric HV and LV coils on one core limb.

- Volt–ampere product (kVA) and power on both sides are equal.
- Ampere–turn product (magnetomotive force) on both sides is equal.

The circuit symbol for transformer with turn ratio $N_1/N_2 = n$ is shown (Figure 6.1) by two coils with *n*:1 placed in between, meaning that for every n turns in the primary coil, there is 1 turn in the secondary coil. Sometimes, vertical lines are placed between two coils to denote the laminated magnetic core. When the turn ratio is not given, it is assumed to be the voltage ratio as per Equation (6.1) without much loss of accuracy.

6.1 TRANSFORMER CATEGORIES

All transformers work on the same principle—Faraday's law of electromagnetic induction—and obey the same voltage, current, power, and ampere-turn relations on two sides as per Equations (6.1) and (6.2). However, transformers are categorized based on their primary purpose in a given application as follows:

1. *Power transformer* has the sole purpose of transferring power from one voltage level on the primary side to another voltage level on the secondary side. We deal only with the power transformer in this chapter, and call it just the transformer. More than 95% of the power transforms have turn ratio $n > 1$, making them step-down transformers. In a power grid, a step-up transformer is used to raise the generator voltage to HV (hundreds of kV) to gain high efficiency in long distance power transmission. On ships, the step-up transformer is often found with a bow thruster motor, which has a high power rating and longer distance from the generators, and which requires high voltage to limit the line voltage drop.
2. *Voltage transformer (VT)* has the sole purpose of stepping down voltage from a very high level to a safe low level suitable for measurement by voltmeter. It is also called a *potential transformer (PT)* and has $n >> 1$. Each VT and

its fuse are mounted in separate steel compartments. The high-voltage side connection to VT is appropriately insulated where it enters the compartment through porcelain bushings.

3. *Current transformer (CT)* has the sole purpose of stepping down current from a very high level to a safe low level suitable for measurement by ammeter. It has $n \ll 1$. The low-current side of CT has high voltage and must have sufficient insulation. Since CT can see high current during short circuit faults, it is designed to withstand the thermal and mechanical stresses for peak current rating of the circuit breaker.
4. *Isolation transformer* has the sole purpose of electrically isolating the secondary side (user side) from the primary side (source side) without changing the voltage level. This is done for safety or other reasons. It has $n = 1$.

The VT and CT are jointly known as the *instrument transformers*. In addition to providing safety in measurements, they provide precise measurements, since their output has highly precise linear relation with input under all loading—unlike the power transformer.

Our main focus in this chapter is the power transformer. We need numerous power transformers because bulk power transmission is economical at HV with low current, but we must use power at various voltage levels: medium voltage for medium power and low voltage for small power for personal safety. A typical land-based utility power system is shown by 1-line diagram in Figure 6.2. Every kVA generated at the central power plant gets transformed five to six times before it is consumed at the user end. For this reason, over 95% of the transformers are step-down transformers.

Both HV and LV coils in the power transformer have tapping points around the rated number of turns. Typical tapping points are +5%, +2.5%, rated, –2.5%, and –5%, permitting some adjustment for low or high line voltage where the transformer is located. The transformer at the low end of the line voltage, say, 5% less than the nominal voltage, can boost the voltage on the load side by connecting to +5% taps on the secondary side, or to –5% taps on the primary side of the transformer. Either way, the turn ratio gets boosted by 5%.

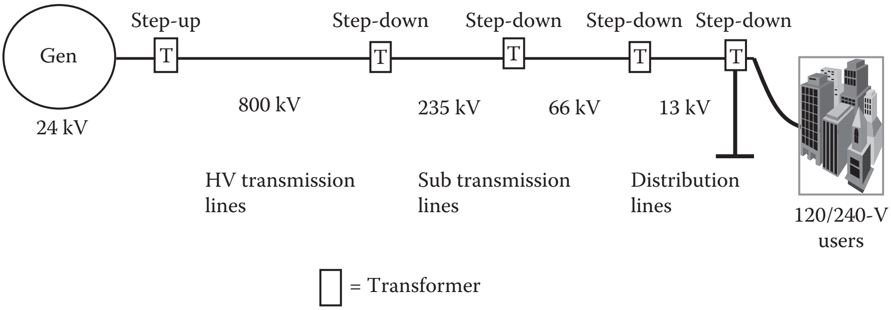
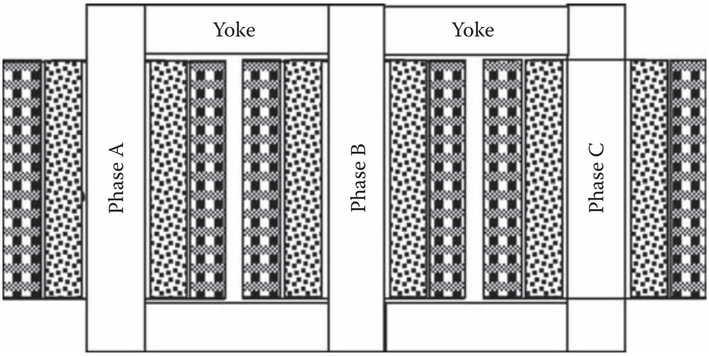


FIGURE 6.2 Typical voltage levels in land-based power grid from power plant to end users.

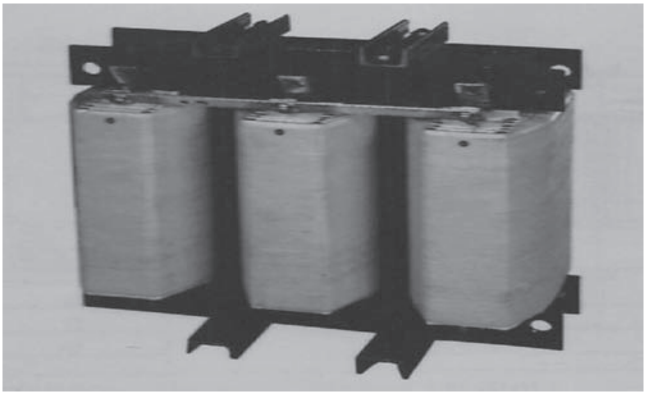
6.2 TYPES OF TRANSFORMERS

Power transformer construction falls into various types. The 1-phase unit is made with two coils (HV and LV, often concentric) on the core as was shown in Figure 6.1. The 3-phase transformer is shown in Figure 6.3, where two coils (HV and LV) in each phase are always concentrically wound and placed on one core limb. All three limbs of the core meet at the top and bottom yokes, where the phasor sum of 3-phase flux is zero due to 120° phase difference from each other; thus requiring no return path for the 3-phase flux. The concentric HV and LV coils for each phase minimize the flux leakage. All three phases wound on a 3-limb core are placed in one enclosure. They need less core material, hence cost less. Almost all transformers used in land-based power grids are 3-phase units except those providing the power to the residential and small commercial customers. Three-phase units are not used on ships for reasons explained later in Section 6.6.

Power transformers (and circuit breakers) are categorized by the insulation medium they use, which also works as the cooling medium that takes the internal



(a) Cross section



(b) Exterior view

FIGURE 6.3 Three-phase transformer with three pairs of concentric HV and LV coils on three core limbs.

power loss to the enclosure surface for dissipation to the ambient air. They can be of the following types:

Dry type transformers use air as the insulation and cooling medium. Since the air is a relatively poor insulation and coolant, these units are bulky and costly. However, they are safer than other types for indoors and shipboard use. The enclosure can be ventilated or sealed where necessary for added safety. They are made up to 35 kV primary side voltage.

Oil-filled transformers have electrical grade mineral oil as the insulation and cooling medium inside a sealed or breathing enclosure. They are extensively used in power grid and outdoor industrial and commercial installations. They are not used indoors or on ships because of possible oil spills and fire hazard following an accidental damage.

Gas-filled transformers work in pressurized gas (SF6 or nitrogen), which is much better insulation than air, requiring less separation between HV and LV parts. Moreover, it provides a better cooling medium. As a result, these transformers are compact and often used where space is at premium, such as indoors in a factory or ship, or outdoors in a center city substation.

Nonflammable oil-filled units are available for indoor or other places where fire risk needs to be minimized. They offer compact low-cost design compared to dry type units.

The key features of these transformers are listed in Table 6.1.

TABLE 6.1
Construction and Performance Features of Various Transformer Types

Performance Feature	Dry Type (Air Ventilated)	Oil-Filled	Pressurized Gas-Filled
Typical location	Indoor, close to the loads	Outdoor in a yard	Substation in city with space at premium.
Construction	Coils impregnated with varnish or cast in epoxy	Coils and core immersed in insulating mineral oil	Coils and core in pressurized SF6 or nitrogen
Fire risk	Low	High due to oil	Low
Cooling mechanism	Open loop air flow through coils by convection and radiation, or forced air by fans	Circulating oil by convection inside, and by air convection and radiation outside, or forced air by fans	Circulating gas by convection inside and by air convection and radiation outside
Average conductor temperature above 40°C ambient	80°C (costs more) to 150°C (costs less)	55°C	80°C (costs more) to 150°C (costs less)
Volume and weight	High	Low	Moderate
Cost	High	Low	High

6.3 SELECTION OF KVA RATING

The transformer size (kVA rating) for a given load profile is determined based on the following factors:

- The peak load it would supply, taking into account the NEC load factors if numerous small loads with intermittent use are connected to it. Large loads are accounted for their individual on and off timing.
- For transformers supplying harmonic-rich power electronics loads, the additional harmonics heating is accounted for by selecting correct *K*-rating (discussed latter).
- About 30% margin is then added for the future growth, which will make the transformer operate at about 1/1.3 = 0.77 or 77% of the rated capacity until the load grows to full capacity. This indirectly makes the transformer operate initially at peak efficiency, since most power equipments have maximum efficiency at about 70%–80% of rated load.

The transformer kVA, voltage, and frequency ratings must meet the IEEE Standard-C57.12.01, and the mechanical and thermal integrity under short circuit faults must meet the IEEE-Standard-45 Section 12.01 and ANSI Standard C57. The compliance to these standards in shipboard transformers must be in accordance to ABS Rules 4.8.5 and 3.7.5(e).

Example 6.1

Determine the voltage and kVA ratings of a Y-Δ connected transformer to power a 3-phase Y-connected load drawing 800 A line current at 480 V from 3-phase, 4160 V source.

SOLUTION

For 3-phase transformer, kVA rating = $\sqrt{3} \times 480 \times 800 \div 1000 = 665$ kVA

3-phase voltage rating = Line voltages = 4160/480 V Y-Δ connection.

Alternatively, this load can also be supplied by three 1-phase transformers, each rated $\frac{1}{3} \times 665 = 221.67$ kVA. The 1-phase transformer voltage rating depends on the transformer connections.

For the three 1-phase units to be connected in Y on the HV side and Δ on the LV side,

HV side voltage rating = $4160 \div \sqrt{3} = 2402$ V
LV side voltage rating = line voltage = 480 V
∴ Transformer voltage rating = 2402/480 V

Since transformers in such small sizes are rarely custom made, which are more expensive, we round the above 1-phase kVA size to the nearest standard rating readily available off the shelf, such as 225 kVA or more if margin is needed. The voltage ratings, however, cannot be rounded, but we can use units with HV rating higher than 2402 V and HV/LV voltage ratio the same as 2402/480 V.

Example 6.2

A 5000 kVA, 3-phase transformer bank steps down line voltage from 12.5 kV to 480 V. The HV side is connected in Δ and the LV side in grounded Y. Determine (a) HV side line and phase currents, and (b) LV side line and phase currents.

SOLUTION

First we work with kVA per phase, which is ⅓ 5000 = 1666.7 kVA/phase for both HV and LV sides.

(a) On HV side in Δ, phase voltage = line voltage = 12,500 V, which gives phase current = 1000 × 1666.7 ÷ 12,500 = 133.33 A, and line current = √3 × 133.33 = 230.94 A.
(b) On LV side in Y, phase voltage = 480 ÷ √3 = 277.1 V, phase current = 1000 × 1666.7 ÷ 277.1 = 6015 A, and line current = phase current = 6014 A.

Alternatively, we can work with 3-phase kVA relations as follows:

(a) On HV side in Δ, 5000 × 1000 = √3 × 12,500 × I_L, which gives line current I_L = 230.94 A, and phase current = 230.94 ÷ √3 = 133.33 A.
(b) On LV side in Y, 5000 × 1000 = √3 × 480 × I_L, which gives I_L = 6014 A, and phase current = line current = 6014 A.

6.4 TRANSFORMER COOLING CLASSES

The power transformers are classed by the method of cooling core and coils, that is, how the internal power loss is dissipated in ambient air to keep the temperature rise within the allowable limit set by the insulation material used in the construction. The temperature rise can be as high as 180° in some dry type transformers widely used on ships, and as low as 55°C in oil immersed transformers widely used on land. In the increasing order of better cooling and compact size for the same kVA rating, the oil-filled transformer cooling classes are as follows:

- OA = Oil–air self-cooled by natural convection and radiation
- FA = Forced-air cooled by fans
- FOA = Forced-oil–air cooled by oil pumps
- OW = Oil–water cooled by water tubes or hollow conductors

Many transformers have multiple kVA ratings with alternative cooling methods, such as OA/FA, OA/FA/FOA, OW, etc.

6.5 THREE-PHASE TRANSFORMER CONNECTIONS

The 3-phase transformer can be in one 3-phase unit or a bank of three 1-phase units. Typical off-the-shelf 3-phase transformer ratings readily available are from 9 to 5000 kVA. Typical 1-phase transformer ratings range from 3 to 2500 kVA.

The primary coils in 3-phase transformer are typically connected in Δ, whereas the secondary coils are connected either in Δ or Y-Gr (Y with neutral grounded). The performance features of these two connection methods are as follows:

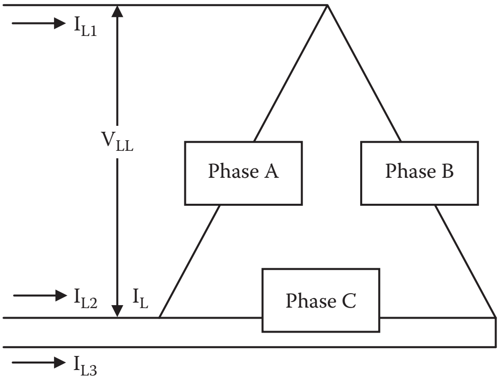
A. Primary Δ–Secondary Y with neutral grounded (common on land)
   No neutral available on the primary side Δ
   Each secondary line and neutral can be used for 1-phase loads
   Introduces 30° phase shift between the primary and secondary line voltages
   Zero-sequence and triplen harmonic currents are blocked in the primary lines, but can flow in the primary phase coils and the secondary lines, causing extra harmonic heating.
   One ground fault on the secondary side causes the short circuit current to flow and trip the circuit breaker.
B. Δ primary–Δ secondary (common on ships):
   No neutral on either primary or secondary side
   First ground fault on either side causes no short circuit
   First ground fault detection is required for reliable operation
   No 30° phase shift between the primary and secondary line voltages
   Zero-sequence and triplen harmonic currents are blocked on both sides

Both the HV and LV side coils in 3-phase transformer can be connected in Y or Δ, but at least one side is generally connected in Δ for the following reason. Since the commercial transformer core is designed near magnetic saturation point, the magnetizing current wave gets a hump near the peak value. Such a nonsinusoidal wave gives rise to the 3rd harmonic current of the 3 × fundamental frequency, superimposed on the fundamental frequency magnetizing current in each phase. The 3rd harmonic currents in each phase are not 3-phase currents with usual 120° phase difference. They have $3 \times 120° = 360° = 0°$ phase difference from each other, that is, they are in phase and require a return or circulating path. The neutral wire in Y connection carries the sum of 3rd harmonic currents in three phase lines back to the source, and the secondary is not impacted. The line voltages on both sides remain sinusoidal. In the absence of neutral wire, the line voltages on LV side get distorted with the 3rd harmonic voltages. One way of suppressing the 3rd harmonic voltages is to have the phase coils either on HV or LV side connected in Δ, which provides an internal path for circulating the 3rd harmonic currents without impacting the outside line voltages. In a typical 3-phase Y-connected source with or without neutral, the HV side of the transformer is usually connected in Y, and the LV side in Δ. On ships, both sides are generally connected in Δ. In addition to suppressing the 3rd harmonic currents, Δ-Δ connection improves the availability of power as explained below.

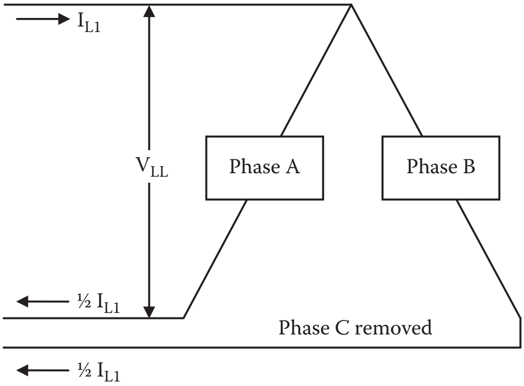
6.6 FULL-Δ AND OPEN-Δ CONNECTIONS

The 3-phase power systems on land generally use 3-phase transformers for economy, with three phases connected in Y or Δ as needed for the application. On ships, however, three separate 1-phase transformers are connected in Δ for reliability. With one of the three phases damaged, the entire 3-phase unit with all phases in one enclosure becomes unavailable until it is repaired or replaced. With three 1-phase units in separate enclosures, the damaged unit can be removed from the bank, leaving two healthy units operating in open-Δ (also called V-V) connection without impacting the line voltages. The kVA capacity of the open-Δ bank, however, gets reduced to 57.7% of the full-Δ capacity until the damaged unit is replaced.

Consider three 1-phase transformers connected in full Δ as shown in Figure 6.4, supplying three line currents of equal magnitudes I_L in all three lines at line-to-line voltages V_{LL}. If say phase C transformer gets damaged and removed, the two remaining transformers in open Δ still maintain the same balanced 3-phase voltages both on the load side as well on the source side. However, the currents get adjusted for the new connections pattern. The phase A and B transformers now carry ½ I_L in open Δ instead of $I_L/\sqrt{3}$ before in full Δ. In order not to overload the feeder line cable, I_L must



(a) Full Δ-system



(b) Open Δ-system

FIGURE 6.4 Full-Δ and open-Δ connections of three 1-phase transformers.

have the same value as before. This degrades the kVA capability of the full-Δ bank when operated in open-Δ as follows:

$\text{kVA}_{\text{full-}\Delta} = \sqrt{3} \times V_{LL} \times I_L$ and $\text{kVA}_{\text{open-}\Delta} = V_{LL} \times \frac{1}{2} I_L$ in phase A + $V_{LL} \times \frac{1}{2} I_L$ in phase B = $V_{LL} \times I_L$. The ratio of the two kVAs is, therefore,

$$\frac{kVA_{\text{open}-\Delta}}{\text{kVA}_{\text{full}-\Delta}} = \frac{V_{LL}\, I_L}{\sqrt{3}\, V_{LL}\, I_L} = 0.577 \;\; \text{pu} = 57.7\% \tag{6.3}$$

The open-Δ kVA loading, therefore, must be reduced to 57.7% of the full-Δ kVA capability in order to maintain the same ampere loading on the transformer feeder line cable. However, the line voltages on the load sides are still balanced 3-phase voltages of full value as before. Just the load current is reduced to limit the primary side line current to its rated value to avoid overheating.

Example 6.3

Three 1-phase transformers, each rated 100 kVA, are connected in Δ-Δ to power a 3-phase 300-kVA load. If one of them is removed for repairs, determine (1) the line voltage on the load side, and (2) maximum kVA load the remaining two transformers can provide.

SOLUTION

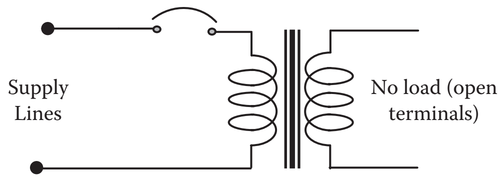
(i) By removing one transformer, the Δ-Δ connected bank of three 1-phase transformers become V-V connected bank, but the line voltage remains the same as in the Δ-Δ connection.

Total kVA load, however, must be reduced to 57.7% in order not to overload the cables.

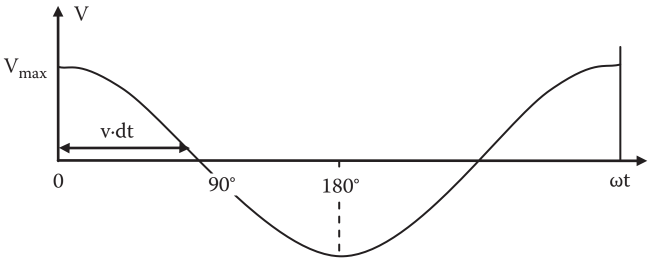
∴ Maximum permissible 3-phase load = 0.577 × 300 = 173.1 kVA

6.7 MAGNETIZING INRUSH CURRENT

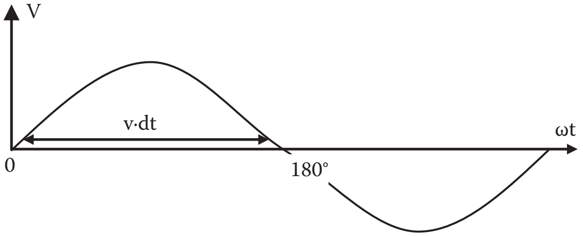
On the instant of closing the breaker on the transformer primary side and connecting to the supply lines, the primary side draws heavy inrush current for initial magnetization of the core even with no load connected to the secondary side (Figure 6.5a). This can be simply explained by Faraday's law, which is (ignoring the – sign) $v(t) = N\, d\varphi/dt$, where $v(t)$ is the sinusoidal supply voltage. The flux build-up in core with N turn coil is then $\varphi = \frac{1}{N}\int v \cdot dt$, where the integral is the area under ($v \times t$) curve. The flux adds under positive voltage, and subtracts when voltage becomes negative. If the circuit breaker is closed on the instant of voltage passing to its natural maximum value as shown in (b), the flux rises from time 0° to 90° to the peak value $\phi_{\text{peak}1} = \frac{1}{\omega N}\int_0^{90} v \cdot d(\omega t)$, and then starts falling back from $\varphi_{\text{peak}1}$. On the other hand, if the circuit breaker is closed on the instant of line voltage passing through its natural zero as shown Figure 6.5 (c), the flux rises from time 0° to 180° to the peak value $\phi_{\text{peak}2} = \frac{1}{\omega N}\int_0^{180} v \cdot d(\omega t)$, and then starts falling back from $\varphi_{\text{peak}2}$. Clearly, $\varphi_{\text{peak}2}$



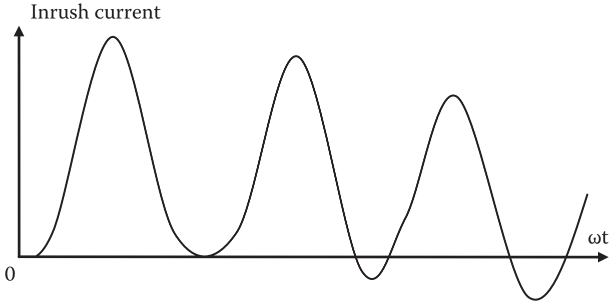
(a) Transformer with line breaker open and no load



(b) Breaker closed at V_{max} instant



(c) Breaker closed at V = 0 instant



(d) Inrush current on connecting primary coil (three cycles)

FIGURE 6.5 Magnetizing inrush current on connecting transformer to supply lines with no load on the secondary side.

covers twice the area compared to φ_{peak1}, giving $\varphi_{\text{peak2}} = 2\ \varphi_{\text{peak1}}$. Normally, the core is designed to carry flux equal to φ_{peak1}, at which level the core is at the saturation knee point on its magnetization (B-H) curve. With φ_{peak2}, the flux density reaches two times the saturation limit, where the magnetic permeability of core is extremely low, matching with that of air, which requires very high magnetizing current. For example, if φ_{peak1} requires the magnetizing current equal to 2% of the rated load current, then φ_{peak2} would require over 1000 times more, that is, 2000% or 20 times the rated load current for a very short time in the first cycle. It then gradually decays to a normal 2% value in several cycles with oscillations that are damped out by the power losses in the core and coil, as shown in (d).

Thus, the inrush magnetizing current can peak up to 10 to 15 times the rated load current, and may trip the breaker or melt the fuse on the primary side. Connecting the transformer to the supply lines is a normal operation. Therefore, such tripping would be a nuisance that can be avoided by using the time-delay fuse or setting the circuit breaker trip time with intentional delay. Although the magnetizing inrush current does not impact the transformer kVA size determination, it does impact the fuse selection and circuit breaker setting to avoid nuisance tripping.

6.8 SINGLE-LINE DIAGRAM MODEL

In the single-line diagram of a 3-phase power system model, an ideal transformer can be represented by an ideal wire with no power loss or voltage drop. Practical transformers are designed with remarkably high efficiency, often approaching 99% in large units. However, small power losses occur in the conductor resistance and magnetic core. And, some flux leakage in the gap between two coils causes a small voltage loss (drop), usually represented by the leakage inductance L. In a continuous one-line diagram, the transformer is then represented by its total coil resistance R and leakage reactance $X = \omega L$ in series, combined in impedance $Z_{Trfr} = R + j\,X$, as shown in Figure 6.6. The magnetizing current and power loss in core are drawn from the source side, and do not enter in the series impedance between the primary and secondary coils for determining the line voltage drop due to Z. However, they are separately accounted for in the efficiency and temperature rise calculations.

Transformer nameplate typically includes kVA, number of phases (i.e., 1-phase or 3-phase), frequency, HV and LV side voltages, Y- or Δ-connected, percentage of taps, cooling class (air, oil, natural or forced), average conductor temperature rise in degrees Celsius, basic insulation level (BIL) against lightning voltages, percentage impedance Z, and sometimes percentage efficiency.

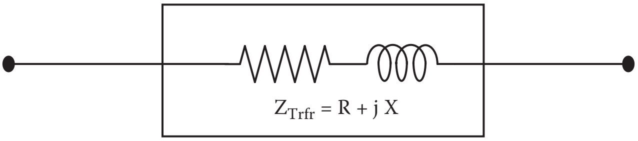


FIGURE 6.6 Equivalent electrical model of transformer in 1-line diagram.

TABLE 6.2
Typical Impedance Values of 1-Phase Transformers

KVA Rating	HV 460–4100 V		HV 4200 V–11 kV	
	% R	% X	% R	% X
10	1.5	1.8	1.5	2.0[a]
50	1.3	2.3	1.3	2.5
100	1.2	2.8	1.2	3.5
250	1	4.7	1	5.2
500	1	4.8	1	5.4
1000	0.9	5.0	0.9	6.0
10,000	0.8	6.0	0.8	7.0[b]

Note: [a] Users of small transformers often specify impedance not less than 4% in order to keep the fault current below the available protection device rating.

[b] Large high voltage transformers have higher impedance, since the insulation gap between the HV and LV coils needs to be wider, resulting in greater leakage flux.

Table 6.2 gives representative values of percentage Z for different kVA and voltage class of 1-phase distribution transformers. We note that smaller transformers have a lower percentage Z than larger units. The transformer is very efficient in power transfer, and its design efficiency improves with size, as seen in Table 6.3.

Since the transformer remains connected to the supply voltage regardless of the load current, the core loss is continuously present. For this reason, a well designed transformer has power loss in core = $\frac{1}{3}$ total power loss and the loss in conductor = $2 \times \frac{1}{3}$ total power loss. With such distribution of the total loss, the maximum efficiency occurs at $\sqrt{½} = 0.707$ or 71% of the rated load when *conductor loss = core loss*. From the nameplate value of impedance Z, the equivalent series resistance R and X can then be estimated from efficiency η (known or estimated) and impedance Z as follows:

$$R = 2 \times \tfrac{1}{3}\,(1 - \eta) \qquad \text{and} \qquad X = \sqrt{\,(Z^2 - R^2)} \tag{6.4}$$

where all values here are in per unit (pu). In percentage values, the number 1 in R above becomes 100.

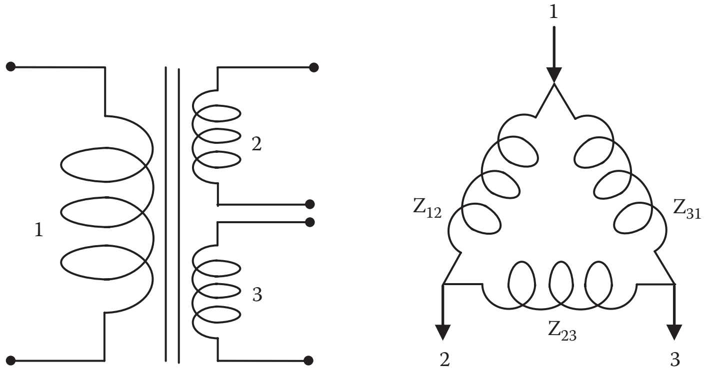
TABLE 6.3
Typical Values of Transformer Efficiency

kVA Rating	100 kVA	1000 kVA	10 MVA or Higher
% Efficiency	96%–97%	97%–98%	98%–99%

6.9 THREE-WINDING TRANSFORMER

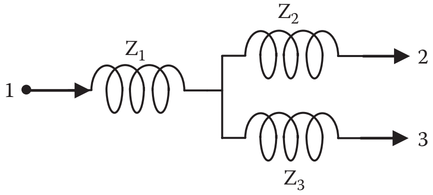
The transformer with one primary coil and two secondary coils, all three wound on a common core, is sometimes used for two purposes: (i) to supply two loads at different voltages—main load at high voltage and auxiliary load at low voltage—or (ii) the third (called tertiary) winding is connected in delta, but no terminals are brought out for load connection (it is buried inside the enclosure). The buried delta-connected tertiary coils carry the internally circulating 3rd harmonic currents, which are all in phase. This way, it prevents the 3rd harmonic currents from entering the external lines and improves the quality of power in *Y*-connected, 4-wire distribution systems.

Dual-voltage output from one transformer is obtained by using two secondary coils as shown in Figure 6.7(a). In such a transformer, one output voltage is influenced by load change in the other via magnetic coupling. A short circuit in one secondary would also impact the output voltage of the other secondary. Analytically predicting such mutual influence can save the prototyping cost and time. The three-winding transformer equivalent circuit shown in (b) can facilitate such analysis. It is based on the premise that between any two coils, there is a leakage flux and the



(a) Physical layout

(b) Leakage reactance between windings pairs



(c) Equivalent reactances in series Y

FIGURE 6.7 Equivalent circuit models of three-winding transformer powering two secondary loads at different voltages.

corresponding leakage reactance X. The equivalent circuit shows the total impedance—made of the winding resistance and the leakage reactance—denoted by Z_{12} between coils 1 and 2, Z_{23} between coils 2 and 3, and Z_{31} between coils 3 and 1. The value of Z_{12} is determined by a short circuit test between coils 1 and 2, and so on. The analysis is significantly simplified by representing the three coil-to-coil impedances in an equivalent Y-circuit shown in (c). Through a circuit analysis, the impedance values in equivalent Y-circuit can be derived as follows:

$$Z_1 = \tfrac{1}{2}\,(Z_{12} + Z_{13} - Z_{23})$$

$$Z_2 = \tfrac{1}{2}\,(Z_{12} + Z_{23} - Z_{13})$$

$$Z_3 = \tfrac{1}{2}\,(Z_{13} + Z_{23} - Z_{12}) \tag{6.5}$$

These calculations become easier by expressing the impedances in percentage of the rated impedance of the transformer, as discussed in the next section. It should be emphasized that these equivalent impedances have no real meaning. Any one impedance can be zero or negative, as long as the sum of impedances of any two branches is equal to the short circuit impedance between those two windings. By using the Y-circuit, it is easy to see that the terminal voltage V_2 depends not only on the secondary load, but also on the tertiary load because of the voltage drop in Z_1. It is also easy to see that when load I_2 is shed, the voltage of the other load will rise by I_2Z_1, and that Z_{31} can influence the short circuit current in winding –2 terminals. The transformer with more than three windings can be similarly analyzed using the same general principle, although with significant complexity.

6.10 PERCENT AND PER UNIT SYSTEMS

When we express any quantity in percent, we must have a base that is considered 100%. In per unit system, we expressed any quantity as a fraction of the base value that is considered 1.0 unit. Since 100% = 1.0 pu, 100 × pu value = percent value, or pu value = percent value/100.

$$\%\,Z\ of\ transformer = \frac{Series\ Z\ of\ transformer\ in\ ohms}{Base\ Z\ of\ transformer\ in\ ohms} \times 100 \tag{6.6}$$

$$Perunit\ Z\ of\ transformer = \frac{Series\ Z\ of\ transformer\ in\ ohms}{Base\ Z\ of\ transformer\ in\ ohms} = \frac{\%\,Z}{100} \tag{6.7}$$

In electrical power system studies, we deal with kVA, volts, amperes, and ohms. Generally the rated values of equipment are taken as the base values in expressing percent or pu values. But, we cannot select all four base values, since they are not independent. If the kVA and voltage bases are defined (rated values or otherwise), then the ampere and ohm bases are indirectly defined from the following relations:

$$\text{In 1-phase system, } I_{\text{base}} = \frac{1000\, kVA_{\text{base}}}{V_{\text{base}}} \qquad \text{A/phase}$$

$$Z_{\text{base}} = \frac{V_{\text{base}}}{I_{\text{base}}} = \frac{V_{\text{base}}^2}{1000\, kVA_{\text{base}}} = \frac{kV_{\text{base}}^2}{MVA_{\text{base}}} \qquad \Omega\text{/phase} \tag{6.8}$$

In large high-voltage, 3-phase systems, it is customary to state 3-phase kVA and line-to-line voltage in kV_{LL}, but the power system analysis is still done on a per phase (line-to-neutral) basis, where

$$\text{In 3-phase system, } I_{\text{base}} = \frac{kVA_{\text{3ph.base}}}{\sqrt{3}\; kV_{\text{LLbase}}} \qquad \text{A/phase}$$

$$Z_{\text{base}} = \frac{1000\; kV_{\text{LLbase}}^2}{kVA_{\text{3ph.base}}} = \frac{kV_{\text{LL.base}}^2}{MVA_{\text{3ph.base}}} \qquad \Omega\text{/phase} \tag{6.9}$$

The values of base volts, base amperes, and base ohms are different on LV and HV sides, but the kVA and percent Z of the transformer remain the same on both sides.

Example 6.4

Determine the base current and base impedance of 1-phase system based on 500 kVA and 277 V.

SOLUTION

$V_{\text{base}} = 277$ V (generally the rated value at the point of interest in the system)

$\therefore I_{\text{base}} = 500 \times 1000 \div 277 = 1805$ A and $Z_{\text{base}} = V_{\text{base}} \div I_{\text{base}} = 277 \div 1805 = 0.1535\ \Omega$

Alternatively, using Equation (6.8) for 1-phase system,

$$Z_{\text{base}} = V^2_{\text{base}} \div (kVA_{\text{base}} \times 1000) = 277^2 \div (500 \times 1000) = 0.1535\ \Omega$$

Example 6.5

A 1-phase, 24 kVA, 480/120 V transformer has $Z = 4\%$ stated on its nameplate. Determine Z in ohms looking from (a) HV side source when connected as step-down transformer, and (b) LV side source when connected as step-up transformer.

SOLUTION

Looking from the HV side source, $V_{\text{base}} = 480$ V $I_{\text{base}} = 24{,}000 \div 480 = 50$ A

$$Z_{\text{base}} = 480 \div 50 = 9.6\ \Omega \ \therefore\ Z_{trfr} = 0.04\ \text{pu} \times 9.6 = 0.384\ \Omega$$

Looking from the LV side source, $V_{\text{base}} = 120$ V $I_{\text{base}} = 24{,}000 \div 120 = 200$ A

$$Z_{\text{base}} = 120 \div 200 = 0.6\ \Omega \ \therefore\ Z_{trfr} = 0.04\ \text{pu} \times 0.6 = 0.024\ \Omega$$

We note that $0.384 \div 0.024 = 16$. This is the same as $(480 \div 120)^2$, which is the voltage ratio squared, as is always the case.

We notice in Example 6.5 that the transformer series impedance on LV side is 0.024 Ω, whereas that on the HV side it is 0.384 Ω. We also notice that the ratio (HV side ohms/LV side ohms) = (HV side volts/LV side volts)^2. Thus, the transformer has the same percent Z series impedance but different ohm values on HV and LV sides.

Example 6.6

A 1-phase, 150 kVA, 460/120 V transformer has 5% impedance and 97% efficiency. Find its series *R* and *X* in percentages and in ohms, looking from the HV side source.

SOLUTION

Since efficiency $\eta = 97\%$, the total power loss is approximately 3% {actually $(1/0.97 - 1) \times 100 = 3.09\%$ to be exact}. About 2/3 of it is typically in resistance *R* of the coils, and the remaining 1/3 in the core. Therefore, we take $R = 2\%$ in the series model of the transformer. Then, from the transformer impedance $Z = 5\%$, we derive the transformer reactance $X = \sqrt{(Z^2 - R^2)} = \sqrt{(0.05^2 - 0.02^2)} = 0.0458$ pu. For this calculation, we can also work in percentages to get percent $X = \sqrt{(Z^2 - R^2)} = \sqrt{(5^2 - 2^2)} = 4.58\%$, as the percentage signs get carried forward with the results.

For base values on the HV side, $V_{HV\text{base}} = 460$ V, $I_{HV\text{base}} = 150 \times 1000 \div 460 = 326$ A, and $Z_{HV\text{base}} = 460 \div 326 = 1.41\ \Omega$. Therefore, the ohm values *looking from the HV side* are

$$Z = Z_{pu} \times Z_{HV\text{base}} = 0.05 \times 1.41 = 0.0706\ \Omega$$

$$R = R_{pu} \times Z_{HV\text{base}} = 0.02 \times 1.41 = 0.0282\ \Omega$$

$$X = X_{pu} \times Z_{HV\text{base}} = 0.0458 \times 1.41 = 0.0656\ \Omega$$

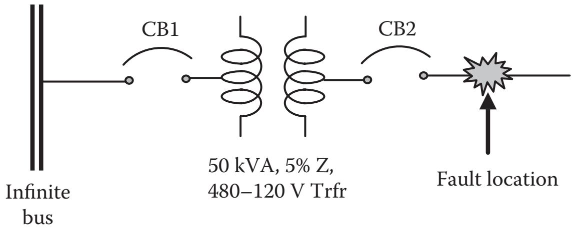
Another advantage of stating the transformer series impedance *Z* in percentages—as opposed to ohms—is that it quickly quantifies the magnitude of the short circuit current in case of a short circuit fault at its secondary terminals. This in turn leads to selecting the short circuit current interruption rating of the circuit breaker or fuse, which is

$$\text{Short circuit current} = \left(\frac{100}{\%Z}\ \text{or}\ \frac{1}{Z_{pu}}\right) \times \text{Rated current} \tag{6.10}$$

Although the rated current on the HV and LV sides are different, the transformer impedance Z in percent or pu remains the same on both sides.

Example 6.7

Determine the rms fault currents on both sides of a 50 kVA, 1-phase, 480/120 V transformer with 5% impedance as shown in the figure below. Assume that all generators behind the bus are large enough to make it an infinite bus and all cable impedances are negligible. Assume 100% voltage before the fault and ignore the normal load current.



SOLUTION

For this transformer,

HV side base (rated) current = 50 × 1000 ÷ 480 = 104.2 A
LV side base (rated) current = 50 × 1000 ÷ 120 = 416.8 A

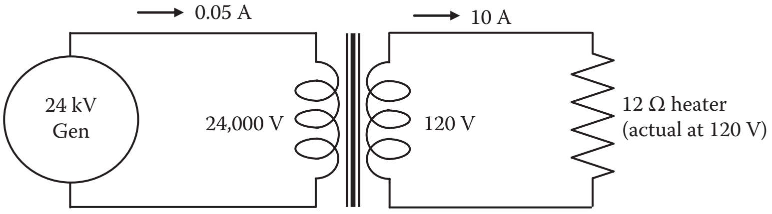
The transformer will see the maximum fault current if it gets a dead short at its output terminals, where the only impedance from the infinite bus to the fault is that of the transformer itself (5% in this case). A fault on the transformer primary side, close to the bus, would produce greater current (theoretically infinite), but that current would not go through the transformer. In fact, such current would not be truly infinite, but limited by the generator and cable impedance behind the bus, which we have ignored in this example.

The rms fault current through the transformer = 100% voltage ÷ 5% impedance = 20 per unit. Note that 20 is pu, as the percentage gets cancelled out from the numerator and denominator, leaving the result in per unit. It is safer to work in pu throughout, such as 1 pu voltage ÷ 0.05 pu impedance = 20 pu current. Thus, the fault current magnitudes on two sides are

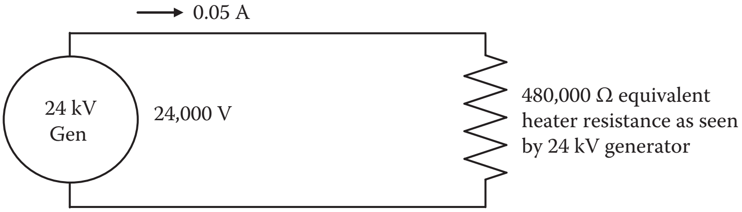
HV side fault current = 20 × 104.2 = 2084 A, and
LV side fault current = 20 × 416.8 = 8336 A

6.11 EQUIVALENT IMPEDANCE AT DIFFERENT VOLTAGE

Consider a 12 Ω heater connected to a 120 V room outlet as shown in Figure 6.8(a). It will draw 10 A current and 120 × 10 = 1200 W. We know that this power eventually comes—via a step-down transformer—from a large regional power plant that generates power at high voltage, say, at 1-phase 24 kV. Now, assume for a moment that no one else is drawing power from this plant except the 12 Ω heater. Using the balance



(a) Actual circuit with 12 Ω resistance at 120 V



(b) Equivalent continuous circuit
for same power from generator

FIGURE 6.8 Equivalent impedance in ohms at different voltage level.

of power, the currant drawn from the 24-kV generator for the sole 1200-watt heater would be 1200 W ÷ 24,000 V = 0.05 A. Therefore, the equivalent heater resistance as seen by the generator is 2400 V ÷ 0.05 A = 480,000 Ω as shown in (b). Thus, the actual 12 Ω resistance connected at 120 V outlet looks like 480,000 Ω at the 24 kV generator terminals, that is, it gets multiplied by 480,000 ÷ 12 = 40,000. The multiplier is also $(24000 \div 120)^2$, which is the voltage ratio squared. This concept is known by different names, such as the *impedance transformation, reflected impedance, or impedance referred to different voltage*. It gives *the equivalent impedance value at different voltage* that will absorb the same power (equivalency in the power system studies always means *for equal power*). The concept is needed in the fault current analysis later in Chapter 9, and can be generalized in the transformer or anywhere else in the power system as follows.

The equivalent value of given impedance in ohms looking from HV or LV side of a transformer changes by the voltage ratio squared; it looks high from HV side and low from LV side. The general expression for such *impedance transformation* of impedance Z_{V1} at voltage level V_1 to Z_{V2} at voltage level V_2 is

$$\frac{Z_{V2}}{Z_{V1}} = \left(\frac{V_2}{V_1}\right)^2 \tag{6.11}$$

This can also be reasoned in another way. The ac power absorbed in impedance Z ohms at voltage level V is $S = V^2 \div Z$. If we wish to convert ohm value of Z_{V1} at voltage V_1 to equivalent ohm value Z_{V2} at voltage V_2 which would absorb the same ac power S (condition for the equivalency), then we must have $S = V_1^2 \div Z_{V1} = V_2^2 \div Z_{V2}$, which leads to the same results as in Equation (6.11).

If the 12-Ω heater load resistance in the above example was expressed in pu of Z_{base} on 1200 W and 120 V base, then $I_{\text{base}LV} = 1200\text{ W} \div 120\text{ V} = 10\text{ A}$, $Z_{\text{base}LV} = 120\text{ V} \div 10\text{ A} = 12\ \Omega$, and $Z_{\text{Load}.pu} = 12 \div 12 = 1.0$ pu. The pu value then remains the same on HV side, which has $I_{\text{base}HV} = 1200\text{ W} \div 24{,}000\text{ V} = 0.05\text{ A}$, and $Z_{\text{base}HV} = 24{,}000\text{ V} \div 0.05\text{ A} = 480{,}000\ \Omega$. The equivalent ohm value of the heater resistance looking from the generator side is 480,000 Ω, which is 480,000 Ω equivalent ÷ 480,000 Ω base = 1.0 pu on HV side, the same pu value as on LV side. The percentage and pu values remaining the same on both HV and LV sides is the main advantage of using percentage or pu values in power system studies.

Example 6.8

A 1-phase 100 kVA, 480/120 V step-down transformer powers a load impedance of 12 Ω on LV side. Determine (a) the current on HV side from conventional transformer calculations, and (b) equivalent load impedance looking from the HV side in continuous equivalent circuit.

SOLUTION

(a) Current on LV side = 120 ÷ 12 = 10 A, and the current drawn from the HV side is then 10 × flip of the voltage ratio, that is, 10 × (120 ÷ 480) = 2.5 A.

(b) Equivalent load impedance looking from the HV side, $Z_{\text{Load }HV} = 12\,(480 \div 120)^2 = 192\ \Omega$. Then, the current drawn from HV side = 480 ÷ 192 = 2.5 A, which is the same as above.

6.12 CONTINUOUS EQUIVALENT CIRCUIT THROUGH TRANSFORMER

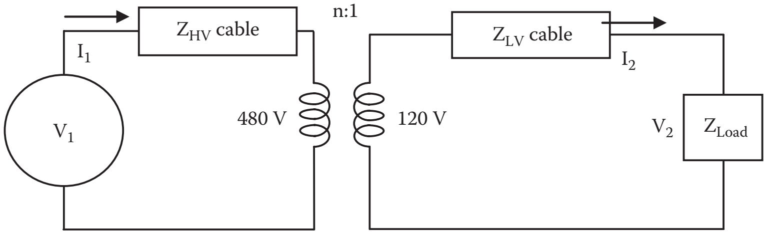
Consider a 480-120 V, 1-phase transformer connected to load impedance Z_{Load} ohms via secondary side cable impedance $Z_{LV\text{cable}}$ ohms and primary side cable impedance $Z_{HV\text{cable}}$ ohms, all connecting to the source as shown in Figure 6.9(a). We can reduce this circuit into continuous circuit with one total impedance powered directly from the HV side source as follows:

The equivalent ohm values of Z_{Load} and $Z_{LV\text{cable}}$ looking from the source side gets transformed by n^2, that is, the voltage ratio squared. The ohm value of $Z_{HV\text{cable}}$ remains the same as given; it does not get transformed since it is already on the source side in physical location. Therefore, the total equivalent impedance looking from the source on HV side is

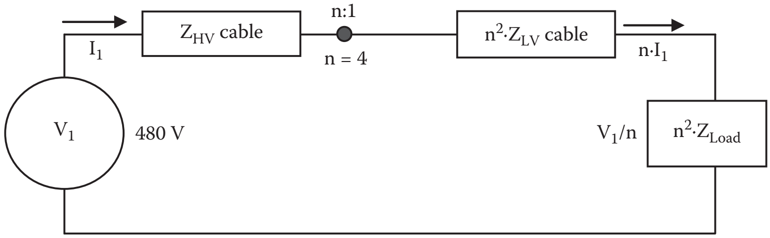
$$Z_{\text{Total}HV\text{side}} = Z_{HV\text{cable}} + n^2(Z_{LV\text{cable}} + Z_{\text{Load}}) \qquad \text{ohms} \tag{6.12}$$

where $n = (V_{HV} \div V_{LV}) = (480 \div 120) = 4$ in this case.

The current drawn from the source is then $I_{HV} = 480\text{ V} \div Z_{\text{Total}HV\text{side}}$ amperes.



(a) Actual circuit with load powered from transformer secondary side (all impedances in ohms)



(b) Equivalent continuous load circuit powered from transformer primary side (all impedances in ohms)

FIGURE 6.9 Equivalent continuous circuit model with electrical continuity through transformer.

If we work in the per unit system—expressing the values in pu of their respective base values—the voltage ratio multiplier n^2 does not apply, and the total impedance looking from the HV or LV side is merely the sum of all pu impedances, that is.

$$Z_{\text{Total.pu}} = Z_{\text{HVcable.pu}} + Z_{\text{LVcable.pu}} + Z_{\text{Load.pu}} \text{ and } I_{\text{pu}} = V_{\text{pu}} \div Z_{\text{Total.pu}} \tag{6.13}$$

$$\text{At full 100\% primary voltage, in per unit system, } I_{\text{pu}} = 1.0 \div Z_{\text{Total.pu}} \tag{6.14}$$

$$\text{In amperes, } I_{\text{HVamp}} = I_{\text{pu}} \times I_{\text{HVbase}} \text{ and } I_{\text{LVamp}} = I_{\text{pu}} \times I_{\text{LVbase}} \tag{6.15}$$

Example 6.9

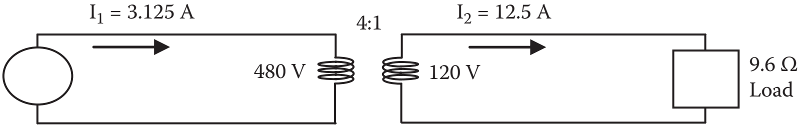
Consider a transformer rated 15 kVA, 1-phase, 480/120 V supplying 9.6 Ω load as shown in the figure below (top). In calculations specific to a given transformer, it is customary to take the rated voltage on two sides as their respective base voltage, and the rated kVA as the base kVA (which remains the same on both sides). Then, on secondary (suffix 2) and primary (suffix 1) sides, we have

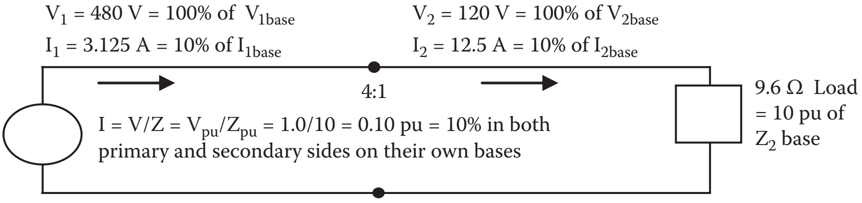
$I_{2\text{base}}$ = 15 kVA x 1000 ÷ 120 V =125 A and $Z_{2\text{base}}$ = 120 V ÷ 125 A = 0.96 Ω
$I_{1\text{base}}$ = 15 kVA x 1000 ÷ 480 V = 31.25 A and $Z_{1\text{base}}$= 480 V ÷ 31.25 A = 15.36 Ω.

By the way, we note that $(480 \div 120)^2 \times 0.96 = 15.36\ \Omega$, which means that the base ohms get transformed by the voltage ratio squared, as we have learned earlier.

Secondary current $I_2 = 120\text{ V} \div 9.6\ \Omega = 12.5\text{ A}$, which is $12.5 \div 125 = 0.10$ pu or 10% of the base current on LV side.

Primary current $I_1 = 12.5 \times (1 \div 4) = 3.125\text{ A}$, which is also $3.125 \div 31.25 = 0.10$ pu or 10% of the base current on HV side.





Alternatively, we express the load impedance 9.6 Ω on LV side as $9.6 \div 0.96 = 10$ pu on LV base. Then, $I = 1.0$ pu $V \div 10$ pu $Z = 0.10$ pu current, which will be the same in pu on both LV and HV sides.

Although volt and ampere are different, the percent voltage and percent current are the same on both LV and HV sides as shown in Figure E6.9 (bottom). It means that the current is continuous in percentage from the source side to the load side, although the base of expressing percentage changes on two sides. Therefore, the entire circuit when expressed in percentages looks as if the two sides were connected by wire at the heavy dots shown in the bottom circuit, with the continuity of percent voltage and percentage current on two sides. This simplifies the system analysis by a great deal when there are multiple transformers in series in the system, which can be reduced to one continuous circuit starting from the generator to the load. Such continuous circuit calculations are possible only if we work in the percentage or pu system.

6.13 INFLUENCE OF TRANSFORMER IMPEDANCE

With fixed primary voltage equal to the supply line voltage, the secondary voltage equals the primary voltage minus the internal voltage drop in the transformer series impedance Z. As the voltage drop varies with load current and power factor, so does the secondary side voltage. The voltage regulation is a measure of how close the secondary voltage remains to the rated value with varying load and power factor. It is desirable to have voltage regulation low, which requires using a transformer with low impedance. It can be calculated at any load, but the full load (worst case) regulation is of most interest to power engineers, which is defined as

$$\%\,Voltage\,\mathrm{Re}\,gulation = \frac{V_{2\mathrm{noload}} - V_{2\mathrm{fullload}}}{V_{2\mathrm{fullload}}} \times 100 \tag{6.16}$$

If the secondary voltage needs to be maintained constant regardless of the load, it must be regulated by using automatic tap changer over the range equal to the percent voltage regulation. Alternatively, the secondary loads must allow that much percentage variation in their input voltage without adversely impacting the performance.

The exact value of the voltage regulation at any load and pf can be determined from Equations (3.14), (3.15), and (3.19). However, for typical loads with pf around 0.85 pf lagging, Equation (3.18) gives fairly approximate voltage drop due to the transformer series impedance,

$$V_{\mathrm{drop}} = I(R \cdot pf + X\sqrt{1 - pf^2}) \qquad \text{and} \qquad \%\ V.R. = \%\ V_{\mathrm{drop}} \tag{6.17}$$

Where R and X are the transformer series impedance derived from the nameplate impedance and efficiency values using Equation (6.4). One can use volt, ampere, and ohm units, or all in percentages or pu.

A low impedance gives a desirable low regulation, but it adversely impacts the short circuit current in case of a dead short on the secondary terminals. The actual short circuit current depends on all the equipment connected in series from the generator to the fault location, including the transformer. However, a conservative estimate of the fault current can be made by ignoring the generator and cable impedances, with only the transformer percent Z in the circuit with 100% voltage at the transformer primary and solid 3-phase short at the secondary terminals. Full (100% or 1.0 pu) source voltage is now available for the short through the transformer impedance, giving $I_{\mathrm{sc}.pu} = 1.0 \div Z_{\mathrm{Trfr}.pu} = 100\% \div \%\ Z_{\mathrm{Trfr}}$. Note that percent divided by percent gives pu, as two percentage signs cancel out, leaving the quotient dimensionless multiplier. The actual sc amperes on the two sides of the transformer are then $I_{HV\mathrm{sc.amp}} = I_{\mathrm{sc}.pu} \times I_{HV\mathrm{base}}$ amperes and $I_{LV\mathrm{sc.amp}} = I_{scpu} \times I_{LV\mathrm{base}}$ amperes, for which the circuit breakers must be rated. The probability of mechanical or thermal damage on the transformer during a dead fault increases directly with the short circuit current, or inversely with the transformer impedance.

Thus, a lower percent Z gives lower voltage regulation on one hand, but gives higher short circuit current on the other hand. These two effects are carefully balanced by the system engineer to meet the voltage regulation requirement while keeping the circuit breaker rating and the short circuit damage risk reasonably low.

Example 6.10

A 5000 kVA transformer has series impedance of $R = 3\%$ and $X = 7\%$. Determine the percentage rise in the output voltage on unloading the transformer from full load at 0.85 pf lagging to zero load, which is also known as full load voltage regulation (VR) of the transformer.

SOLUTION

Full load means $I = 1.0$ pu and 0.85 power factor lagging means and $\theta = \cos^{-1}0.85 = 31.8°$, so $\tilde{I} = 1.0\angle{-31.8°}$ pu. The full load voltage is taken as 100% or 1.0 pu. We will work out this example three ways.

1. Transformer series impedance $\tilde{Z} = 0.03 + j\ 0.07 = 0.076\angle 66.8°$ pu. Taking $V_{2\text{fulllaod}} = 1.0\angle 0°$ pu as the reference phasor, and $\tilde{V}_{2\text{nolaod}} = \tilde{V}_{2\text{fulllaod}} + \tilde{I} \times \tilde{Z} = 1.0\angle 0° + 1.0\angle{-31.8°} \times 0.076\angle 66.8 = 1.0\angle 0° + 0.076\angle 35° = 1.063\angle 2.35°$.
   Using the voltage magnitudes, we get VR = (1.063 − 1) ÷ 1 = 0.063 pu or 6.3%.
2. We can also use exact Equation (3.14) in pu system to obtain

   $$V^2_{\text{no.laod}} = V_S^2 = (V_L + I{\cdot}R\cos\theta + I{\cdot}X\sin\theta)^2 + (I{\cdot}X\cos\theta - I{\cdot}R\sin\theta)^2$$

   $$= (1.0 + 1 \times 0.03 \times 0.85 + 1 \times 0.07 \times 0.527)^2 + (1 \times 0.07 \times 0.85 - 1 \times 0.03 \times 0.527)^2 = 1.063$$

   And then, VR = (1.063 – 1.0) ÷ 1.0 = 0.063 pu or 6.3%, the same as above.
3. Alternatively, using approximate Equation (3.18) with pu value, we obtain $\mathit{Voltage\,drop} = 1.0(0.03\,x\,0.85 + 0.07\sqrt{1-0.85^2}) = 0.0624$ *pu or* 6.24 %, which compares well with the exact value of 6.3%.

   In this percentage range, the secondary voltage must be regulated by using automatic tap charger if constant load voltage is desired. Otherwise, the secondary loads must allow this much percentage change in their input voltage and still deliver specified performance.

PROBLEMS

Problem 6.1: Determine the kVA and voltage ratings of a Δ-Y connected transformer to power a 3-phase Y-connected load drawing 600 A line current at 460 V from a 3-phase, 4160 V source. Using (i) one 3-phase transformer, and (ii) three i-phase transformers.

Problem 6.2: A 3000 kVA, 3-phase transformer bank steps down line voltage from 12.5 kV to 480 V. The HV side is connected in Δ and the LV side in grounded Y. Determine (a) the HV side line and phase currents, and (b) the LV side line and phase currents.

Problem 6.3*:* Three 1-phase transformers, each rated 75 kVA, are connected in Δ-Δ to power a 3-phase 225 kVA load. If one of them is removed for repairs, determine (i) the line voltage and line current on the load side, and (ii) the maximum kVA load the remaining two transformers can provide.

Problem 6.4: Determine the base current and base impedance of a 1-phase system based on 50 kVA and 120 V.

Problem 6.5: A 1-phase, 36 kVA, 480/120 V transformer has series impedance $Z = 5\%$ stated on its nameplate. Determine Z in ohms looking (a) from HV side source when connected as a step-down transformer, and (b) from LV side source when connected as a step-up transformer.

Problem 6.6: A 1-phase, 225 kVA, 460/120 V transformer has 6% impedance and 96% efficiency. Find its series R and X in percentages and in ohms, looking from the HV side source.

Problem 6.7: Determine the rms fault currents on both sides of a 100 kVA, 1-phase, 460/120 V transformer with 6% impedance as shown in Figure P6.7. Assume the generator bus infinite and all cable impedances are negligible.

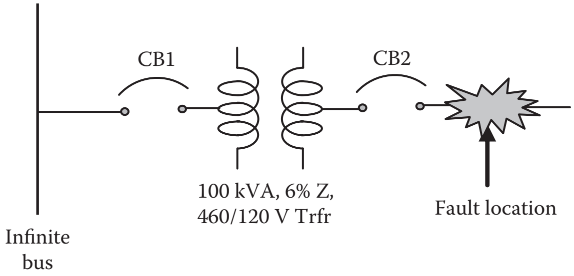


FIGURE P6.7

Problem 6.8: A 1-phase 100 kVA, 480/120 V step-down transformer powers a load impedance of 8 Ω on LV side. Determine (a) the current on HV side from conventional transformer calculations, and (b) equivalent load impedance looking from the HV side in the continuous equivalent circuit.

Problem 6.9: A 25 kVA, 1-phase, 460/120 V transformer powers 6 Ω load as shown in Figure P6.9. Determine the HV and LV side currents in amperes and also in% based on the rated voltage on two sides as their respective base.

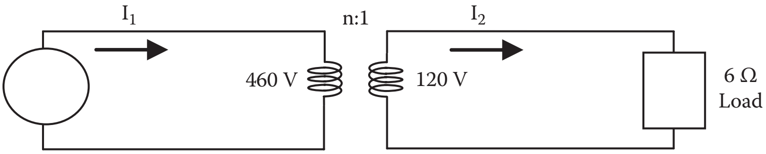


FIGURE P6.9

Problem 6.10: A 2-MVA, 4160/480 V transformer has series impedance of $R = 2\%$ and $X = 6\%$. Determine the (i) full load voltage regulation of the transformer at 0.85 pf lagging, and (ii) secondary output voltage on unloading the transformer completely.

Problem 6.11: A 3-phase. 500 kVA, 4160/480 V transformer bank has $R = 1.0\%$ and $X = 5\%$. Using Equation (3.15) or (3.18) with pu values, determine the% voltage rise in the secondary voltage on unloading from the following load conditions: (i) 80% load at unity power factor, (ii) full load at 0.85 power factor lagging, and (iii) full load at 0.90 pf leading. Assume that the secondary voltage was equal to the rated voltage in all load conditions.

Problem 6.12: A 1-ph, 100 kVA, 460/120 V transformer with 3% *R* and 6% *X* is powering a 3-kW heater. Determine the total continuous circuit impedance in ohms looking from the source side, and the current drawn from the source.

Problems 6.13: Determine a 3-phase 480 V/208Y-120 V transformer kVA rating if the daily variations in the secondary load current are 150 A for 5 h, 170 A for 1 h, and 130 A for 18 h, taking into account the service factor of 1.15 for 2 hours. Allow 30% margin for future growth.

QUESTIONS

Question 6.1 Explain the difference between power transformer, voltage (potential) transformer, and current transformer.

Question 6.2 Why doesn't the 3-phase transformer need the 4th core leg for the return flux?

Question 6.3 Which type of transformers are used on shipboard power system and why? How do they compare in cost, volume, and weight with other types commonly used on land?

Question 6.4 What are the instrument transformers, and how do they differ from the power transformer?

Question 6.5 Why are electrical equipments using pressurized gas (or vacuum) as the insulating mediums more compact than the dry type equipment?

Question 6.6 If you place a dry type transformer in an oil tank, it can deliver greater kVA load and withstand higher lightening voltages. Discuss why.

Question 6.7 The 3-phase transformer costs less than three 1-phase transformers in a 3-phase system. Even then, three 1-phase transformers are used in Δ-Δ connected to form the 3-phase bank in shipboard electrical power system. Explain why.

Question 6.8 Which part of the system would get overloaded if the kVA loading is not reduced after one of the three 1-phase transformers connected in Δ-Δ is removed for service? What will be the heat generation rate in that part?

Question 6.9 Why is the magnetizing current on the instant of connecting the transformer primary to the supply lines very high even when there is no short or no load connected on the secondary side?

Question 6.10 Explain in your own words the concept of reflected impedance or the impedance retransformation.

FURTHER READING

Smith, S. 1985. *Magnetic Components*. New York: Van Nostrand Reinhold.
Chapman, S.J. 1999. *Electric Machinery Fundamentals.* Boston: McGraw Hill.

7 Power Cable

The term *wire* generally means one or more insulated conductors (solid or stranded for flexibility) in small size, whereas the cable means one or more insulated conductors of large size grouped in a common insulation jacket, often with a ground shield. Figure 7.1 shows a single-conductor, 2 kV class cable, whereas Figure 7.2 is a 3-conductor, 3-phase cable suitable for 5 to 15 kV applications.

7.1 CONDUCTOR GAGE

Each conductor size is measured in American wire gage (AWG), British wire gage (BWG), or in metric gage designated by the conductor cross section area in mm^2. The AWG and BWG gage number are log-inverse measures of the conductor diameter. For example, the AWG is set on log scale as

$$AWG = 20\,\mathrm{Log}\left(\frac{0.325}{dia_{\mathrm{inch}}}\right) \quad \text{or} \quad dia_{\mathrm{inch}} = \frac{0.325}{10^{\frac{AWG}{20}}} \tag{7.1}$$

where dia = conductor diameter in inches, bare solid or one equivalent diameter of the conducting area of all strands combined. Decrease in one gage number increases the diameter by a factor of 1.1225 and area by a factor of 1.26. The diameter doubles every six gages down and the area doubles every three gages down. One can visualize the AWG gage number as the approximate number of wires that can be placed side by side in 1 in. width. For example, 14 bare conductors of AWG 14 can be placed side by side in 1 in. width, or the AWG 14 conductor has approximately 1/14 in. diameter.

The conductor is typically made of numerous thin strands for flexibility in handling and bending in installation. The strand diameter varies from 10 to 22 mils (1 mil = 1/1000th in. = 25.4 μm) depending on the wire gage. The strands are generally tin-plated to avoid oxidation that normally occurs on pure uncoated copper. The thin film of lubricant used on strands in the cable-making machine acts as the inter-strand insulation that keeps the skin effect (discussed later in this chapter) confined to the individual strand.

The cable sizes heavier than AWG 4/0 (also written as 0000) are designated by the net conductor cross section area in kilocircular mils (kcmil), which is also known as MCM (the first M for 1000 in Roman numerals), where

$$kcmil = \frac{(wire\ diameter\ in\ mils)^2}{1000} = 1000 \times (wire\ diameter\ in\ inch)^2 \tag{7.2}$$

The IEEE-45 standard uses kcmil to specify all conductor sizes, which increase linearly with the conductor cross section area. The European and international standards

Conductor

Soft annealed flexible stranded tinned copper per IEEE 1580 Table 11.

Insulation/Jacket

GEXOL® cross-linked flame retardant polyolefin, meeting the requirements for Type P of IEEE 1580 and Type X110 of UL 1309/CSA 245. 2000V/IEC 1000V.

Armor (Optional)

Basket weave wire armor per IEEE 1580 and UL 1309/CSA 245. Bronze standard. Aluminum or tinned copper available by request.

Sheath (Optional)

A black, arctic grade, flame retardant, oil, abrasion, chemical and sunlight resistant thermosetting compound meeting UL 1309/CSA 245 and IEEE 1580.

FIGURE 7.1 Single-conductor, 2 kV class power cable showing strands, insulation jacket, and outer sheath/shield (with permission from Gexol-insulated marine shipboard cable, a product of AmerCable, Inc.).

measure wire in metric gage equal to mm^2 of the net conductor area. The AWG, IEEE-45, and metric gages are all listed in Table 7.1, along with the number of strands, the strand diameter, and the total uninsulated conductor diameter in the last column.

The most common conductor material is copper, although aluminum finds special applications for its light weight and low cost. The physical properties of copper and aluminum are compared in Table 7.2. For the same electrical resistance per meter length, aluminum conductor will weigh in the (resistivity × mass density) ratio, which is $(2.83 \times 2.7) \div (1.724 \times 8.89) = 0.50$ or 50% compared to copper.

7.2 CABLE INSULATION

A variety of insulations are used in manufacturing wires and cables depending on the temperature rating desired in various applications. Major insulation materials and their operating temperature ranges are given in Table 7.3. Obviously, higher temperature rating of the insulation allows higher current carrying capacity of the conductor. In addition to an appropriate operating temperature rating of the insulation,

37-105

Type MMV Medium Power Cable

Single Conductor: 5kV – 15kV, 100% & 133% Insulation Levels. Rated 90°C
Multi-Conductor: 5kV – 15kV, 100% & 133% Insulation Levels. Rated 90°C

AmerCable INCORPORATED
Oil & Gas Cables

Conductors
Soft annealed flexible stranded tinned copper per IEEE 1580 Table 11.

Insulation
Extruded thermosetting 90°C Ethylene Propylene Rubber (EPR), meeting UL 1309 (Type E), IEEE 1580 (Type E), ICEA S-68-516 and UL 1072.

Metallic Shield
Composite shield consisting of 0.0126" tinned copper braided with nylon providing 60% copper Shielded coverage meeting UL 1309, IEEE Std. 1580, ICEA S-68-516 and UL 1072. The nylon is colored for easy phase identification (three conductor = black, blue, red) without the need to remove the shield to find an underlying colored tape.

Conductor Shield
A combination of semi-conducting tape and extruded thermosetting semi-conducting material meeting UL 1309, IEEE 1580, ICEA S-68-516 and UL1072.

Insulation Shield
Semi-conducting tape, with overlap, for fast and easy termination meeting UL 1309, IEEE 1580, ICEA S-68-516 and UL 1072.

Grounding Conductor (optional)
One uninsulated soft annealed flexible stranded tinned copper conductor per ASTM B 33 and sized according to Table 21.1 of UL 1072.

Jacket
A black, arctic grade, flame retardant, oil, abrasion, chemical and sunlight resistant thermosetting compound meeting UL 1309, IEEE 1580, ICEA S-68-516 and UL 1072. Colored jackets for signifying different voltage levels are also available on special request (ie. yellow = 5kV, orange = 8kV and red = 15kV).

Armor (optional)
(Optional) 0.0126" bronze braid providing 88% minimum coverage meeting UL 1309 and IEEE Std. 45-1998.

Sheath (optional)
A black, arctic grade, flame retardant, oil, abrasion, chemical, and sunlight resistant thermosetting compound meeting UL 1309, IEEE 1580, ICEA S-68-516 and UL 1072. Colored jackets for signifying different voltage levels is also available on special request (ie. yellow = 5kV, orange = 8kv and red = 15kV).

Applications

AmerCable's Type MMV marine medium voltage cables are for use aboard commercial ships, mobile offshore drilling units (MODUs), and fixed or floating offshore facilities.

Features

- These cables utilize flexible stranded conductors, braided shields and a braided armor (when armored) which make them very suitable for applications involving repeated flexing and high vibration.
- These cables have a small minimum bending radius (6xOD for unarmored cables and 8xOD for armored cables) for easy installation.
- Optional uninsulated grounding conductors sized per UL 1072.
- The increased flexibility of this cable allows for termination of one end and coiling on multiple module offshore platforms. Then coiling and terminating other end when modules are mated at sea thereby reducing installation time.
- Passes IEC 332-3 Category A and IEEE 1202 flame tests.

Ratings & Approvals

- UL Listed as Marine Shipboard Cable (E111461)
- American Bureau of Shipping (ABS)
- Det Norske Veritas (DNV) Pending
- Lloyd's Register of Shipping (LRS) Pending
- 90˚C Temperature Rating
- Voltage Rating – 5kV to 15kV

FIGURE 7.2 Three-phase medium voltage power cable with ground conductor, shield, armor, and outer sheath for 5 to 15 kV applications (with permission from Gexol-insulated marine shipboard cable, a product of AmerCable, Inc.).

the shipboard cable also needs high moisture resistance to withstand the damp and even wet conditions normal on ships. The industry has devised letter designations to indicate the temperature and moister resistance of the cable insulation as listed in Tables 7.3 and 7.4. Some examples of using the letter designations follows:

RH cable is made from rubber that can withstand high temperature up to 70°C (158°F)
RHH cable is made from rubber that can withstand very high temperature up to 90°C (194°F)
TW cable is made from thermoplastic that is moisture-resistance and can operate under wet conditions up to 60°C (140°F)
XHHW cable is made from cross-linked polyethylene that can withstand very high temperature in wet conditions up to 90°C (194°F)

TABLE 7.1
AWG, IEEE, and Metric Conductor Sizes with Stranding Profile

Size AWG/kcmil	Number of Strands	Individual Strand Diameter (inches)	Closest IEEE 45 Standard Size (kcmil)	Equivalent Metric Size (mm²)	Uninsulated Conductor Diameter (inches)
18	19	0.0100	2	0.96	0.049
16	19	0.0117	3	1.32	0.059
14	19	0.0147	4	2.08	0.074
12	19	0.0185	6	3.29	0.093
10	37	0.0167	10	5.23	0.113
8	37	0.0201	16	7.57	0.136
6	61	0.0201	26	12.49	0.175
4	133	0.0177	41	21.11	0.258
2	133	0.0223	66	33.51	0.324
1	209	0.0201	83	42.79	0.361
1/0	266	0.0201	106	54.45	0.407
2/0	342	0.0201	133	70.01	0.461
3/0	418	0.0201	168	85.57	0.510
4/0	532	0.0201	212	108.91	0.575
262	646	0.0201	262	132.25	0.654
313	777	0.0201	313	159.06	0.720
373	925	0.0201	373	189.36	0.785
444	1110	0.0201	444	227.23	0.860
535	1332	0.0201	535	272.68	0.941
646	1591	0.0201	646	325.70	1.029
777	1924	0.0201	777	393.87	1.132
1111	2745	0.0201	1111	561.94	1.354

TABLE 7.2
Copper and Aluminum Conductor Properties

Characteristic	Copper	Aluminum
Resistivity Ωm at 20°C	$1.724\ 10^{-8}$	$2.830\ 10^{-8}$
Mass density gram/cm^3	8.89	2.70
Specific heat J/kg°C at 20°C	377	900
Thermal conductivity W/m °C	395	211
Temperature coefficient of resistance α per °C	$3.93\ 10^{-3}$	$3.90\ 10^{-3}$
Melting point °C	1083	660
Flex life (relative)	1	0.5
Thermal coefficient of expansion (relative)	1	1.4
Creep rate at 65°C (relative)	1	1000

TABLE 7.3
Cable and Wire Insulations and Operating Temperatures

Cable and Wire Insulation (Material Identifying Letter)	Operating Temperature Range	
	°C	°F
Thermoplastic (T)	–40°C to 60°C	–40˚F to 140°F
Rubber (R)	–40°C to 75°C	–40°F to 167°F
Vinyl	–20°C to 80°C	–4°F to 176°F
Cross-linked polyethylene (X)	–60°C to 80°C	–76°F to 176°F
Neoprene	–30°C to 90°C	–22°F to 194°F
Polypropylene	–20°C to 105°C	–4˚F to 221°F
Fluorinated ethylene propylene (PF)	–40°C to 150°C	–40°F to 302°F
Teflon	–70°C to 200°C	–94°F to 392°F
Silicon rubber (S)	–70°C to 200°C	–94°F to 392°F

7.3 CONDUCTOR AMPACITY

Each conductor in the cable is sized to meet the required current carrying capacity (ampacity) and the voltage drop limitation under normal and inrush currents. The cable ampacity is limited by the continuous operating temperature limit, which in turn is limited by the insulation type. The cable surface temperature is typically limited to 60°C to 75°C for safety that depends on the application. The ampacity depends also on the ambient air temperature that can vary from 40°C to 65°C depending on the cable routing (e.g., inside cabins, on deck, in engine room, etc.), and on grouping in the raceway because of the mutual heating of the neighboring conductors. A cable with three or four conductors in one jacket can carry less current than a single conductor of the same size, and several cables side-by-side in a raceway can carry even less current. Both the higher ambient temperature and the raceway grouping require derating of the wire ampacity from the normally rated values given in the vendor technical data sheets.

Ampacity of three-phase cables with various insulation temperature ratings in 30°C (86°F) ambient cooling air are listed in Table 7.5 for low-voltage cables. At higher ambient air temperature, the amperes rating of a given cable must be reduced, and vice versa. Such derating factors are listed in Table 7.6.

TABLE 7.4
Letter Designation for Operating Conditions

Operating Conditions	Letter Designation
High temperature 70°C (158°F)	H
Very high temperature 90°C (194°F)	HH
Wet or damp	W
Oil resistance	M
Flexible (stranded)	F

TABLE 7.5
Permissible Ampacity of Selected Low-Voltage 3-Phase Insulated Copper Cables in Dry Location in Ambient Air at 30°C (86˚F)

	Temperature Rating of Insulation			
AWG or kcmil*	60°C (140°F) Type T, TW, RUW	90°C (194°F) Type TA, FEP, THHN, XHHW*	125°C (257°F) Insulation	200°C (392°F) Special Use
14	15	25	30	30
10	30	40	50	55
6	55	70	85	95
4	70	90	115	120
2	95	120	145	165
1/0	125	155	200	225
2/0	145	185	230	250
3/0	165	210	265	285
4/0	195	235	310	340
250	215	270	335	—
350	260	325	420	—
500	320	405	500	—
750	400	500	620	—
1000	455	585	730	—
1500	520	700	—	—
2000	560	775	—	—

[*] kcmil = 1000 circular mils = MCM in old usage (first M for 1000 in Roman numerals)

TABLE 7.6
Cable Derating Factors on Table 7.5 Ampacity for Ambient Air Temperature Higher than 30°C (86˚F)

Ambient temperature		Temperature rating of insulation			
°C	°F	60°C (140°F)	90°C (194°F)	125°C (257°F)	200°C (392°F)
40	104	0.82	0.90	0.95	—
50	122	0.58	0.80	0.89	—
60	140	—	0.67	0.83	0.91
70	158	—	0.52	0.76	0.87
80	176	—	0.30	0.69	0.84
90	194	—	—	0.61	0.80
100	212	—	—	0.51	0.77
140	284	—	—	—	0.59

Source: Adapted from NEC®.

Example 7.1

A cable made with polypropylene insulation has nominal ampacity of 200 A in standard 40°C ambient air. Determine its ampacity for use in the ship's engine room uptake where the ambient air is 70°C.

SOLUTION

As per Table 7.3, polypropylene insulation is good up to 105°C, and that leaves 105 – 40 = 65°C temperature rise with nominal ampacity in 40°C ambient air. If we use this cable in 70°C ambient air, we must limit the conductor temperature rise to 105 – 70 = 35°C, instead of 65°C. We write Equation (3.23) in a ratio form to cancel out the cable heat dissipation geometry constant that does not change with the ambient temperature,

$$\frac{\Delta T_{\text{new}}}{\Delta T_{\text{old}}} = \frac{35}{65} = \frac{K\left(I_{\text{new}}^2 R\right)^{0.8}}{K\left(I_{\text{old}}^2 R\right)^{0.8}} = \left\{\frac{I_{\text{new}}}{I_{\text{old}}}\right\}^{1.6} \text{, which gives}$$

$$I_{\text{new}} = 200 \times \left(\frac{35}{65}\right)^{\frac{1}{1.6}} = 200 \times 0.68 = 136\ A$$

This cable can be used only for 136 A maximum in the engine room uptake, that is, a derating factor of 136 ÷ 200 = 0.68 factor or 68% of its nominal ampacity of 200 A.

Alternatively, Table 7.6 can be used with some interpolation. For example, for 105°C class polypropylene insulation, we read between 90°C and 125°C columns, which for 70°C ambient air gives derating factor between 0.52 and 0.76. The 105°C being approximately the mid-point in the 90°C–125°C range, the derating factor would be ½ (0.52 + 0.76) = 0.64 approximately, which is a close match with 0.68 derived above from the heat transfer fundamentals.

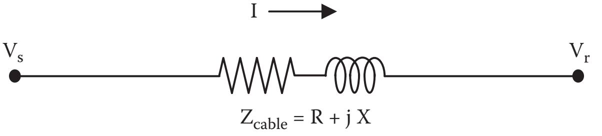
7.4 CABLE ELECTRICAL MODEL

In the electrical model of a power system, each phase conductor of the cable is represented by its resistance *R* and leakage reactance *X* placed in series in the single-line diagram (Figure 7.3). The reactance $X = \omega L$ where inductance *L* is due to the magnetic flux leakage in air between the current carrying conductors. The cable also has a capacitance to ground, which is small and is generally ignored.

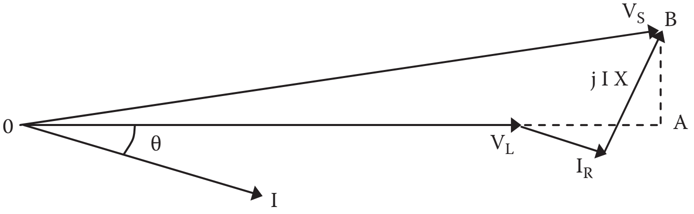
The cable *R* and *X* values are often provided by the cable manufacture, or can be found from tables in reference books, or from the general formulas given below.

The conductor resistance varies directly with length and inversely with cross section area, that is,

$$R = \frac{\mathit{Resistivity} \times \mathit{Length}}{\mathit{Conductor\ cross\ section\ area}}$$



(a) Cable impedance between source and load



(b) Phasor diagram for voltage drop

FIGURE 7.3 Cable electrical model and phasor diagram.

or

$$R = K\frac{\textit{Length in feet}}{\textit{Area in circular mils}} \text{ ohms} \tag{7.3}$$

For dc or low frequency currents at 20°C, $K = 10.372$ for copper and 18.046 for aluminum, both of electrical grade. The conductor resistance changes with temperature. Its value R_2 at higher temperature T_2 increases from R_1 at lower temperature T_1 as follows:

$$R_2 = R_1 \times \{1 + \alpha\,(T_2 - T_1)\} \tag{7.4}$$

where α = temperature coefficient of resistance = 0.0039 per °C for both copper and aluminum.

The leakage reactance of 3-phase cable shown in Figure 7.4 cross section is given by

$$X = 52.9\frac{f}{60}\log_{10}\left(\frac{GMD_\phi}{GMR_\phi}\right) \ \mu\Omega \text{ per phase per foot of cable} \tag{7.5}$$

where

f = supply line frequency

$GMD_\varphi = (D_{ab}D_{bc}D_{ca})^{1/3}$ = geometrical mean distance between phase conductors separated by center-to-center distance of D_{ab}, D_{bc}, D_{ca} between phase conductors a-b, b-c and c-a, respectively

$GMR_\varphi = (r_1\ r_2 \ldots r_n)^{1/n}$ = geometrical mean radius of one phase conductor made of n strands placed at radial locations r_1, r_2, r_3, … r_n (not shown) from the conductor center

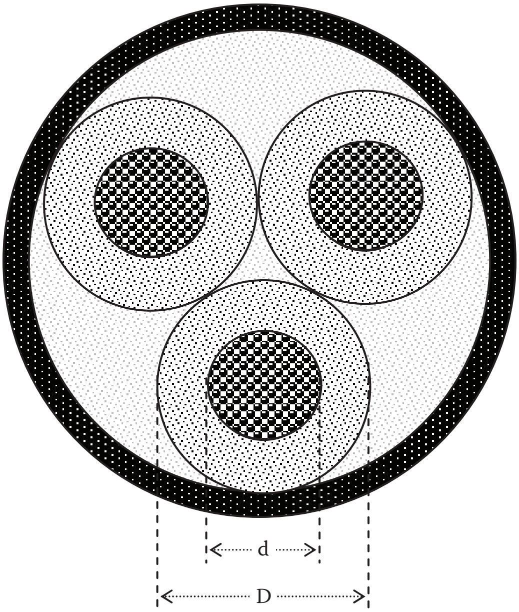


FIGURE 7.4 Three-phase cable cross section and GMD and GMR definitions.

The GMD and GMR can be in any unit (cm or inches), as they are in a ratio. For cable sizes used in shipboard power ranges, GMD_φ and GMR_φ can be approximated as follows:

$GMD_\varphi = D$, where D = outer diameter of each insulated phase conductor in 1-phase (2-wire) or 3-phase (3-wire or 4-wire) cable, and
$GMR_\varphi = 0.375 \times d$, where d = outer diameter of bare phase conductor

Equations (7.3) and (7.5) values are per phase that are used in balanced 3-phase *Y*-connected system analysis, where only one wire per phase carries the load current. There is no return current, so the neutral wire (even if provided) does not contribute in the voltage drop. In 1-phase system, Equations (7.3) and (7.5) values are multiplied by two to account for the lead and return conductors carrying the same current.

For twisted pairs of small wires touching each other, $D = (d + 2 \times \text{wire insulation thickness})$. Since the insulation thickness is proportional to the wire radius for mechanical reasons, the GMD/GMR ratio remains approximately constant, making the cable inductance somewhat insensitive to the wire gage. For AWG 4/0 to AWG 30 wires, for example, the leakage inductance remains in the range of 0.5–0.7 μH/m length of round-trip twisted pair up to hundreds of kHz frequency. Twisting does not reduce the inductance per meter, but adds 10%–15% in the length.

7.5 SKIN AND PROXIMITY EFFECTS

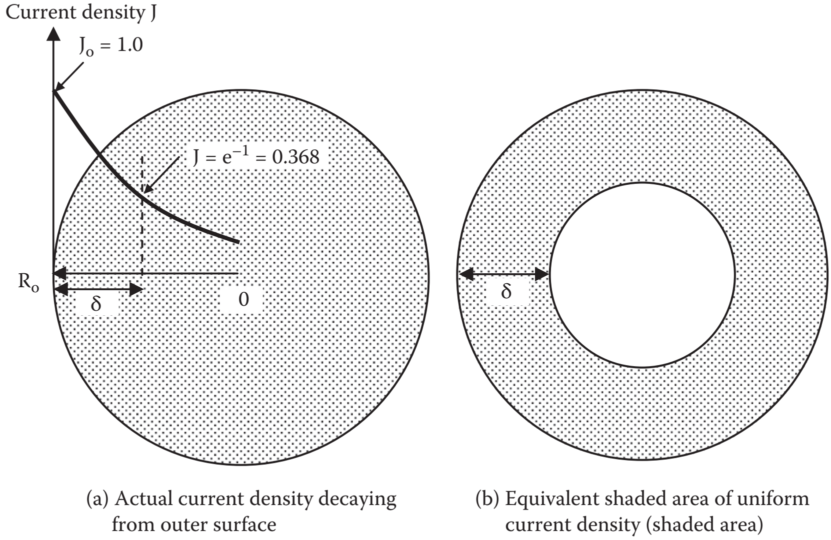
Equation (7.3) gives dc resistance of conductor in which the current is uniformly distributed. High frequency current distribution is not uniform over the conductor cross section. It concentrates near the conductor skin as shown in Figure 7.5 for a round conductor, leaving the inner cross section not fully utilized in carrying the current. This effectively increases the conductor resistance and hence the I^2R loss and heating. The higher the frequency, the thinner the skin depth, which is given by

$$\text{Skin thickness in SI units, } \delta = \sqrt{\frac{\rho}{\mu \pi f}} \tag{7.6}$$

where ρ = electrical conductivity of wire, μ = magnetic permeability of flux medium, and f = frequency, all in SI units.

For copper at typical working temperature, the skin depth is approximately 10 mm (0.394 inch) at 60 Hz, 3.87 mm (0.152 in.) at 400 Hz, 0.5 mm (0.020 in.) at 20 kHz, and 0.25 mm (0.010 in.) at 100 kHz. Thinner skin with concentrated current means less cross section area for the current and proportionally higher conductor resistance. For this reason, the ac resistance is always higher than the dc resistance depending on the frequency. For example, a 5 mm (0.20 inch) thick bus bar would have the ratio $R_{ac}/R_{dc} = 1.1$ at 60 Hz, 1.32 at 400 Hz, and 20 at 100 kHz.

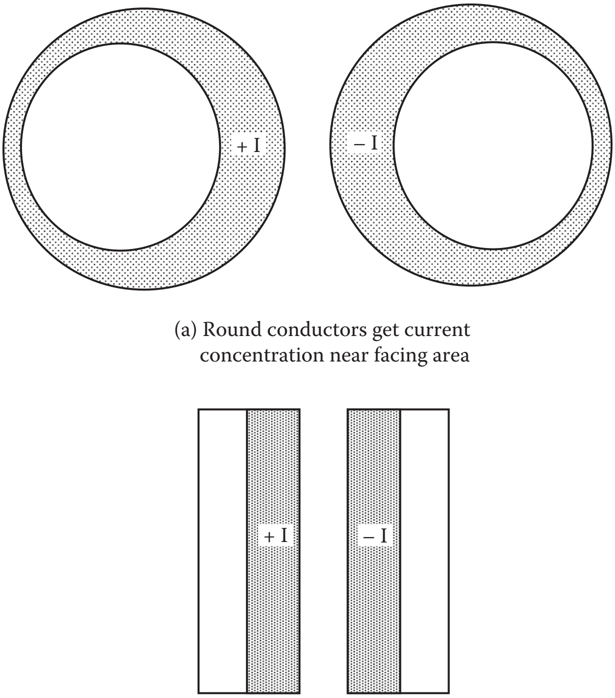
The conductor adds power loss due to its own skin effect, and also due to eddy currents induced by the leakage flux of neighboring current carrying conductors. In the case of two conductors carrying 1-phase current in opposite directions, the current



(a) Actual current density decaying from outer surface

(b) Equivalent shaded area of uniform current density (shaded area)

FIGURE 7.5 Skin effect and resulting nonuniform current distribution in round conductor.



(a) Round conductors get current concentration near facing area

(b) Bus bars with facing flats get current concentration near facing strips

FIGURE 7.6 Proximity effect in two adjacent current carrying conductors.

in both conductors concentrates near their adjacent faces as shown in Figure 7.6. The virtual effect of such current concentration is to increase the effective resistance of the conductor working in proximity of other conductors. The combined skin and the proximity effects are generally accounted for by multiplying factors listed in Table 7.7 for various conductors at 60 Hz and 50 Hz, from which

$$R_{ac} = R_{dc} \times \text{Skin and proximity factor from Table 7.7} \tag{7.7}$$

7.6 CABLE DESIGN

The cable design primarily requires selecting the conductor size with required ampacity at the operating temperature that will also meet the voltage drop limitation under steady-state and the motor starting inrush current over the feeder length. Both calculations require the values of the conductor current I and power factor, resistance R, and leakage reactance X, all values per phase. Then, the analysis is similar to that in Section 3.7, where Equations (3.15) and (3.17) gave

$$V_{drop} = I \times (R\cos\theta + X\sin\theta) = I \times Z_{eff} \tag{7.8}$$

$$\text{where } Z_{eff} = (R\cos\theta + X\sin\theta) \tag{7.9}$$

TABLE 7.7
Combined Skin and Proximity Effect on ac/dc Resistance Ratio at 60 Hz

Conductor Size AWG or kcmil[a]	Cable with Nonmetallic Shield in Air or Nonmetallic Conduit		Cable with Metallic Shielded or Metallic Raceway	
	Copper	**Aluminum**	**Copper**	**Aluminum**
Up to AWG #1 (84 kcmil)	1.000	1.000	1.000	1.000
250 kcmil	1.005	1.002	1.06	1.02
500 kcmil	1.018	1.007	1.13	1.06
1000 kcmil	1.067	1.026	1.30	1.19
1500 kcmil	1.142	1.058	1.53	1.36
2000 kcmil	1.233	1.100	1.82	1.56

Note: 60 Hz resistance = DC resistance × factor above = approximate 50 Hz resistance

[a] kcmil = 1000 circular mils = MCM in old usage (first M for 1000 in Roman numerals).

Source: Selected data from NEC®.

All calculations can be done using volts, amperes, and ohms per phase, or in pu or percent values, and taking θ *positive for lagging pf* and negative for leading pf.

Equation (7.9) is a convenient definition of Z_{eff}, because the product of I and Z_{eff} simply gives the voltage drop magnitude. Or, the voltage drop per ampere in the cable,

$$V_{\text{drop/amp}} = Z_{\text{eff}} \text{ volts/ampere} \tag{7.10}$$

Many cable manufactures list Z_{eff} as the cable impedance at typical pf of 0.85 lagging. Therefore, cable Z values listed in manufacturer's catalog are typically

$$Z_{\text{cable}} = R \times 0.85 + X \times \{1 - 0.85^2\}^{1/2} = 0.85\,R + 0.527\,X\ \Omega/\text{phase} \tag{7.11}$$

Alert: If the pf is much different than 0.85 lagging, Equation (7.10) is not valid. Equation (7.8) gives better results for a wide range of power factors from 0.2 lag to 0.9 lead if $R \ll X$.

The cable size is usually selected to meet the ampacity with 20% to 30% margin that will also limit the steady state voltage drop generally below 3%–5% from the controlled switchboard to the load that may include a transformer.

Many cable vendors provide ac resistance R_{ac} and leakage reactance X_L values for various cable sizes, as illustrated in Table 7.8 that includes the skin and proximity effects and the conduit type as well. The skin and proximity effects reduce the cable leakage inductance slightly. The difference between dc and high frequency inductance is small and is generally ignored in power system studies.

TABLE 7.8

Impedance of 600 V 3-Phase Cables with Copper Conductors at 60 Hz and 75°C (167°F), Including Skin and Proximity Effects

Ohms Per Phase (Line-to-Neutral) per 1000 ft Copper Wires at 60 Hz*, 75°C, with Skin and Proximity Effects

	Reactance X_L		R_{ac} for Copper Conductors		Z_{eff} at 0.85 pf Lagging**		
AWG or kcmil	**PVC or Alum. Conduit**	**Steel Conduit**	**PVC Conduit**	**Steel or Alum. Conduit**	**PVC Conduit**	**Alum. Conduit**	**Steel Conduit**
10	0.050	0.063	1.2	1.2	1.1	1.1	1.1
4	0.048	0.060	0.31	0.31	0.29	0.29	0.30
1	0.046	0.057	0.15	0.16	0.16	0.16	0.16
00	0.043	0.054	0.10	0.10	0.11	0.11	0.11
0000	0.041	0.051	0.062	0.065	0.074	0.078	0.080
250	0.041	0.052	0.052	0.055	0.066	0.070	0.073
300	0.041	0.051	0.044	0.047	0.059	0.063	0.065
400	0.040	0.049	0.033	0.037	0.049	0.053	0.056
500	0.039	0.048	0.027	0.031	0.043	0.048	0.050
750	0.038	0.048	0.019	0.023	0.036	0.040	0.043
1000	0.037	0.046	0.015	0.019	0.032	0.036	0.040

* Approximate 50 Hz values $R_{50Hz} = R_{60Hz}$ and $X_{L50Hz} = (50/60)\ X_{L60HZ}$

** $Z_{eff} = R \cos\theta + X \sin\theta$, where $\cos\theta$ = pf of the cable load. The cable voltage drop per phase (line-to-neutral) is approximately equal to $I_{Line} \times Z_{eff}$. The table values above are for 0.85 power factor lagging.

Source: Modified from NEC data.

Example 7.2

A 1 kV, AWG-4 copper conductor has dc resistance of 0.30 Ω per 1000 ft at 25°C. Using the tables provided in this chapter, determine its 60 Hz resistance at operating temperature of 110°C.

SOLUTION

For 1000 feet length of AWG 4 conductor, Table 7.8 gives $R_{ac} = 0.31\ \Omega$ at 60 Hz and 75°C. We correct it for 110°C using Equation (7.4),

$$R_{ac} \text{ at } 110°\text{C} = 0.31\ \{1 + 0.0039\ (110 - 75)\} = 0.352\ \Omega$$

Alternatively, Table 7.7 gives R_{ac}/R_{dc} ratio of 1.00. With temperature correction factor of $\{1 + 0.0039\ (110 - 25)\} = 1.3315$, we have R_{ac} at 110°C = $0.30 \times 1.00 \times 1.3315 = 0.399\ \Omega$.

Seeing such difference (0.352 vs. 0.399 Ω per 1000 ft) is possible, as data come from different sources with somewhat different considerations. In such a situation, the engineer must use conservative value or in accordance with specific standard cited in the contract.

Example 7.3

A 3-phase, Y-connected, 4.16 kV_{LL} feeder has $R = 50$ mΩ and $X = 30$ mΩ, both per phase per 1000 feet length. Determine voltage drop in the feeder at (i) 0.8 power factor lagging, and (ii) unity power factor. Establish the V_{drop} per 1000 ft run for 1 MVA load so that it can be easily reused for various MVA loading and feeder lengths.

SOLUTION

$V_{LN} = 4160 \div \sqrt{3} = 2400 \angle 0° \text{ V}$

For 1-MVA 3-phase load, phase current $I = 1000 \text{ kVA} \div (\sqrt{3} \times 4.16 \text{ kV}) = 138.8$ A/ph

For 1000 feet run, $R = 0.05$ Ω/ph and $X = 0.07$Ω/ph

Using Equation (3.15) at 0.8 power factor lagging, that is, $\cos\theta = 0.8$ and $\sin\theta = 0.6$,

$V_{drop} = 138.8\,(0.05 \times 0.8 + 0.07 \times 0.6) = 11.38$ V/phase per MVA per 1000 feet cable.

At unity pf, $\cos\theta = 1.0$, $\sin\theta = 0$, and

$V_{drop} = 138.8\,(0.05 \times 1.0 + 0.07 \times 0) = 6.94$ V/phase per MVA per 1000 feet cable.

At leading pf, it would be even less than 6.94 V/phase, and can be zero at a certain leading pf. This illustrates the effects of pf on the voltage drop.

7.7 MARINE AND SPECIAL CABLES

The shipboard and ocean environments pose severe challenges to power cables with heat, vibration, salt corrosion, mud, and mechanical stress. The cable must be selected to operate reliably in such harsh environments, which the marine cable manufacturers take in to account. A few technical data sheets of flexible marine power cables are given in the following tables with permission from Gexol-insulated marine shipboard cable, a product of AmerCable, Inc.:

Table 7.9 Two-conductor power cable with 1 kV, 110°C class insulation
Table 7.10 Three-conductor power cable with 1 kV, 100°C class insulation
Table 7.11 Three-conductor medium voltage cable with 5 kV class insulation
Table 7.12 Three-conductor medium voltage cable with 8 kV class insulation
Table 7.13 Three-conductor medium voltage cable with 15 kV class insulation

The cable manufacturers also offer special designs to meet specific requirements in certain applications, for example, for variable frequency drives, shore-to-ship power (cold ironing), mobile substation, magnetic crane for loading and unloading ship cargo, etc. Some such requirements are listed in Table 7.14.

TABLE 7.9
Two-Conductor Power Cable with 1 kV, 110°C Class Insulation

Flexible Power Cable—Two Conductor

			Unarmored		Armored (B)		Armored and Sheath (BS)						Ampacity		
Size AWG/ Kcmil	mm²	Part No. 37-102	Nominal Diameter (inches)	Weight (lbs/1000 ft.)	Nominal Diameter (inches)	Weight (lbs/1000 ft.)	Nominal Diameter (inches)	Weight (lbs/1000 ft.)	DC Resistance at 25°C (ohms/1000 ft.)	AC Resistance 110°C, 60 Hz (ohms/1000 ft.)	Inductive Reactance (ohms/1000 ft.)	Voltage Drop 110°C (volts/ Amp/1000 ft.)	110°C	100°C	95°C
16	1.3	–501	0.350	75	0.400	141	0.540	202	4.610	6.121	0.039	8.511	20	19	20
14	2.1	–507	0.380	84	0.430	165	0.561	230	2.907	3.859	0.036	5.379	33	31	27
12	3.3	–515	0.420	111	0.470	190	0.601	263	1.826	2.424	0.034	3.390	43	40	32
10	5.2	–553	0.460	146	0.510	230	0.641	307	1.153	1.530	0.032	2.151	53	49	43
8	7.6	–209	0.600	221	0.650	327	0.781	416	0.708	0.940	0.034	1.336	69	64	58
6	12.5	–210	0.690	308	0.730	424	0.903	559	0.445	0.590	0.032	0.850	91	85	77
4	21	–594	0.887	516	0.937	664	1.110	835	0.300	0.399	0.029	0.582	118	110	103
1/0	54	–216	1.243	1128	1.293	1334	1.466	1562	0.117	0.156	0.028	0.245	213	199	184
4/0	109	–219	1.593	2003	1.643	2271	1.878	2680	0.059	0.080	0.026	0.138	329	307	285

Note: Cable diameters shown as nominal are subject to a ±5% manufacturing tolerance.

Source: From Gexol-insulated marine shipboard cable, a product of AmerCable, Inc. With permission.

TABLE 7.10

Three-Conductor Power Cable with 1 kV, 100°C Class Insulation

Flexible Power Cable—Three Conductor

Size AWG/ Kcmil	mm2	Part No. 37-102	Unarmored		Armored (B)		Armored and sheath (BS)		DC Resistance at 25°C (ohms/1000 ft.)	AC Resistance 110°C, 60 Hz (ohms/1000 ft.)	Inductive Reactance (ohms/1000 ft.)	Voltage Drop 110°C (volts/ Amp/1000 ft.)	Opt. Uninsulated Grounding Cond. Size AWG	Ampacity		
			Nominal Diameter (inches)	Weight (lbs/1000 ft.)	Nominal Diameter (inches)	Weight (lbs/1000 ft.)	Nominal Diameter (inches)	Weight (lbs/1000 ft.)						110°C	100°C	95°C
16	1.3	–502	0.369	65	0.419	127	0.519	181	4.610	6.121	0.039	8.511	—	17	16	16
14	2.1	–508	0.401	102	0.451	176	0.583	228	2.907	3.859	0.036	5.379	—	27	25	22
12	3.3	–516	0.445	133	0.495	212	0.626	276	1.826	2.424	0.034	3.390	—	33	31	27
10	5.2	–308	0.488	189	0.538	281	0.669	352	1.153	1.530	0.032	2.151	—	44	41	36
8	7.6	–309	0.637	274	0.687	385	0.818	477	0.708	0.940	0.034	1.336	—	56	52	48
6	12.5	–310	0.723	390	0.773	519	0.946	650	0.445	0.590	0.032	0.850	8	75	70	64
4	21	–312	0.942	678	0.992	843	1.165	1004	0.300	0.399	0.029	0.582	8	99	92	85
2	34	–314	1.084	987	1.134	1160	1.307	1374	0.184	0.244	0.028	0.366	6	131	122	113
1	43	–315	1.206	1234	1.256	1458	1.431	1675	0.147	0.195	0.028	0.299	6	153	143	131
1/0	54	–316	1.326	1448	1.376	1781	1.550	2015	0.117	0.156	0.028	0.245	6	176	164	152
2/0	70	–317	1.422	1945	1.472	2082	1.645	2424	0.093	0.125	0.027	0.200	6	201	188	175
3/0	86	–318	1.528	2379	1.578	2720	1.814	3106	0.074	0.100	0.027	0.166	4	234	218	202
4/0	109	–319	1.765	2864	1.815	3233	2.050	3652	0.058	0.080	0.026	0.138	4	270	252	235
262	132	–320	1.980	3452	2.030	3880	2.266	4434	0.048	0.067	0.026	0.119	3	315	294	267
313	159	–321	2.131	4023	2.181	4434	2.418	4919	0.040	0.056	0.026	0.105	3	344	321	299
373	189	–322	2.231	4772	2.281	5219	2.517	5718	0.034	0.047	0.025	0.092	3	387	361	334

444	227	−323	2.394	5670	2.444	6176	2.680	6864	0.028	0.041	0.025	0.083	2	440	411	372
535	273	−324	2.637	6784	2.687	7492	2.986	8250	0.024	0.035	0.026	0.075	2	498	443	418
646	326	−326	2.958	7961	3.008	8414	3.301	9258	0.020	0.030	0.026	0.068	1	553	516	470
777	394	−327	3.168	9573	3.218	10065	3.511	10945	0.016	0.026	0.026	0.063	1	602	562	529

Source: From Gexol-insulated marine shipboard cable, a product of AmerCable, Inc. With permission.

TABLE 7.11
Three-Conductor Medium Voltage Cable with 5 kV Class Insulation

Three Conductor Type MMV Marine Medium Voltage – 5kV, 100/133% Insulation Level for MMV Stranding Profile												
			Unarmored		Armored and sheathed (BS)		Ampacity					
Size AWG/ Kcmil	mm2	Part No. 37–105	Nominal Diameter (inches)	Weight (lbs./1000 ft.)	Nominal Diameter (inches)	Weight (lbs./1000 ft.)	In Free Air (amps)	Single Banked in Trays (amps)	DC Resistance at 25°C (ohms/1000 ft.)	AC Resistance at 90°C, 60 Hz (ohms/1000 ft.)	Inductive Reactance (ohms/1000 ft.)	Voltage Drop (Volts per amp per 1000 ft.)	AWG Size of Optional Grounding Conductor
8	7.6	–301	1.137	781	1.369	1218	66	56	0.708	0.885	0.048	1.275	8
6	12.5	–302	1.226	955	1.457	1424	88	75	0.445	0.556	0.044	0.815	6
4	21	–303	1.402	1307	1.625	1824	116	99	0.300	0.376	0.039	0.560	6
2	34	–304	1.538	1690	1.824	2372	152	129	0.184	0.230	0.036	0.356	6
1	43	–305	1.626	1974	1.911	2692	175	149	0.147	0.184	0.035	0.291	4
1/0	54	–306	1.783	2423	2.081	3232	201	171	0.117	0.147	0.034	0.239	4
2/0	70	–307	1.913	2884	2.210	3749	232	197	0.093	0.117	0.033	0.196	4
3/0	86	–308	2.007	3315	2.305	4220	266	226	0.074	0.094	0.032	0.163	3
4/0	109	–309	2.140	3937	2.438	4899	306	260	0.058	0.075	0.031	0.136	3
262	132	–310	2.310	4619	2.608	5654	348	296	0.048	0.063	0.030	0.118	3
313	159	–311	2.453	5319	2.796	6549	386	328	0.040	0.053	0.029	0.104	2
373	189	–312	2.589	6107	3.000	7402	429	365	0.034	0.045	0.029	0.092	2

444	227	−313	2.818	7280	3.161	8684	455	387	0.028	0.039	0.028	0.083	1
535	273	−314	2.974	8463	3.317	9964	528	449	0.024	0.033	0.028	0.074	1
646	326	−315	3.164	9814	3.507	11407	584	496	0.020	0.028	0.027	0.067	1
777	394	−316	3.385	11526	3.729	13226	647	550	0.016	0.025	0.027	0.062	1/0

Source: From Gexol-insulated marine shipboard cable, a product of AmerCable, Inc. With permission.

TABLE 7.12
Three-Conductor Medium Voltage Cable with 8 kV Class Insulation

Three Conductor Type MMV Marine Medium Voltage – 8kV, 100% Insulation Level

			Unarmored		Armored and sheathed (BS)		Ampacity						
Size AWG/ Kcmil	mm2	Part No. 37–105	Nominal Diameter (inches)	Weight (lbs./1000 ft.)	Nominal Diameter (inches)	Weight (lbs./1000 ft.)	In Free Air (amps)	Single Banked in Trays (amps)	DC Resistance at 25°C (ohms/1000 ft.)	AC Resistance at 90°C, 60 Hz (ohms/1000 ft.)	Inductive Reactance (ohms/1000 ft.)	Voltage Drop (Volts per amp per 1000 ft.)	AWG Size of Optional Grounding Conductor
6	12.5	–317	1.338	1094	1.561	1589	88	75	0.445	0.556	0.046	0.818	6
4	21	–318	1.514	1462	1.799	2134	116	99	0.300	0.376	0.041	0.562	6
2	34	–319	1.650	1970	1.998	2725	152	129	0.184	0.230	0.038	0.357	6
1	43	–320	1.800	2263	2.085	3054	175	149	0.147	0.184	0.037	0.293	4
1/0	54	–321	1.859	2617	2.181	3454	201	171	0.117	0.147	0.036	0.241	4
2/0	70	–322	2.025	3100	2.310	3989	232	197	0.093	0.117	0.034	0.198	4
3/0	86	–323	2.119	3531	2.404	4458	266	226	0.074	0.094	0.033	0.165	3
4/0	109	–324	2.252	4162	2.537	5140	306	260	0.058	0.075	0.032	0.138	3
262	132	–325	2.422	4864	2.707	5913	348	296	0.048	0.063	0.031	0.119	3
313	159	–326	2.565	5581	2.914	6884	386	328	0.040	0.053	0.030	0.105	2
373	189	–327	2.704	6392	3.054	7760	429	365	0.034	0.045	0.030	0.093	2
444	227	–328	2.930	7582	3.280	9059	455	387	0.028	0.039	0.029	0.084	1
535	273	–329	3.096	8806	3.439	10366	528	449	0.024	0.033	0.029	0.075	1
646	326	–330	3.267	10137	3.611	11780	584	496	0.020	0.028	0.028	0.068	1
777	394	–331	3.512	11959	3.855	13708	647	550	0.016	0.025	0.028	0.063	1/0

Source: From Gexol-insulated marine shipboard cable, a product of AmerCable, Inc. With permission.

TABLE 7.13
Three-Conductor Medium Voltage Cable with 15 kV Class Insulation

Three Conductor Type MMV Marine Medium Voltage – 15kV, 100% Insulation Level

			Unarmored		Armored and Sheathed (BS)		Ampacity						
Size AWG/ Kcmil	mm2	Part No. 37–105	Nominal Diameter (inches)	Weight (lbs./1000 ft.)	Nominal Diameter (inches)	Weight (lbs./1000 ft.)	In Free Air (amps)	Single Banked in Trays (amps)	DC Resistance at 25°C (ohms/1000 ft.)	AC Resistance at 90°C, 60 Hz (ohms/1000 ft.)	Inductive Reactance (ohms/1000 ft.)	Voltage Drop (volts per amp per 1000 ft.)	AWG Size of Optional Grounding Conductor
2	34	–346	2.157	2759	2.443	3697	156	133	0.184	0.230	0.042	0.361	6
1	43	–347	2.239	3073	2.524	4045	178	151	0.147	0.184	0.040	0.296	4
1/0	54	–348	2.335	3466	2.620	4477	205	174	0.117	0.147	0.039	0.244	4
2/0	70	–349	2.461	3991	2.810	5242	234	199	0.093	0.117	0.037	0.201	4
3/0	86	–350	2.559	4466	2.910	5764	269	229	0.074	0.094	0.036	0.168	3
4/0	109	–351	2.691	5150	3.041	6513	309	263	0.058	0.075	0.035	0.141	3
262	132	–352	2.749	5749	3.152	7348	352	299	0.048	0.063	0.034	0.122	3
313	159	–353	2.881	6483	3.287	8184	389	331	0.040	0.053	0.033	0.107	2
373	189	–354	3.021	7331	3.365	8856	432	367	0.034	0.045	0.032	0.095	2
444	227	–355	3.183	8380	3.527	9983	456	388	0.028	0.039	0.031	0.086	1
535	273	–356	3.357	9599	3.701	11285	528	449	0.024	0.033	0.031	0.077	1

Source: From Gexol-insulated marine shipboard cable, a product of AmerCable, Inc. With permission.

TABLE 7.14
Cable Requirement in Selected Special Applications

Application	Special Cable Requirements
Variable frequency drives	Braided shield to limit EMI emission Multiple ground conductors to reduce voltage unbalance and common mode noise back to VFD Lower dielectric constant of the insulation to reduce reflected wave peak voltages Insulation with higher breakdown voltage strength to resist 2 × rated voltage spikes
Magnetic cranes	Thinner strands for greater flexibility
Mobile substation	Greater protection against abrasion, impact, heat, oil, alkali, and acid
Portable power	
Shore-to-ship power (Cold ironing)	Thinner strands for greater flexibility Greater abrasion resistance and tear strength in wet conditions

Example 7.4

Select a 3-phase 4.6 kV cable to carry 240 A/phase in single-banked tray and determine its outer sheath diameter and weight per 1000 feet. The cable must limit the voltage drop below 30 V/phase per 1000 feet at operating temperature of 90°C.

SOLUTION

As the first trial, we use Table 7.11 for 5 kV class cable and choose AWG 4/0 that has 260 A ampacity in single-banked tray and voltage drop of 0.136 V/A. This amounts to the actual voltage drop of 240 A (our actual current) × 0.136 = 32.64 V per 1000 ft. The ampacity of this cable meets the requirement, but the voltage drop does not.

We, therefore, select the next higher size 262 kcmil, 5 kV class cable that has the ampacity of 296 A in single-banked tray and voltage drop of 0.118 V/1000 ft, amounting to 240 × 0.118 = 28.32 V, now meeting the requirement.

For this 262 kcmil cable, Table 7.11 gives nominal diameter 2.608 in. with outer sheath and weight 5654 lb per 1000 ft. The overhead trays should be designed to support this weight.

Example 7.5

Select 3-phase, 11 kV cable to power 20,000 hp propulsion motor that has 97% efficiency and 95% pf lagging. Determine the percentage of voltage drop if the distance from the switchboard to the motor is 200 feet. You may place more than one cable in parallel, if needed.

SOLUTION

The line current in the motor lines is derived from

$$\text{Motor input } kVA_{3\text{ph}} = \frac{20{,}000 \times 0.746}{0.97 \times 0.95} = \sqrt{3} \times 11 \times I_L, \text{ which gives } I_L = 850 \text{ A.}$$

For 11 kV operating voltage, the next standard insulation class is 15 kV. Therefore, we use Table 7.13. The heaviest cable in the table is 535 kcmil with ampacity of 449 A in trays. Therefore, we must use two cables in parallel, each carrying ½ × 850 = 425 A, which is less than the cable ampacity of 449 A.

For 535 kcmil cable, Table 7.13 gives V_{drop} of 0.077 volts/ampere per 1000 ft. For 200 ft cable carrying 425 A, $V_{\text{drop}} = 0.077 \times 425 \times (200 \div 1000) = 6.545$ V/ph. The motor line-to-neutral voltage is $11{,}000 \div \sqrt{3} = 6351$ V/ph, and $V_{\text{drop}} = 6.545 \div 6351 = 0.001$ pu or 0.1%.

Although the voltage drop in this feeder is negligibly small, we cannot go to thinner cable to save cost, since this cable design is determined by the ampacity and not by the voltage drop.

7.8 CABLE ROUTING AND INSTALLATION

Cables are often assembled in raceways that provide support and mechanical protection for the cable conductor and insulation. The most common raceways are conduits (metallic or nonmetallic) and rectangular trays, open from the top or totally enclosed. Cables in a flat bundle can also be supported from a ceiling by hooks without any enclosure. Cables should be adequately supported to withstand the peak mechanical force under the worst-case short circuit current.

Synthetic hooks for suspending cables in ships, off-shore, and chemical plants can improve safety. More than one-half of the shipyards in the Netherlands have introduced such hooks in the last five years. Synthetic hooks are designed to suspend cables, extension cords, wires and hoses away from the floor on decks, stairs and other structures. These are *s*-shape hooks made from nonconductive and nonflammable reinforced polyester. A recent introduction of these hooks under the trade name Cablesafe™ glow in dark to provide reference points and route to safety in case of power loss.

The physical routing of cables may require many bends. The cable bending radius in the installation is limited to certain value that depends on the cable type and diameter. Table 7.15 lists the maximum allowable bend radius in order to limit the bending stress below the damaging level. In general, cables should not be bent in a radius less than 6 to 8 times the cable diameter.

PROBLEMS

Problem 7.1: A cable with Neoprene insulation has nominal ampacity of 250 A in 30°C ambient air. Determine its ampacity for use in the ship's engine room where the ambient air is 50°C.

TABLE 7.15
Cable Bend Radius Limit in Installations as per Various Industry Standards

Industry Standard	Unshielded Cable	Shielded Cable
IEEE-45	6 × diameter for all cables	8 × diameter for all cables
IEC-92	4 × diameter for <1 inch (25 mm) diameter cable 6 × diameter for >1 inch (25 mm) diameter cables	8 × diameter for all cables
Transport Canada	4 × diameter for <1 inch (25 mm) diameter cable 6 × diameter for >1 inch (25 mm) diameter cables	6 × diameter for all cables

Problem 7.2: A 1 kV, 750 kcmil copper cable in metallic tray has dc resistance of 0.016 Ω per 1000 ft at 25°C. Using the tables provided in this chapter, determine its 60 Hz resistance at operating temperature of 110°C. What change would you expect in R_{ac} at 400 Hz?

Problem 7.3: A 3-phase, Y-connected, 4.16 kV_{LL} feeder has $R = 200$ mΩ and $X = 35$ mΩ, both per phase per 1000 feet length. Determine the V_{drop} per 1000 feet per MVA load so that it can be easily reused for various feeder lengths and MVA loadings at (i) 0.85 power factor lagging, and (ii) unity power factor.

Problem 7.4: Select a 3-phase 6.6 kV cable to carry 525 A/phase in single-banked tray and determine its (i) voltage drop per 100 feet at operating temperature of 90°C, (ii) outer sheath diameter, and (iii) weight per 100 feet.

Problem 7.5: Select a 3-phase, 13.8 kV cable to power 25,000 hp propulsion motor that has 96% efficiency and 90% pf lagging. Determine the percentage of voltage drop if the switchboard is 250 ft away from the motor. Place more than one cable in parallel, if needed.

Problem 7.6: Select a 3-phase, 460 V power distribution cable for 100 hp induction motor that has 0.95 efficiency and 0.85 pf lagging. The motor is located on the deck 20 m from the switchboard. Determine the steady-state voltage drop in the cable.

Problem 7.7: A special 3-phase cable is assembled as shown in Figure 7.4 using three one conductor cables, each with the conductor diameter of 1 in. and the insulated outer diameter of 1.5 in. The net copper area of 20 mil thin strands is 75% on 1 in. diameter base. Assuming R_{ac}/R_{dc} ratio of 1.1, determine the cable R and X values per 1000 ft.

QUESTIONS

Question 7.1 Between AWG 30 or AWG 10 conductors, which is the heavier conductor and how many of each can be placed in a 1-inch width?

Question 7.2 In specifying the conductor size, what do the terms MCM and kcmil stand for?

Question 7.3 List two major factors that limit the cable ampacity.

Question 7.4 Give two reasons why heavy conductors are made of thin strands as opposed to solid copper.

Question 7.5 Explain the difference between the skin effect and the proximity effect. How do they increase the conductor ac resistance over the dc resistance?

Question 7.6 The term *effective* is used for the rms value of ac voltage or current, and also for the cable impedance. Explain the term *effective impedance* and its use.

FURTHER READING

Ghandakly, A.A., R.L. Curran, and G.B. Collins, 1990. Ampacity ratings of bundled cables for heavy current applications. IEEE Industry Applications Society Conference Records, Oct.1990 Vol.2, 1334–39.

Neher, J.H. and M.H. McGrath, Calculation of the Temperature Rise and Load Capability of Cable Systems. 2000. IEEE Paper *ieeexplore.ieee.org/iel2/497/3988/00152358.pdf.*

8 Power Distribution

The shipboard power distribution, particularly in a cruise ship, is very much like the land-based power distribution in a small township. The generator switchboard in the ship works like the township substation switchyard, from where the power is distributed to all users via multiple feeders going to various parts of the township.

8.1 TYPICAL DISTRIBUTION SCHEME

Figure 8.1 depicts a typical power distribution system. The township power comes from transmission lines via a step-down substation transformer shown in the dotted box, whereas the shipboard power comes from main generators located in the dotted box. In a large electric cruise ship, the propulsion power gets distributed by dedicated high power lines (not shown in Figure 8.1) at high voltage directly from the generators to the propulsion motors. Figure 8.1 shows four 3-phase feeders running *radially* from the central distribution station switchboard to various loads via circuit breakers. Feeders 1, 2, and 4 are shown in a single-line diagram, whereas Feeder 3 is shown in full details with 4-wire configuration (3-phase lines A, B and C with neutral N). The lateral feeders are tapped off from each phase (line-to-neutral) of the main feeder in a way that 1-phase service loads are about equally distributed on all three phases of the system. For example, the first two top most lateral feeders—one going to the left and one going to the right—take power from phase A, the next two lateral feeders take power from phase B, and the next two from phase C.

The 3-phase loads are, of course, connected to all three phases as shown at the end of Figure 8.1. If the voltage drop at the end of main feeder exceeds the allowable limit with a cable that meets the ampacity requirement but not the voltage drop requirement, 3-phase capacitors may be connected to improve the pf and henceforth reduce the voltage drop as per Equation (3.18). The power loss in capacitors can be minimized by switching them off during a light load when the voltage drop remains within the limit, and switching them back on only during heavy load demand. The capacitors can be remotely or automatically switched on and off in response to the feeder voltage at the far end.

If there are more continuing loads in a large system, the pattern shown in the upper part of Figure 8.1 is repeated after the sectionalizing switch that disconnects the lower part of the loads from the upper part if needed for maintenance or to isolate a fault.

Multiple voltages, where desired, can be obtained from the distribution transformer in two ways shown in Figure 8.2. In the 3-phase system shown in (a), the Y-connected transformer with 208 V line-to-line secondary with neutral can

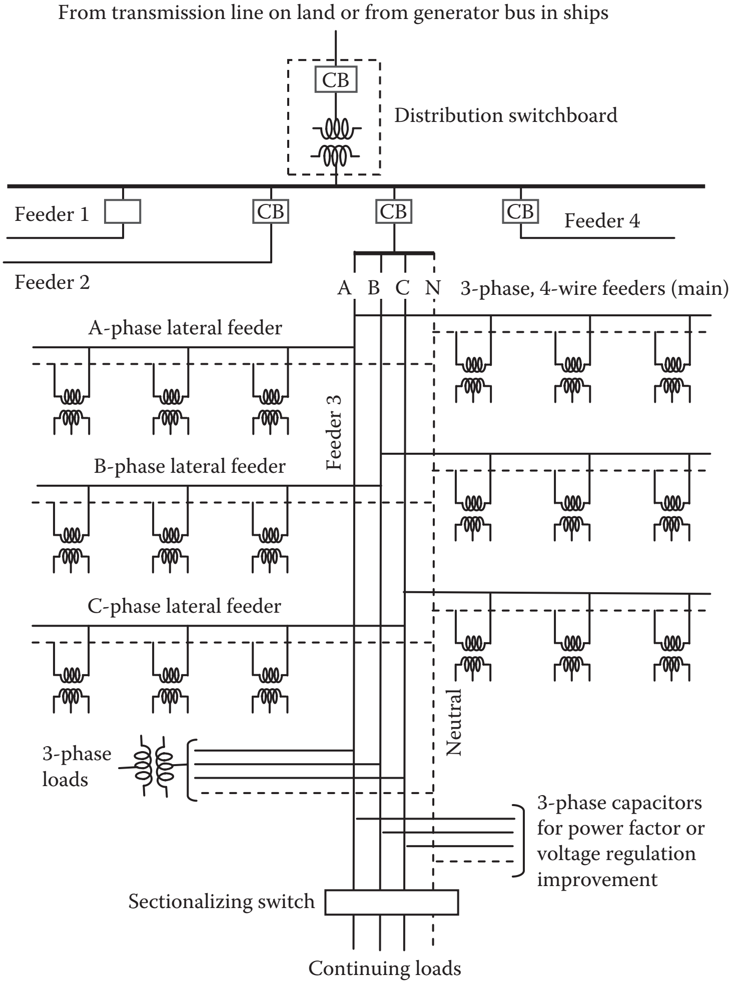
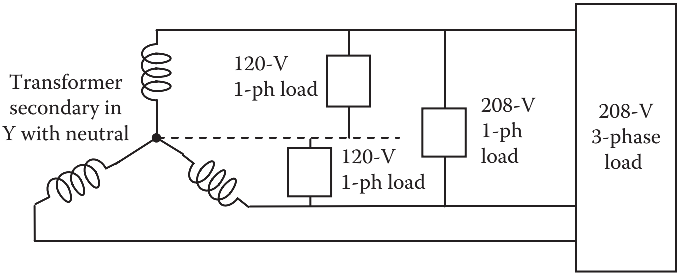


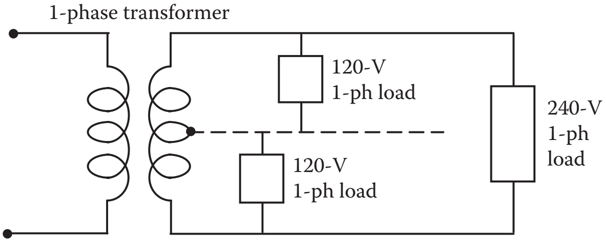
FIGURE 8.1 Typical distribution systems from township substation or ship generator switchboard.

provide three different voltages; 120 V from line to neutral for small 1-phase loads, 208 V for high power 1-phase loads connected from line to line, and 208 V 3-phase for large 3-phase loads. Figure 8.2(b) is typical for servicing small 1-phase service loads on ships or township homes, where the distribution transformer with 240 V center-tapped secondary winding provides 120 V for low power loads (lights and small appliances), and 240 V for high power loads (space heater, cloth dryer, and kitchen stove).

Voltages below 120 V are required in some hands-on operations, such as arc welding and metal cutting. They are obtained by special step-down transformers designed with high internal series impedance, which would cause the transformer output voltage to drop heavily in case of an accidental short circuit.



(a) 208-V 3-phase Y-connected transformer secondary for 120-V 1-phase and 208-V 3-phase loads (primary not shown)



(b) 480-V/240-V 1-phase transformer with center-tapped secondary for 120-V and 240-V 1-phase loads

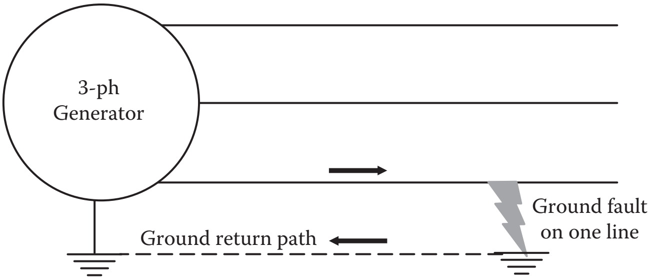
FIGURE 8.2 Multiple-voltage distribution systems for 1-phase and 3-phase loads.

8.2 GROUNDED AND UNGROUNDED SYSTEMS

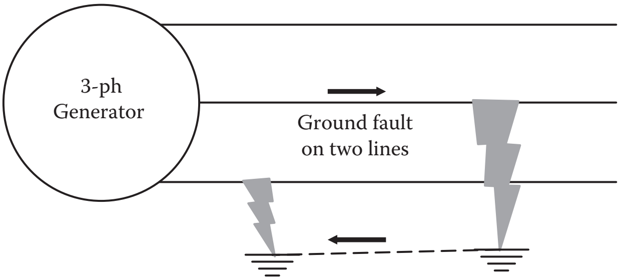
The 3-phase power distribution system can be 3-wire without neutral or 4-wire with neutral that can be either grounded or ungrounded. Shipbuilding standards recommend (but do not require) an ungrounded distribution system coming out of the generator for reliability, but a grounded distribution system for low-voltage service loads for personnel safety. Navy ships generally require an ungrounded system from the generators. As for the neutral, the following three alternatives may be considered:

- Solidly grounded neutral via ground strap of negligible resistance and leakage reactance ($Z_{\text{ground}} \approx 0$).
- High-impedance ground via a resistance and/or reactance inserted in series with the ground strap to control the ground fault current matching with the protection system.
- Ungrounded system where the neutral—although running full length along the phase wires—is not connected to the ground.

In comparing the pros and cons of grounded versus ungrounded systems, we must recognize that about 90% of faults in 3-phase systems are between one of the phases and the ground. In the grounded system shown in Figure 8.3(a), a ground fault on one phase in the first instant results in high fault current that will trip the circuit breaker.



(a) Grounded 3-phase systems results in high current on first line to ground fault



(b) Ungrounded 3-phase system is single-fault tolerant (second line to ground fault causes high current)

FIGURE 8.3 Ground faults in grounded and ungrounded (single fault tolerant) power systems.

However, in the ungrounded system shown in (b), the first ground fault does not complete the ground loop. Only the second ground fault completes the loop and results in high fault current that will trip the circuit breaker. Thus, the ungrounded system is a single-fault tolerant system. It offers an improved availability of power, provided that we detect the first ground fault, locate it, and clear it before the second one occurs.

There is another reason why the ungrounded system is preferred in navy ships. With the generator neutral grounded, the triplen harmonic high-frequency currents due to power electronics loads and zero sequence currents due to unbalance 3-phase loads—in phase in all three phases—must flow in the neutral to form the return path. In some systems, particularly with electric propulsion, the neutral current can exceed the fundamental frequency phase current. This may cause interference to sensitive low-frequency radio and sonar signals, which could compromise the navy operation.

The power system can be grounded or ungrounded. In the grounded system, grounding the neutral of Y-connected system makes the system grounded, whether or not the neutral is running along with the lines. It is also important to understand the difference between grounding the neutral and grounding the power equipment

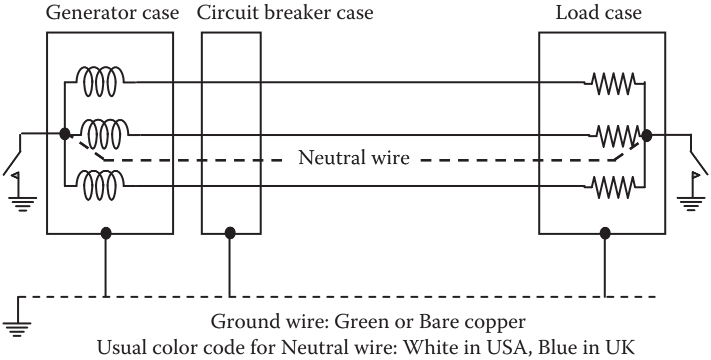


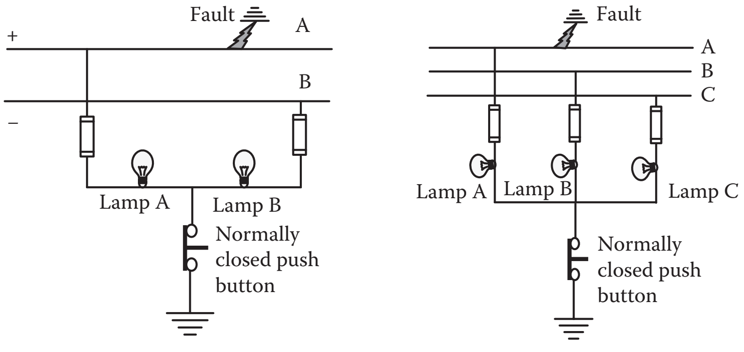
FIGURE 8.4 Neutral and chassis ground wires run separately and serve different purposes.

chassis, as they are not the same. Although both are often erroneously called ground wires, the former is correctly called the *neutral ground* and the latter is called the *chassis ground*. The chassis ground is for human safety, and the power distribution neutral ground is for the equipment safety and desired performance. The system shown in Figure 8.4 becomes a grounded system when the switch in the neutral is closed so that the neutral wire is grounded, otherwise the system remains ungrounded. Regardless whether the system neutral is grounded or ungrounded, the equipment enclosures (chasses) are always grounded for personnel safety. The ground and neutral wires are of different colors and run separately as shown in Figure 8.4. The ground wire is always independent of the neutral wire. It basically keeps the enclosures at zero voltage under all operating conditions, even if an internal live conductor touches the enclosure wall. The general color code of the ground wire is green or bare copper, and the neutral wire color is white in the United States and blue in the United Kingdom and Europe.

The neutral current is zero with balance 3-phase load with no harmonics. However, large power electronics loads typically found in modern power systems draw high triplen-harmonic currents that require the neutral wire to be as heavy as the phase conductor, or even heavier in some cases. The industry standards, therefore, requires that generator neutral wire must be sized to be equal to the largest phase conductor supplying the system with all generators connected in parallel.

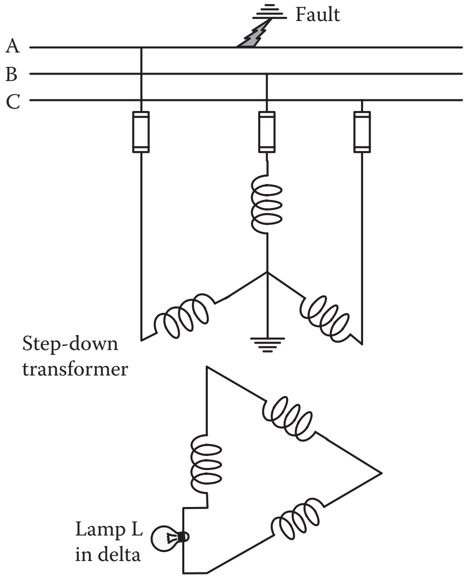
8.3 GROUND FAULT DETECTION SCHEMES

In the ungrounded system, the first ground fault must be detected immediately and corrected before the second ground fault trips the generator. The generally used ground fault detection schemes are shown in Figure 8.5. The scheme (a) is typically used in dc or 1-phase ac system, where a ground fault on line A causes lamp A connected to line A to lose voltage and turn dark, while lamp B remains lit. Pressing the push-button opens the normally closed contactor, breaks the ground loop, and brings lamp A back to light. The engineer can ascertain the ground fault by pushing the button on and off and seeing the lamp A off and on in response. The scheme (b)



(a) 1-phase or DC system

(b) 3 phase low-voltage system



(c) 3-phase high voltage system

FIGURE 8.5 Ground fault detection schemes in LV and HV distribution systems.

is for low voltage systems to which the lamps can be directly connected. The lamp A will turn dark if line A gets grounded, which can again be ascertained by pressing the pushbutton on and off. The scheme (c) is for high voltage systems, where a single lamp is connected inside the Δ of secondary coils of a suitable step-down transformer. When all phases are healthy, the phasor sum of all phase voltages in the Δ is zero, keeping the lamp dark. If phase C on the primary side gets grounded, the phase C of the secondary side also loses the voltage, and the phasor sum of the remaining two voltages in the Δ is not zero. This results in lamp L being lit, indicating a ground fault in one of the lines, although it cannot indicate which line is faulted. Locating the fault in any of these schemes requires trouble-shooting skills mixed with good logic.

8.4 DISTRIBUTION FEEDER VOLTAGE DROP

The distribution system has automatic voltage regulation schemes at one or more control stations, such as the switchboard on ships or substations on land, to maintain constant voltage regardless of the load current. Various feeder cables deliver power from this controlled *sending end* to the user load at the *receiving end*, with the voltage gradually dropping from a maximum value near the sending end to the minimum value at the receiving end. Both the maximum and minimum values must be within the allowable limits for all users in the system. For this reason, the distribution system is sized not only for the required ampacity, but also to stay below the allowable voltage drop limits under steady-state rated current, and during transients such as inrush current during motor starting that generally put the distribution system under stress. If the load current $\tilde{I}$ of lagging pf cos θ flows from the switchboard that is controlled to maintain constant bus voltage $\tilde{V}_s$ at sending end to the load at the receiving end at voltage V_r, and the total series impedance of all transformers and cables between points s and r is $R + jX$ ohms/phase, then the magnitude of voltage drop between points s and r is approximately given by Equation (3.15) for all practical power factors, that is,

$$V_{drop} = I\ \{R\cos\theta + X\sin\theta\} \tag{8.1}$$

All calculations above are done using amperes, volts, and ohms per phase, and taking θ *positive for lagging pf* and negative for leading pf. One can also use R and X values in per unit based on the receiving end kVA and voltage. Then, $V_r = 1.0$ pu, and $I = 1.0$ pu when delivering rated load, or $I = 0.8$ pu when delivering 80% load, etc. Equation (8.1) gives per unit voltage drop if pu values of current, resistance, and reactance were used, or the percent voltage drop if percentage values were used.

8.4.1 VOLTAGE DROP DURING MOTOR STARTING

The power factor of motor starting current is very low, 0.2 to 0.3 lagging. If $R \ll X$, the phasor diagram of $\tilde{V}_r = \tilde{V}_s - \tilde{I} \times (R + j\,X)$ in Figure 8.6 with such poor pf shows that the voltage drop magnitude $(V_s - V_r) = I \times Z$, is approximately in phase with the

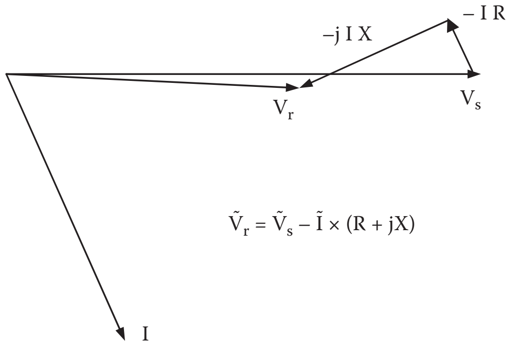


FIGURE 8.6 Feeder voltage phasor diagram under motor starting inrush current.

source voltage. Therefore, V_{drop}, under motor starting inrush current in a cable with impedance Z in ohms, is simply given by

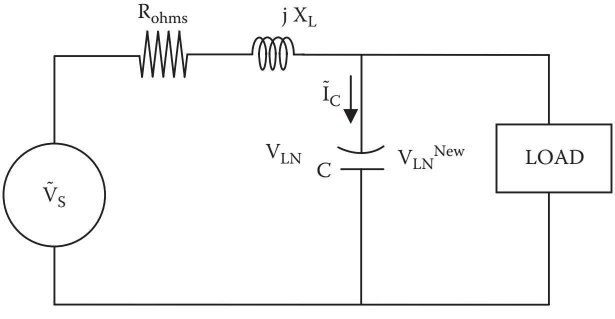
$$V_{drop} = I_{start} \times Z \tag{8.2}$$

Here, the cable impedance Z is not Z_{eff} often found in the vendor's data sheets, but is the actual $Z = \sqrt{(R^2 + X^2)}$. The voltage drop can further be approximated as $I \times X$ in large cables where $X >> R$, or as $I \times R$ in small cables where $R >> X$.

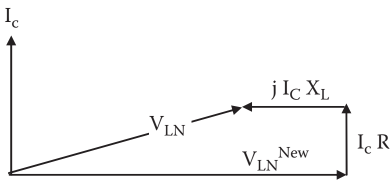
8.4.2 VOLTAGE BOOST BY CAPACITORS

We note that the system voltage drop depends on the load current and also on load power factor. A power factor of 0.85 lagging is common in many systems. Capacitors can improve the power factor close to 1.0, and in turn reduce the voltage drop by eliminating $X\sin\theta$ term from Equation (8.1) and also by reducing the line current. The voltage drop at unity pf is then simply $I \times R$. A leading power factor has a $-\sin\theta$ term that can result in negative voltage drop, resulting in a higher voltage at the receiving end than at the sending end.

The voltage boost by the capacitors placed at the end of feeder having the total system resistance R and leakage reactance X_L (inductive) is shown in Figure 8.7(a), where all values are per phase. The new phase voltage $\tilde{V}_{LN}^{new} = \tilde{V}_{LN} - \tilde{I}_c \times (R + j\,X_L)$ or $\tilde{V}_{LN} = \tilde{V}_{LN}^{new} + \tilde{I}_c \times (R + jX_L)$, the phasor diagram of which is shown in (b). The voltage boost magnitude is then approximately equal to $V_{boost} = \tilde{V}_{LN}^{new} - V_{LN} = I_c \times X_L$.



(a) Capacitor added at feeder end



(b) Load voltage change with added I_C

FIGURE 8.7 Feeder voltage boost with pf improvement capacitors.

The capacitor bank's 1-phase $kVAR_{cap}$ rating = $V_{LN} \times I_c$, giving $I_c = kVAR_{cap \cdot 1ph} \div \mathrm{kV}_{LN}$, which leads to

$$V_{boost} = \frac{kVAR_{cap.1ph}}{kV_{LN}} \cdot X_L \text{ volts/phase} \quad \text{and} \quad \%\, V_{boost} = \frac{V_{boost}}{V_{LN}} \times 100 \tag{8.3}$$

Ignoring the favorable effect of line current reduction with pf improvement, the voltage drop as per Equation (8.1) can be zero for leading pf that gives ($R\cos\theta - X\sin\theta) = 0$ or $R/X = \tan\theta$ (when θ reverses the sign and enters in the leading region). Therefore, the receiving-end voltage will be constant when the leading pf angle $\theta = \tan^{-1}(R/X)$. The R/X ratio for a typical power distribution cable is around 0.2 to 0.3. Using its average value of 0.25, the pf angle $\theta = \tan^{-1}(0.25) = 14°$ leading, or pf = $\cos\theta = 0.97$ leading, gives a flat receiving end voltage, regardless of the load current. Capacitors are typically used for improving poor lagging pf by bringing it close to unity, up to about 0.95 lagging, beyond which they give a diminishing rate of economic return on the capacitor capital cost. Therefore, making the pf 0.97 leading is generally uneconomical for pf improvement.

Example 8.1

A 3-phase, 460 V, 60 Hz, 200 hp induction motor has design efficiency of 90% and pf of 0.85 lagging. The cable reactance from the control center to the motor is 0.015 Ω/ph. If the motor pf were corrected to unity by placing capacitors next to the motor, determine the percentage of voltage rise at the motor terminals.

SOLUTION

$$\text{Motor full load current} = \frac{200 \times 746}{0.90 \times 0.85 \times \sqrt{3} \times 460} = 244.8\ A/ph$$

Phase voltage = 460 ÷ √3 = 265.6 V = 0.2656 kV_{LN}, and $X_L = 0.015$ Ω/ph

Motor power triangle has 3-phase power $P = 200 \times 0.746 = 149.2$ kW, $S = 149.2 \div 0.85 = 175.5$ kVA, and $Q = \sqrt{\{S^2 - P^2\}} = 92.5$ kVAR

For unity pf, $kVAR_{cap/ph} = \frac{1}{3} \times 92.5 = 30.83$, and using Equation (8.3),

$V_{boost} = \frac{30.83}{0.2656} \times 0.015 = 1.74$ V/phase

$\%\, V_{boost} = 1.74 \div 265.6 = 0.00656$ pu or 0.656 %

8.4.3 SYSTEM VOLTAGE DROP ANALYSIS

The voltage drop analysis for the entire system with multiple feeders is carried out using the method we just developed for one feeder. The drop in each feeder cable must be accordingly calculated to ascertain that it meets the percentage limitation imposed by the system requirements. The system voltage drops data are usually compiled in a table. Table 8.1 is an example of such compilation for three feeder cables from three control centers 1, 2, and 3.

TABLE 8.1
Performa Table for Voltage Drop Analysis in Various Distribution Cables

Feeder Line <	From Motor Control Center #1	From HVAC Control Center # 2	From Service Load Center # 3
HP or kW rating			
Cable type			
Cable rated voltage			
Cable rated ampacity			
Full load amperes			
Conductor area			
Cable length			
Resistance *R* ohms			
Reactance X ohms			
Starting current			
Amperes trip			
Voltage drop volts			
Voltage drop percentage			

Since the voltage drop analysis is a major task in designing a power distribution system, we summarize below the important approximation that can be used for quick screening of the alternatives.

If the feeder delivers load current I at a certain pf between two points, A and B, separated by total resistance R and inductive reactance X of all equipment in series, the approximate voltage drop between points A and B is given by

$$V_{\text{drop per amp}} = R \cdot pf + X \cdot \sqrt{1 - pf^2} \quad \text{for all practical power factors}$$

This equation results in the following approximations:

$V_{\text{drop}} = I \times R$ at unity or very near unity pf (such as with pf improvement).
$V_{\text{drop}} = I \times Z$ at pf typical during motor starting inrush current $\{Z = \sqrt{(R^2 + X^2)}\}$.
$V_{\text{drop}} = I \times Z_{eff}$ in cables with current at 0.85 pf lagging (Z_{eff} from cable vendor data sheet).

Remember that these are good approximations for quick designs. The exact voltage drop calculations that may be required to show the compliance with contractual standards or in litigation cases come from Equation (3.14) in Section 3.7.

Example 8.2

A 3-phase, 100 hp, 460 V, letter code G induction motor is started directly from 480 V lines. The internal (Thevenin) source impedance and the cable impedance add to a total of $0.01 + j\,0.02\ \Omega$/phase. If the starting pf of the motor is 0.35 lagging, determine the exact and approximate voltage drops in percentage of the line voltage.

SOLUTION

Letter code G motor draws 5.6 to 6.3 kVA/hp on starting as per Table 5.3. Therefore, the worst-case starting current is $6.3 \times 100 \times 1000 \div (\sqrt{3} \times 460) = 791$ A at $\theta = \cos^{-1}0.35 = 69.5°$ lagging. The phase voltage at the source is $480 \div \sqrt{3} = 277.1$ V.

Using exact Equation (3.13), the voltage at motor terminals is

$277.1\angle 0° - 791\angle -69.5° \times (0.01 + j\,0.02) = 259.4\angle 0.2°$ V/ph

Voltage drop magnitude = (277.1 – 259.4) = 17.7 V/ph

$\% V_{drop} = (17.7 \div 277.1) \times 100 = 6.39$ %.

By approximate Equation (3.15), $V_{drop} = 791\ (0.01 \times 0.35 + 0.02 \times 0.937) = 17.6$ V/ph, a close match with the exact value of 17.7 V.

By approximate Equation (8.2), $V_{drop} = I \times Z_{Total} = 791 \times \sqrt{(0.01^2 + 0.02^2)} = 17.69$ V, which is even closer to 17.7 V derived from the exact formula.

This indicates that the approximate Equation (3.15), which is common for many voltage drop calculations, gives a fairly accurate result for a wide range of power factors.

8.5 BUS BARS ELECTRICAL PARAMETERS

Heavy power in tens of megawatts requires high current that is often distributed by bus bars of rectangular cross section shown in Figure 8.8. For two bus bars with facing flats as in (a), the *R* and *L* parameters per meter run of two bars combined (1 m for lead plus 1 m for return) are given below in SI units:

$$\text{For lead and return bars combined, } R = \frac{2\rho}{a \cdot b}\ \Omega/\text{m} \tag{8.4}$$

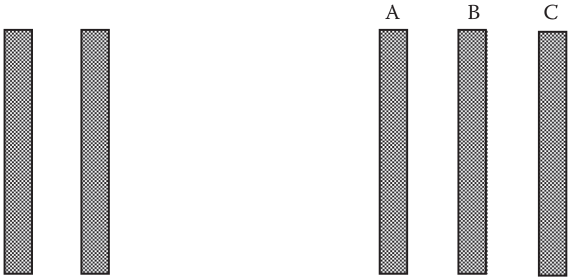
$$\text{Leakage inductance between bars, } L = \frac{4\pi \times 10^{-7}}{b}\left(d + \frac{2a}{3}\right)\ \text{H/m} \tag{8.5}$$

where ρ = conductor resistivity = 0.01724 μ Ω.m for copper and 0.0282 μΩ.m for aluminum both at 20°C, a = bar thickness, b = bar width, d = separation between bar faces. These equations are valid for bars thinner than the skin depth at low operating frequency (50 or 60 Hz). For higher frequency, *R* will be higher and L will be slightly lower due to skin and proximity effects.

Copper bus bars with facing flats, 10 cm wide × 5 mm thick, separated by 1 cm insulation would typically have, $R = 69$ μΩ/m at 20°C and $L = 0.1675$ μH/m.

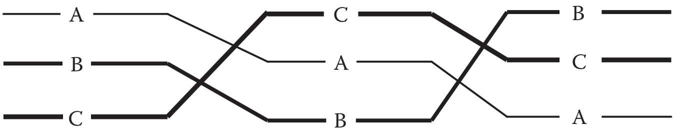
Long 3-phase bus bars shown in Figure 8.8(b) may be used to carry high current from the source to a distant load, for example, high-power propulsion motors or weapon loads. However, the unequal spacing between phase bars A-B, B-C, and C-A in this configuration results in unbalanced leakage reactance per phase, causing unbalanced voltage drops and unbalanced voltages at the load end. Such imbalances can be eliminated by transposing the bus bars as shown in Figure 8.8(c) such that each bar occupies all three possible positions relative to each other, with different positions in each third of the distribution length.

Sometimes, bus bars in HV equipments are encapsulated in epoxy and assembled with facing edges (Figure 8.9) to withstand high voltages. Such configuration results



(a) Rectangular bus bars for 1-phase ac or dc with facing flats

(b) Rectangular bus bars for 3-phase power



(c) Transposition of 3-phase bus bars for balanced reactance in each phase

FIGURE 8.8 Rectangular bus bars: 1-phase and 3-phase with transposition at one-third and two-thirds length.

FIGURE 8.9 High voltage bus bars in switchgear through porcelain bus ducts: cast epoxy insulated with joints covered by removable boots (with permission form Myers Power Products, Inc.).

in high reactance per meter, which generally does not matter due to short distance within the enclosure.

8.6 HIGH-FREQUENCY DISTRIBUTION

High-frequency distribution systems, such as 400 Hz in aircraft and 20 kHz used in some U.S. Navy systems, are more difficult to design for low voltage drop. Since the leakage reactance $X = 2\pi f \times L$ increases with frequency, the cable has high reactance at high frequency. The R also increases at high frequency due to skin and the proximity effects. The result is a high voltage drop per ampere. For a high power bus, copper tubes with wall thickness less than the skin depth at operating frequency are often used to reduce ac resistance. Copper tube bus bars have added advantages of possible water-cooling and structural integrity, compared to slender solid rectangular bars. However, around conductor results in high leakage reactance, which increases the voltage drop per ampere.

In a design where a low reactance is desired, Figure 8.8(a) bars can be made wider and thinner for the same cross section. For a very low reactance in a very high frequency distribution system, each phase bus bar can be made in a virtually concentric configuration shown in Figure 8.10. The central bar carries the lead current to the load, and the two outer thinner bars carry the return current. The inductance in this case gets reduced by approximately half compared to that with two parallel bars. In SI units, it is

$$L = \frac{1}{2}\frac{4\pi \times 10^{-7}}{b}\left(d + \frac{2a}{3}\right) \text{ H/m} \tag{8.6}$$

where a = lead (center) bar thickness, b = bar width, and d = separation between center bar and one of the outer bar faces (not center-to-center).

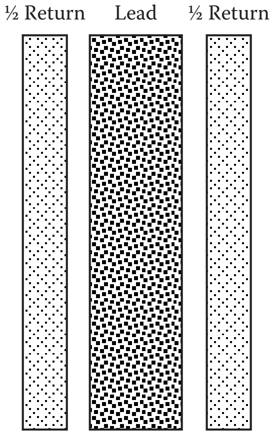


FIGURE 8.10 Virtually concentric low-reactance bus bars for high frequency power distribution.

Example 8.3

Two rectangular copper bus bars, each ¼ in. thick × 4 in. wide with facing flats are separated by ½ in. space between the flats. Determine the per foot run of bus bars (i) resistance and leakage inductance at 75°C, and (ii) mechanical force under fault current of 50 kA peak, assuming the bar shape factor $K = 0.7$.

SOLUTION

(i) Conductor cross section area = ¼ × 4 = 1 in^2, which has the equivalent diameter = √(4 × 1 ÷ π) = 1.128 inch or $1128^2 = 1.2732 \times 10^6$ circular mils = 1273.2 kcmil.

Using Equation (7.3) in British units, $R = 10.372 \times 2 \div (1.2732 \times 10^6) = 16.29$ μΩ/ft at 20°C.

In SI units, bus thickness a = ¼ inch = 6.35 mm = 0.00635 m, width b = 4 in = 101.6 mm = 0.1016 m, separation d = ½ in = 12.7 mm = 0.0127 m, and length L = 1 ft = 304.8 mm = 0.3048 m. Then, using Equation (8.4), we have

$$R = \frac{2 \times 0.01724 \times 0.3048}{0.00635 \times 0.1016} = 16.29\ \mu\Omega/\text{ft at } 20°\text{C (same as above)}.$$

At 75°C, Equation (7.4) gives $R = 16.29\,[1 + 0.0039\,(75 - 20)] = 16.29 \times 1.2145 = 19.78$ μΩ/ft

Using Equation (8.5), flat bus bar inductance—independent of temperature—is given by

$$L = \frac{4\pi \cdot 10^{-7}}{0.1016}\left(0.0127 + \frac{2 \times 0.00635}{3}\right) = 0.21 \times 10^{-6} = 0.21\ \mu\text{H/m}$$

Or, $L = 0.21 \times 0.3048 = 0.0638$ μH/ft.

Using Equation (3.3), the mechanical force between the bars

$$F = \frac{5.4 \times 0.7 \times 50{,}000 \times (-50{,}000)}{0.5} \times 10^{-7} = -1890\ \text{lbf/ft}$$

The negative sign indicates repulsive force. The bus bar must be braced to support this force without exceeding the allowable bending stress or deflection between supports to avoid mechanical damage following a short circuit fault.

Very high frequency power is often distributed with *Litz cable*, which is made from numerous thin strands of magnet wire. Thin conductor with hard polyimide type film insulation often used for winding coils for electromagnets is known as the *magnet wire*. The strands in a Litz cable are continuously transposed so as to occupy every possible position in the cable over one transposition pitch. This way, the current distribution in the cable is forced to be uniform regardless of the cable diameter. An arbitrarily large diameter Litz cable can have R_{ac}/R_{dc} ratio near 1.0 even at very high frequency if the individual strand diameter is thinner than the skin depth. The leakage reactance is another matter. Round cable, conventional or Litz, would have high leakage reactance due to high frequency, since $X = 2\pi fL$. The leakage inductance can be reduced by making the Litz cable with rectangular straps with facing flats.

FIGURE 8.11 Virtually concentric Litz cable for 20 kHz power distribution (from M.R. Patel, NASA Report CR-175071, 1986).

The leakage inductance can further be reduced by using 3-strap design as was shown in Figure 8.10. A virtually concentric Litz cable (Figure 8.11) for 20 kHz high power distribution in Space Station Freedom was designed, manufactured, and tested by Induction General, Inc. for NASA.

8.7 SWITCHBOARD AND SWITCHGEAR

The switchboard (SWBD) or switchgear (SWGR) is a general term that includes the assembly of circuit breakers, disconnect switches, fuses, relays, meters, instruments, controllers, and other equipment associated with the generation and distribution of electrical power. The examples of switchboard are the generator power panels, motor control center, or any other central power distribution center. One such switchboard's exterior view is shown in Figure 8.12. The switchboard in a large ship can be extensive to fill a large room as shown in Figure 8.13. The following terms are often used in designing and discussing the switchboard.

FLA, full load ampere: The nameplate rated load current of the motor or any other load.

CAR, cable ampacity requirement: It determines the cable size in circular mils of conductor that is required to carry the rated load current with some margin without overheating.

CM, circular mills: Square of the diameter of conductor in mils (1/1000th of an inch). It represents the conductor area available to carry the current.

AT, ampere trip: The trip current setting of circuit breaker that protects the cable by tripping out.

FIGURE 8.12 A 15 kV generator switchgear in self-contained enclosure. (With permission form Myers Power Products, Inc.)

AF, ampere frame: The peak fault-current withstand capability of the circuit breaker without mechanical or thermal damage.

LVR, low voltage release: It automatically reconnects the circuit when power is restored after a temporary loss. This is provided to auxiliary equipment critical to ship propulsion equipment, where automatic restarting would not pose hazard.

LVP, low voltage protection: It will permanently shut down the circuit if power is lost, and will not start again by itself when the power is restored. All large motors drawing very high starting current for long time use LVP.

Red-V, reduced-voltage starting. It reduces the power drawn from generator while starting a large load such as a heavy motor.

FLS, full-voltage starting: It connects small loads directly to the line at full voltage.

8.7.1 AUTOMATIC BUS TRANSFER

The automatic bus transfer (ABT) uses various types of circuit breakers and relays for automatic transfer of power feeder from the main source to an alternate source

FIGURE 8.13 Typical main switchboard on large cargo ship. (From Raul Osigian, U.S. Merchant Marine Academy.)

when the main source detects a low or zero line voltage (Figure 8.14). All ABTs have a local manual transfer capability also, overriding the automatic operation. Some ABTs may have remote bus transfer (RBT) capability that can be operated from a remote location manually (but not automatically).

8.7.2 DISCONNECT SWITCH

It is used to disconnect the circuit only under zero current after the circuit current is removed. It ensures safety during maintenance work by manually disconnecting the power source from the line and keeping it locked open. It is not a circuit breaker;

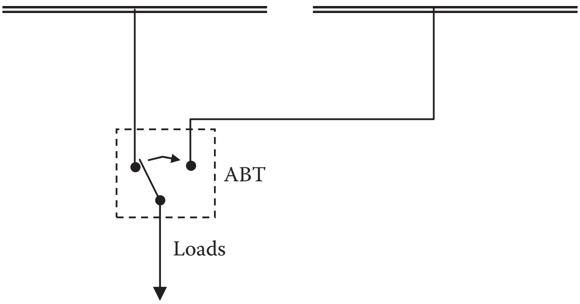


FIGURE 8.14 Automatic bus transfer switch.

it cannot break the load current. If the disconnect switch is opened while carrying load current, it will surely spark heavily, may catch on fire, or even explode, posing a safety risk.

PROBLEMS

Problem 8.1: A 3-phase, 460 V, 60 Hz, 500 hp induction motor has a design efficiency of 92% and pf of 0.80 lagging. The cable reactance from the control center to the motor is 0.015 Ω/ph. If the motor pf were corrected to unity by placing capacitors next to the motor, determine the percentage voltage rise at the motor terminals.

Problem 8.2: A 3-phase, 156.5 hp, 480 V, letter code L induction motor is started directly from 480 V lines. The source impedance and the cable impedance add to a total of $0.01 + j\,0.02$ Ω/phase. If the starting pf of the motor is 0.25 lagging, determine the exact and approximate voltage drops in percentage of the line voltage.

Problem 8.3: Two rectangular copper bus bars, each 8 mm thick × 100 mm wide with facing flats are separated by 10 mm space between the flats. Determine per meter run of the bus bars (i) resistance and leakage inductance at 75°C, and (ii) mechanical force under fault current of 35 kA peak, assuming the bar shape factor of 0.75.

Problem 8.4: Determine the percentage of voltage regulation of a 100 kVA, 480/120 V distribution system having total impedance $Z_{Total} = (2 + j\,9)\%$ delivering 90% load at 0.8 pf lagging. Make your calculations using per unit values.

Problem 8.5: A 60 Hz 1-phase cable having inductance of 0.3 μH/m and resistance of 2.5 mΩ/m is delivering 100 kW at 2400 V and 0.85 pf lagging. The load is 40 m away from the generator. Determine the generator terminal voltage.

Problem 8.6: A new 1000 kW, Δ-connected, 440 V, 60 Hz, induction motor is proposed on your ship. On a direct line start, it would draw 10 × rated current and cause transient voltage sag of 15%, which is unacceptable. Determine the line voltage sag with a Y-Δ starter.

Problem 8.7: A power distribution center serves 1-phase load via cable and 50 kVA 277/120 V step down transformer. Based on the transformer kVA rating, the cable impedance is $2 + j\,1\%$ and the transformer impedance is $1 + j\,5\%$. The equipment is drawing 75% load at 86.6% power factor lagging. Using pu values, determine the percentage voltage regulation from the distribution center to the load point.

Problem 8.8: In a power distribution system from the generator to a pump, various equipment efficiencies are: generator 95%, transformer 97%, cable 99%, motor 94%, and pump 93%, all in series. If the pump output is 1000 hp, determine the contribution of this pump on the prime mover's output in kW.

Problem 8.9: In a cruise ship that is 1000 ft long and 300 ft wide, a 3-phase, 4160 V main feeder from the generator switchboard distributes 1200 kVA hotel load to 12 sectors fed by 1-phase, 100 kVA, 120 V lateral feeders as shown in Figure P8.9. Each junction of the main feeder and the lateral feeder has a 1-phase, 100 kVA, step-down transformer for 120 V cabin loads. Determine: (i) the size of the main and lateral feeders, (ii) voltage available to the last lateral feeder at point E, and (iii) voltage available to the last cabin at point EE. Assume (i) 0.85 pf lagging load, and (ii) both feeders have uniform size from the start to the end of the line (versus it can have tapered cross section as the load current decreases from maximum at the start to the minimum at the end).

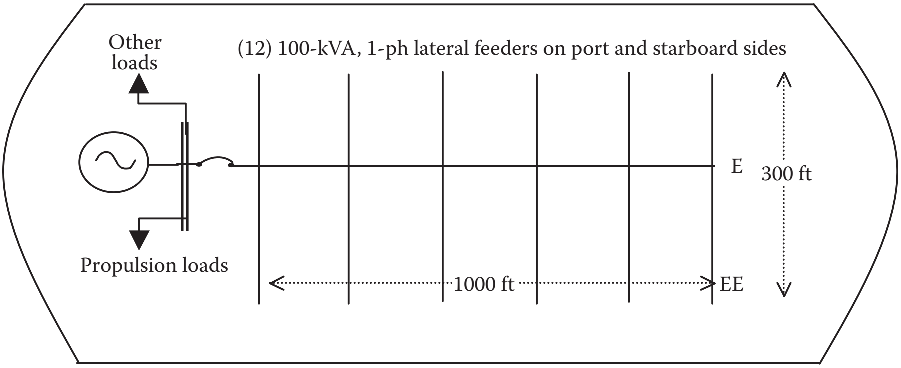


FIGURE P8.9

Problem 8.10: A 20 kHz high-power cable has 10 mm conductor diameter. Determine its approximate R_{ac}/R_{dc} ratio if it is (i) made of one solid conductor, (ii) made of numerous thin conductor strands that are not insulated from each other, and (iii) made of Litz cable. Use the concept of skin depth in your calculations, and explain the reason why Litz cable gives the best results at such high frequency.

QUESTIONS

Question 8.1 How are two voltages, 120 V and 240 V, obtained in residential power distribution from single secondary coil of a 1-phase step-down transformer?

Question 8.2 How many voltages can be obtained from a 3-phase Y-connected transformer?

Question 8.3 Explain the advantage of an ungrounded system that is usually preferred in ships.

Question 8.4 Explain the function of neutral ground and chassis ground.

Question 8.5 On detecting a ground fault on ship's ungrounded system, how would you make sure that it is indeed a ground fault and not an anomaly?

Question 8.6 Which bus bar configuration—facing flats or facing edges—results in lower voltage drop per ampere per meter length?

Question 8.7 How do capacitors improve the voltage regulation (i.e., reduce the voltage drop)?

Question 8.8 Sketch the magnetic flux pattern of two parallel bus bars and virtually concentric bus bars for 1-phase distribution system.

Question 8.9 In dealing with high power at high frequency (e.g., 10 MW at 10 kHz), what type of the distribution line (round hollow tubes or rectangular bus bars) would you consider to keep the voltage drop low, and why?

Question 8.10 Identify the difference between the circuit breaker and the disconnect switch.

FURTHER READING

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9 Fault Current Analysis

The short circuit (called *fault* in this chapter) is defined as a live conductor touching another live conductor or ground in a grounded system. It can happen accidentally or when the insulation in-between breaks down. The result is an abnormally high current that can cause mechanical and thermal damage to all equipments in series leading to the fault. The magnitude of prospective fault current at a given location in the system determines the current interrupting capability of the fuse or circuit breaker that must be placed between the power source and the fault location in order to protect the system from a potential damage. For this reason, the power system engineer invests a great deal of efforts to analyze the worst-case fault current at key locations in the entire system.

9.1 TYPES AND FREQUENCY OF FAULTS

The types of fault that can possibly occur in a 3-phase, 4-wire distribution system with neutral grounded are, in the order of the frequency of occurrence:

L-G One line shorting to the ground (not a fault in an ungrounded system)
L-L Two lines shorting together but not to the ground
L-L-G Two lines shorting together and to the ground
L-L-L All three lines shorting together but not to ground (3-phase symmetrical fault)
L-L-L-G All three lines shorting together and to ground (3-phase symmetrical fault to ground)

The L-L-L and L-L-L-G faults are called symmetrical faults, because they involve all three lines symmetrically, resulting in fault currents that are balanced symmetrical 3-phase currents. The other three types are called unsymmetrical faults, where the fault currents are not symmetrical in all three lines. The frequency of occurrence of electrical faults on ships is not readily available, but their approximate percentages in land-base systems are shown in Figure 9.1. About 70% of faults in 3-phase systems start as single-line-to-ground (L-G) fault. However, the subsequent heat generation breaks down the insulation between other lines in the cable as well, soon leading to 3-phase symmetrical L-L-L-G fault. Similarly, faults that start as L-L (15%) and L-L-G (4%) also soon lead to a symmetrical fault. For this reason, the fault current analysis presented below is focused on the 3-phase symmetrical fault.

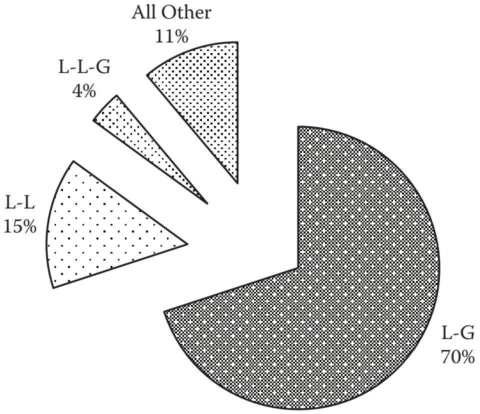


FIGURE 9.1 Types and frequency of occurrence of short circuit faults in 3-phase systems.

9.2 FAULT ANALYSIS MODEL

The system fault analysis model is developed by connecting the electrical models of all series components involved in the fault loop in a 1-line diagram. Each component is represented by its equivalent series resistance *R* and leakage reactance *X*. Typically, small *R* and *X* values of components like circuit breaker, fuse, relay contacts, etc., are ignored to simplify the analysis. Since the analysis is carried out on a per phase basis in Y-connection (actual or equivalent), Δ-connected impedance values (if any) are divided by 3 to convert into equivalent Y-connected values. It is much simpler to carry out the fault current analysis in percentages or with the pu system, since the generator and transformer percent *Z* are usually given on the equipment nameplate. The cable impedance, generally given in ohms per 1000 ft on the manufacturer's data sheet, is converted to pu *Z* by dividing it by the base impedance per phase, which is

$$Z_{\text{base}} = \frac{V_{\text{base}}}{I_{\text{base}}} = \frac{V_{\text{base}}}{V_{\text{base}}}\frac{V_{\text{base}}}{I_{\text{base}}} = \frac{V_{\text{base}}^2}{VA_{\text{base}}}\ \Omega/\text{phase} \tag{9.1}$$

Using kVA or MVA per phase and line-to-neutral voltage V_{LN} or kV_{LN}, we have

$$Z_{\text{base}} = \frac{V_{LN.\text{base}}^2}{1000 \times kVA_{\text{1ph.base}}} = \frac{kV_{LN.\text{base}}^2}{MVA_{\text{1ph.base}}}\ \Omega/\text{phase} \tag{9.2}$$

In a 3-phase system, using line-to-line voltage kV_{LL}, we have

$$I_{\text{base}} = \frac{kVA_{\text{3ph.base}}}{\sqrt{3}\ kV_{LL.\text{base}}}\ \text{A/phase} \tag{9.3}$$

$$Z_{\text{base}} = \frac{1000\ kV_{LL.\text{base}}^2}{kVA_{\text{3ph.base}}} = \frac{kV_{LL.\text{base}}^2}{MVA_{\text{3ph.base}}}\ \Omega/\text{phase} \tag{9.4}$$

For fault current analysis of the entire system involving many components in series, there can be only one base kVA and one base voltage, which is generally taken as the rated voltage and rated kVA of the first upstream transformer feeding the fault from the source side. All component impedance values either in ohms or in percentages are converted into this common base selected for the system study. If the system impedances are all given in ohms, the conversion to a common voltage base (called the *impedance transformation* in Section 6.11) is done in the voltage square ratio, that is,

$$\text{In ohm values, } \frac{Z_{\text{base}2}}{Z_{\text{base}1}} = \left(\frac{V_{\text{base}2}}{V_{\text{base}1}}\right)^2 \ \Omega/\text{phase} \tag{9.5}$$

With lower kVA base, the base current is lower and the base impedance is higher, hence percent Z gets lower in the same proportion as the kVA base. Therefore, the percent Z value changes linearly with the kVA base. If the percent Z value on Base 1 is known, then its equivalent value on Base 2 is

$$\frac{\%Z_{\text{base}2}}{\%Z_{\text{base}1}} = \left(\frac{kVA_{\text{base}2}}{kVA_{\text{base}1}}\right) \tag{9.6}$$

Equations (9.4) and (9.5) can be used with kVA_{1ph} or kVA_{3ph} and with V_{LN} or V_{LL} values, since the conversion from one system to another are expressed in the ratios.

Example 9.1

Determine the base current and base impedance in a 3-phase power system on the base of 10 $\text{MVA}_{3\text{-}ph}$ and 6.6 kV_{LL}, first using 1-phase formulas, and then using 3-phase formulas, recognizing that both 1-phase and 3-phase formulas give the base current and base impedance values *per phase* (there is no such thing as 3-phase amperes and ohms).

SOLUTION

From the fundamental definitions in Equation (9.1) on per phase basis,

$kVA_{\text{base}} = \frac{1}{3}\ 10{,}000 = 3333.3$ kVA and $\text{kV}_{\text{base}} = 6.6 \div \sqrt{3} = 3.81$ kV

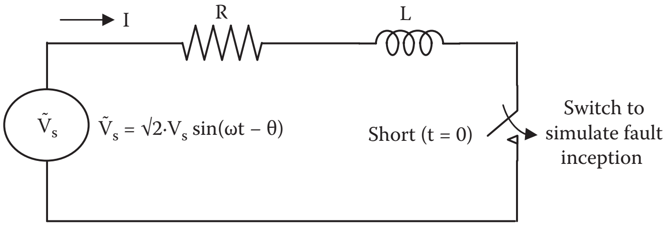
$\therefore I_{\text{base}} = 3333.3 \div 3.81 = 874.8$ A, and $Z_{\text{base}} = V_{\text{base}} \div I_{\text{base}} = 3810 \div 874.8 = 4.355\ \Omega/\text{ph}$

Using 1-phase Equation (9.2), $Z_{\text{base}} = kV_{LN}^2 \div \text{MVA}_{1\text{ph}} = 3.81^2 \div 3.3333 = 4.355\ \Omega/\text{ph}$

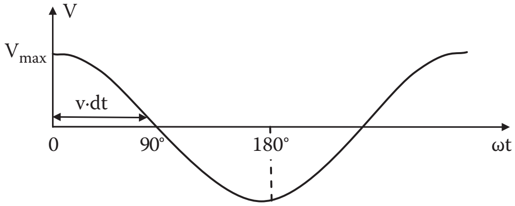
Using 3-phase Equation (9.3), $Z_{\text{base}} = kV_{LL}^2 \div \text{MVA}_{3\text{ph}} = 6.6^2 \div 10 = 4.355\ \Omega/\text{ph}$

9.3 ASYMMETRICAL FAULT TRANSIENT

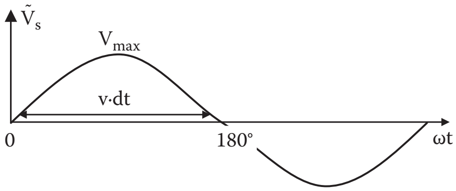
Consider a system model from the source to the fault location that has been reduced to a total equivalent R-L circuit shown in Figure 9.2(a). The initial load current, being small compared to the prospective fault current, can be ignored for simple but fairly accurate analysis. The fault on the system is applied by shorting the switch on the



(a) R-L circuit of fault loop shorted at t = 0



(b) Fault incepts at $\theta = 90°$ when $\tilde{V}_s = V_{max}$



(c) Fault incepts at $\theta = 0°$ when $\tilde{V}_s = 0$ (worst case)

FIGURE 9.2 Transient current in R-L circuit after sinusoidal voltage switched on.

instant when the sinusoidal source voltage is $\sqrt{2}\, V_s \sin(\omega t + \theta)$, where t = time after the inception of fault and θ = voltage phase angle on the instant of closing the switch (fault inception angle on the sinusoidal cycle). The value of $\theta = 90°$ implies that the fault occurs precisely when the circuit voltage is passing through its natural peak as in (b), and $\theta = 0°$ implies that the fault occurs precisely when the sinusoidal circuit voltage is passing through its natural zero as in (c).

9.3.1 SIMPLE PHYSICAL EXPLANATION

In practical power systems, the leakage inductance L dominates the resistance R. A simple physical insight to the fault current can be developed by ignoring R and writing the basic law of physics that gives the voltage drop in the inductor as

$$v(t) = L\, \mathrm{d}i/\mathrm{d}t \text{ or } i = \frac{1}{L}\int v \cdot dt \qquad (9.7)$$

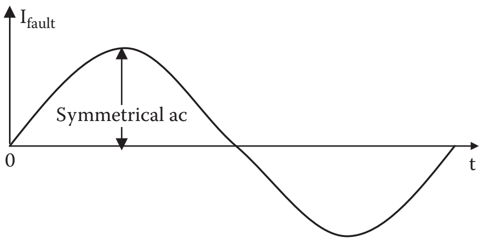
Equation (9.7) expresses the circuit current as the integral of voltage with time, that is, the area under the v-t curve. If the fault incepts when the sinusoid voltage is passing through its peak (Figure 9.2b), then it stays positive for only a quarter cycle. The current in this case keeps rising only during 0° to 90°, reaching a peak value of $I_{1\text{peak}} = \frac{1}{\omega L}\int_0^{90} v \cdot d(\omega t)$, and then decreases to zero before moving to the equal peak on the negative side. On the other hand, if the fault incepts when the sinusoid voltage is passing through its natural zero (Figure 9.2c), then it stays positive for a half cycle (twice as long as before), pumping twice the energy in the circuit. The current in this case keeps rising from 0° to 180°, reaching the peak value of $I_{2\text{peak}} = \frac{1}{\omega L}\int_0^{180} v \cdot d(\omega t)$, and then decreasing to zero, but not moving to the negative side at all. Obviously, $I_{2\text{peak}} = 2 \times I_{1\text{peak}}$, that is, the fault current is pushed twice as high as shown in Figure 9.3(b), which can be seen as the symmetrical ac superimposed on a dc of initial value equal to the peak value of ac. We have seen similar analysis for the magnetizing inrush current in transformer in Section 6.7. It is analogous to the inertial energy stored in a large moving mass being dumped on a spring, or the *water-hammer* building up the pressure in water pipe on the instant of suddenly shutting off the valve.

9.3.2 RIGOROUS MATHEMATICAL ANALYSIS

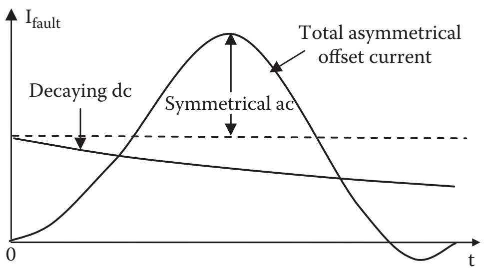
For rigorous transient analysis of the *R-L* circuit of Figure 9.2(a), we apply Kirchhoff's voltage law to write

$$\sqrt{2}\, V_s \sin(\omega t + \theta) = Ri + L\frac{di}{dt} \quad \text{for } t > 0 \tag{9.8}$$

with the initial conditions $i(0) = 0$ and $v(0) = \sqrt{2}\, V_s \sin\theta$ at $t = 0$ (9.9)



(a) Symmetrical fault current if fault incepts at θ = 90°



(b) Asymmetrical fault current if fault incepts at θ = 0° (worst case)

FIGURE 9.3 Transient current with fault inception angle θ = 90° and θ = 0° (worst case).

The particular solution to Equation (9.8) with the initial condition (9.9) is the following transient fault current,

$$i(t) = \sqrt{2} I_{\text{rms}} \left(\sin(\omega t + \theta - \theta_z) - \sin(\theta - \theta_z) e^{-\frac{R}{X}\omega t} \right) \tag{9.10}$$

where R = total resistance in the fault circuit, $X = \omega L$ = total reactance, $\theta_Z = \tan^{-1}(X/R)$ = impedance angle, and I_{rms} = rms value of the ac component superimposed on the exponentially decaying transient term. The fault current, therefore, is a complex function of the switch closing (fault inception) angle θ and the fault circuit time constant $\tau = L / R = X / (\omega R)$.

The fault current given by Equation (9.10) has the maximum peak value when θ = 0, that is, if the fault incepts when the source voltage is passing through its natural zero during the sinusoidal cycle. On the other hand, the fault current peak is the least when θ = 90°, that is, if the fault incepts when the source voltage is passing through its natural sinusoidal peak. Most faults incept when θ = 90°, that is, when the system voltage is at its sinusoidal peak, because the insulation is more likely to break down at the peak voltage rather than at zero voltage. Fortunately, nature is on our side here.

The worst-case (θ = 0°) transient current is closely (although not exactly) given by

$$i(t) = \sqrt{2} I_{\text{rms}} \left(e^{-\frac{R}{X}\omega t} - \cos \omega t \right) \tag{9.11}$$

9.4 FAULT CURRENT OFFSET FACTOR

The plots of $i(t)$ in Figure 9.3 with θ = 0 and with θ = 90° indicate that the ratio of their respective first peaks, $K_{\text{peaks}} = \frac{\textit{First peak value if } \theta = 0^\circ}{\textit{First peak value if } \theta = 90^\circ}$, can approach 2.0 when $R = 0$. This ratio is known as the fault current *asymmetry factor or offset factor or peak factor*, which is 2.0 when θ = 0 (worst-case fault inception time and with $R = 0$). By plotting it for various X/R ratios, it can be seen that K_{peaks} factor is 1.0 for $X/R = 0$ (that is, $X = 0$ or purely resistive circuit) and 2.0 for $X/R = \infty$ ratio (that is, $R = 0$ or purely inductive circuit). For X/R ratio in between, the K_{peaks} value is between 1.0 and 2.0.

It is customary to calculate the rms value of short circuit current for sizing the circuit breaker, which trips after the transient has somewhat subsided. However, the mechanical structure of the equipments and bus bars must be designed for the mechanical force at the first peak of the worst-case fault current. To connect these two currents of the design engineer's interest, the factor K is defined as

$$\therefore K = \frac{\textit{First asymmtrical peak of worst-case fault current}}{\textit{Symmetrical rms value of fault current}} \tag{9.12}$$

The values of K for various X/R ratios are listed in Table 9.1. It shows that a circuit with negligible reactance having $X/R = 0$ gives no asymmetry in the fault current,

TABLE 9.1
First fully offset asymmetrical peak factor *K*

System *X*/*R* ratio	Offset factor K^a	Time of first peak in 60 Hz system (msec)
0	$1 \times \sqrt{2} = 1.414$ (zero asymmetry)	¼ (1/60) sec 4.167 ms
1	1.512	6.1
2	1.756	6.8
3	1.950	7.1
5	2.19	7.5
7	2.33	7.7
10	2.456	7.9
20	2.626	8.1
50	2.743	8.2
100	2.785	8.3
Infinity	$2 \times \sqrt{2} = 2.828$ (full asymmetry)	½ (1/60) 8.333 ms

[a] $K = \dfrac{\textit{First fully offset assymetrical peak value}}{\textit{Symmetrical AC rms value}}$.

and the first peak is just the usual √2 of the sine wave, which occurs at a quarter of the 60 Hz period (i.e., a quarter of 16. 667 ms). On the other hand, in a highly inductive circuit having *X*/*R* > 100, the first peak is 2√2, which is twice the usual sine wave peak and occurs at half of the 60 Hz period (i.e., half of 16.667 ms). This means that with a large *X*/*R* ratio, the initial dc component is as great as the first symmetrical sinusoidal ac peak.

The asymmetrical fault current shown in Figure 9.3 can be characterized by (i) the sum of steady ac and decaying dc that starts from a high initial value, that is, I_{asym} ac = I_{sym} ac + Exponentially decaying dc, (ii) the decay rate of dc component increases with increasing *R*/*X* ratio, and (iii) asymmetry (offset) factor *K* depends on the instant in the cycle at which the short circuit initiates.

9.5 FAULT CURRENT MAGNITUDE

Different faults identified in Section 9.1 produce different fault current magnitudes, with the worst-case values given by the symmetrical 3-phase fault. The analytical method of calculating those magnitudes varies in complexities.

9.5.1 SYMMETRICAL FAULT CURRENT

Figure 9.4 is a 1-line representation of a 3-phase symmetrical fault (L-L-L or L-L-L-G) in a grounded or ungrounded Y-connected system with short at location F, where voltage collapses to zero. The fault current from the source returns to the source

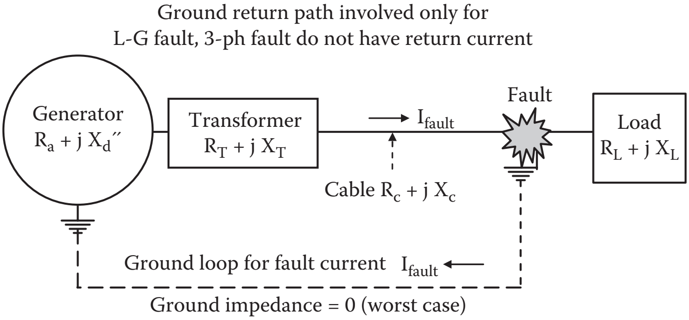


FIGURE 9.4 Short circuit fault with generator, transformer, and cable in series.

via the path of least resistance through the short (zero resistance), thus completely bypassing the load. Therefore, the load is not considered at all in the fault current calculations. All of the source voltage is now consumed to circulate the fault current in the fault loop impedance. In rms values, KVL gives

$$I_{\text{fault.rms}} = \frac{\textit{rms source voltage per phase}}{\textit{sum of all impedances per phase in fault loop}} \tag{9.13}$$

Since the generator, transformer, and cable are involved in series from the source to the fault location, $I_{\text{fault}} = V_{\text{gen}} \div (Z_{\text{gen}} + Z_{\text{trfr}} + Z_{\text{cable}})$,

$$\therefore \mathrm{I}_{\text{fault.rms}} = \frac{V_{LN.\text{gen.rms}}}{(R_{\text{gen}} + \mathrm{R}_{\text{trfr}} + \mathrm{R}_{\text{cable}}) + j(X_{\text{gen}} + \mathrm{X}_{\text{trfr}} + \mathrm{X}_{\text{cable}})} \text{ A/ph} \tag{9.14}$$

If the generator is very large compared to the transformer, it behaves very stiff with effective $Z_{\text{gen}} = 0$, that is, R_{gen} and X_{gen} equal to zero in Equation (9.14). In other situations, where the system from the source up to the fault location can be represented by Thevenin equivalent source model with V_{Th} and Z_{Th} known from calculations or by test, then the fault current is simply

$$I_{\text{fault.rms}} = \frac{V_{\text{Th.rms}}}{Z_{\text{Th}}} \tag{9.15}$$

9.5.2 Asymmetrical Fault Current

Equations (9.14) and (9.15) give the symmetrical rms current. The worst-case fault current occurs if the fault inception angle $\theta = 0°$, with its first peak magnitude given by the offset factor K from Table 9.1 times the above symmetrical value, that is,

$$I_{\text{peak.asym}} = I_{\text{fault.rms}} \times \text{Offset factor } K \text{ from Table 9.1} \tag{9.16}$$

For 3-phase symmetrical fault, with or without the ground involved, the worst-case ($\theta = 0°$) fault current is given by Equation (9.16). However, only one of the three phases (say, Phase A) can have $\theta = 0°$, which would see the fully offset asymmetrical peak. The other two phases, phase B and C, would have $\theta = 120°$ and $240°$, respectively. Therefore, the peaks of fault currents in phases B and C would have lower offsets with lower peak magnitudes than that in phase A. The average asymmetrical peak current in 3-phase fault is defined as

$$I_{\text{avg.assym.peak}} = \frac{1}{3}(\textit{Sum of first peaks in all three phases}) \tag{9.17}$$

This average value may be useful in estimating the total heat in the 3-phase equipment before the fault current is interrupted by the circuit breaker. It is emphasized here that the mechanical force between conductors is proportional to the current squared, which is extremely high at the first fully offset asymmetrical peak of the fault current. The mechanical structure of the generator and transformer coils, bus bars, circuit breakers, etc., must be designed (braced) to withstand this force without exceeding the allowable bending stress and deflection between supports. Also, the equipment must be designed to limit the temperature rise under fault current within the permissible limit, assuming that the heat accumulates adiabatically in the conductor mass until the circuit breaker clears the fault in several cycles.

Example 9.2

A 3-phase 13.8 kV distribution center has the source (Thevenin) impedance of $0.5 + j\,2.5\ \Omega$/phase. Determine the symmetrical rms and the worst-case asymmetrical first peak value of the fault current in a balance 3-phase fault at the distribution center bus.

SOLUTION

Since the source impedance at the bus is already given in ohms, we will work with ohm values. The bus impedance of the source behind the distribution center,

$$Z_{\text{source}} = \sqrt{(0.5^2 + 2.5^2)} = 2.55\ \Omega/\text{ph}$$

The rms symmetrical fault current per phase, $I_{\text{sym}} = (13{,}800 \div \sqrt{3}) \div 2.55 = 3125$ A

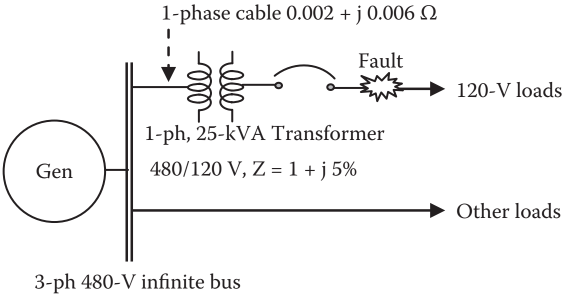
The X/R ratio of the system is 2.5/0.5 = 5. From Table 9.2, the asymmetrical offset peak factor $K = 2.19$.

∴ The worst-case first asymmetrical peak current = 2.19 × 3125 = 6843 A

All electrical equipment and bus bars in the distribution center must be designed to withstand 6843 A peak current without mechanical damage. The thermal withstand capability, however, depends on how fast the current is interrupted, which can be around 100 ms (several cycles).

Example 9.3

A 3-phase, 480 V generator powers a load via 1-phase feeders and 25 kVA, 1-phase 480/120 V step-down transformer with series impedances of $1 + j\,5\%$ as shown in below. The 1-phase feeder on the generator side has $0.002 + j\,0.006\ \Omega$ impedance. The generator is much larger than the 25 kVA transformer rating, making the generator bus infinitely stiff with zero internal impedance. Determine the symmetrical rms current rating of the transformer circuit breaker on the 120 V load side, assuming the fault location close to the breaker LV terminals.



SOLUTION

We ignore the LV cable impedance due to its negligible length. Since the HV cable impedance is given in ohms and the transformer impedance is given in percentage, we must convert both either in ohms or in percentage. Here, we arbitrarily choose to convert the transformer impedance in ohms based on 120 V side. Therefore, $V_{\text{base}} = 120\,V,\ I_{\text{base}} = \frac{25{,}000}{120} = 208.3\,A,\ Z_{\text{base}} = \frac{120}{208.3} = 0.576\ \Omega$

$$\text{Transformer impedance in ohms} = Z_{pu} \times Z_{base}$$

$$= (0.01 + j\,0.05) \times 0.576 = 0.00576 + j\,0.0288\ \Omega$$

$$\text{Total impedance from the generator to the 120 V load side fault} = Z_{\text{cable}} + Z_{\text{trfr}}$$

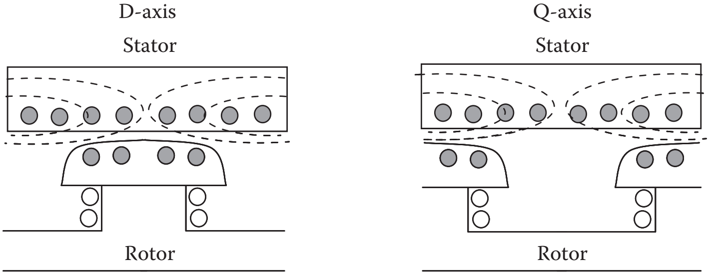
$$= (0.002 + j\,0.006) + (0.00576 + j\,0.0288) = 0.00776 + j\,0.0348\ \Omega$$
$$= 0.03565\ \Omega \text{ on 120 V base}$$

Symmetrical rms fault current on transformer secondary side = 120 ÷ 0.03565 = 3365.6 A

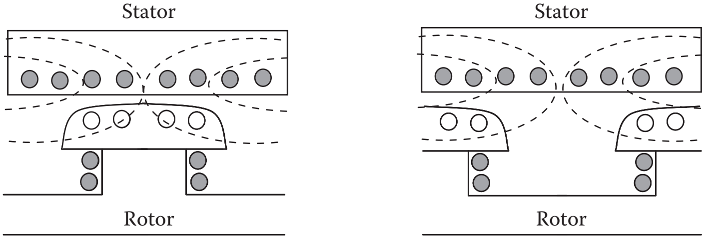
Circuit breaker rating must be the next standard step higher, say 3500 A_{rms} symmetrical.

9.5.3 TRANSIENT AND SUBTRANSIENT REACTANCE

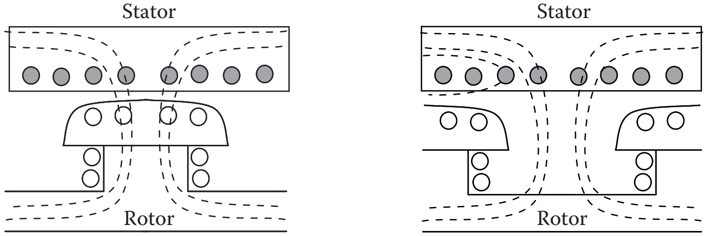
When the generator reactance has a significant effect in determining the fault current, the initial value of the generator transient reactance should be used in the fault current analysis. During the fault, the generator reactance changes over a wide range, which can be explained by the constant flux theorem we covered in Section 3.1.



(a) Subtransient flux path only in air



(b) Transient flux path partially in pole face and partially in air



(c) Steady state flux path wholly in pole body

FIGURE 9.5 Generator subtransient and transient reactance and corresponding flux paths.

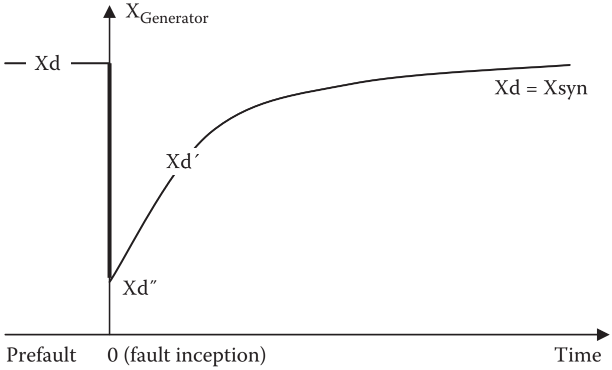
On the instant of fault, the stator coils get shorted and the stator current starts rising rapidly with a corresponding sudden and fast increase in the flux. However, in reference to Figure 9.5, damper bars on the generator pole faces prevent the sudden increase in the flux with opposing current in the damper bars, forcing the stator flux in the air gap to take the path mostly in air as shown in Figure 9.5(a). In salient pole machines, the direct and quadrature axes (*d*- and *q*-axes) have different magnetic structure; hence, both *d*- and *q*-axis are shown. As the damper bar current decays exponentially in five subtransient time constants, the stator flux gradually penetrates deeper in the rotor pole surface as in (b), but not through the rotor field coil. This decay rate is much quicker than what follows, and is known as the subtransient phase of the short circuit. At the end of the subtransient phase starts the transient phase, in which the stator flux is maintained constant by the opposing current induced in the rotor field coil, which decays exponentially in five transient time constants. During the transient phase, the stator flux gradually penetrates the rotor field coil. At the end of the transient phase, the stator flux in the stator and rotor cores and the air gap is established in the normal pattern corresponding to the steady state magnetic circuit shown in (c). During these three regions of time, the rotor conductors shown with shadow in Figure 9.5 (a), (b), and (c) carry the induced current to push back the rising stator flux.

Since the inductance of the coil is a measure of the flux linkage per ampere, and the flux in turn depends on the magnetic flux path, the effective stator inductance and reactance is the least in the subtransient phase shown in (a), small in the transient phase shown in (b), and the normally high synchronous reactance in steady state phase shown in (c). These inductances multiplied by the angular frequency $\omega = 2\pi f$ give the subtransient, transient, and synchronous reactance X_d'', X_d', and X_d, respectively. The X_d here is the normal steady state synchronous reactance on the d-axis. The transient time constant T_d'' in typical synchronous generator is very short, between 5% and 10% of the transient time constant T_d'. The salient pole machine with different magnetic structure on q-axes has different values of X_q'', X_q', and X_q. In large solid rotor turbo-generators, in absence of the discrete damper bars, the current induced in the solid pole faces virtually provides the damper bar effect. These reactances are not physically identifiable, but are concepts formulated to deal analytically with the complex nature of the machine during sudden short circuit. Table 9.2 lists various d- and q-axis reactance and time constants for large solid rotor turbo-generators and medium-size salient pole generators with damper bars on the pole faces.

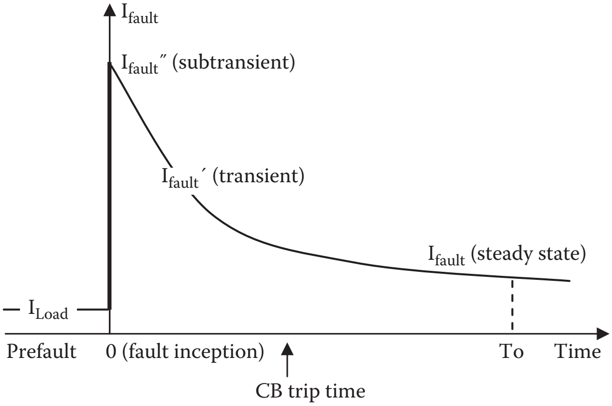
The change from subtransient to transient to steady-state time regions is gradual with exponential rise in reactance and corresponding decay in current. As shown in Figure 9.6(a), the reactance drops from its steady state synchronous value of X_d to X_d''

TABLE 9.2
Synchronous generator reactance (pu) and time constants (sec)

Parameter value	Large solid round rotor turbogenerators	Medium size salient pole generators with dampers
Synchronous reactance	$X_d = X_q = 1.0–2.5$	$X_d = 0.8–1.5$ $X_q = 0.5–1.0$
Transient reactance	$X_d'' = X_q'' = 0.2–0.35$	$X_d' = 0.2–0.3$ $X_q' = 0.2–0.8$
Subtransient reactance	$X_d'' = X_q'' = 0.1–0.25$	$X_d'' = 0.1–0.2$ $X_q'' = 0.2–0.35$
Negative sequence reactance	$X_2 = 0.1–0.35$	0.15–0.50
Zero sequence reactance	$X_o = 0.01–0.05$	0.05–0.20
Time constant τ_{dc}	0.1–0.2 sec	0.1–0.2 sec
Transient time constant τ’	1.5–2.5 sec	1.0–1.5 sec
Subtransient time constant τ”	0.03–0.10 sec	0.03–0.10 sec



(a) Step fall and then rising leakage reactance of generator



(b) Step rise and then decaying envelope of fault current peaks

FIGURE 9.6 Time varying subtransient and transient reactance and fault current magnitudes.

on the instant of the fault, and then gradually rises to X_d' and eventually returns to the normal value of X_d. Similar changes take place on the q-axis. The fault current magnitude, which is initially high with low value of X_d'' decays with rising reactance, and eventually reaches the steady state value as shown in Figure 9.6 (b).

The actual asymmetrical current, however, has two components, a symmetrical sinusoidal superimposed on a decaying dc. It is shown in Figure 9.7 in three different regions of time during the fault, the subtransient, transient, and steady state. The peak value at the beginning of each region is equal to

$$I_{\text{peak}} = \frac{\sqrt{2}\,E_f}{d\text{-axis } \textit{reactance at beginning of the time region}} \tag{9.18}$$

where E_f = prefault field excitation voltage of the generator.

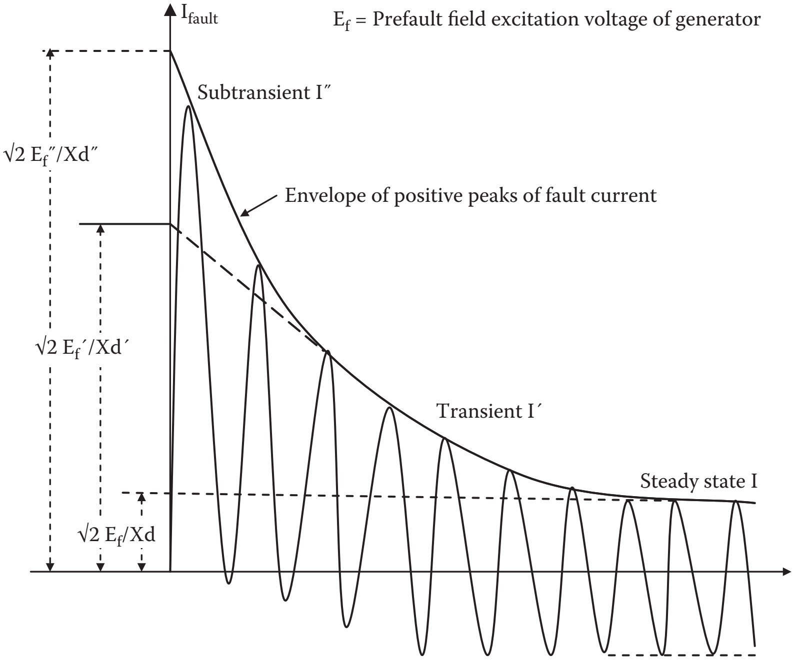
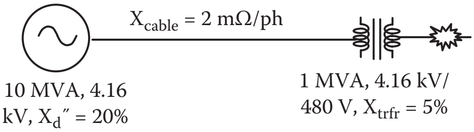


FIGURE 9.7 Exponentially decaying asymmetrical fault current and envelope of peak values.

In Figure 9.7, the circuit breaker tripping time is typically 0.1 to 0.2 sec (6 to 12 cycles in 60 Hz systems, and 5 to 10 cycles in 50 Hz systems). The time T_o for the fault current to reach steady state symmetrical value is less than 1 sec for 1 ßßMVA machine, and as high as 10 sec in a 1000 MVA machine, because of the large rotor field coil inductance.

Example 9.4

A 3-phase, 10 MVA, 4.8 kV generator with subtransient reactance X_d''= 20% feeds a 3-phase, 1 MVA, 4.8 kV/480 V transformer with X_{trfr} = 5% via a cable that has the leakage reactance X_{cable} = 2 mΩ/phase as shown below. Neglecting all resistances, determine the symmetrical rms and the worst-case first offset (asymmetrical) peak current in 3-phase fault at the transformer output terminals using (a) all reactance values in percentages, and (b) all reactance values in ohms.



(a) For percentage bases, we take the transformer secondary ratings of 1 MVA and 480 V as base.

Using Equation (9.3), $I_{\text{base}} = 1000 \div (\sqrt{3} \times 0.480) = 1202.8$ A/ph, and

$$Z_{\text{base}} = (480 \div \sqrt{3}) \div 1202.8 = 0.2304\ \Omega/\text{phase}$$

Or, using Equation (9.4), $Z_{base} = 0.480^2 \div 1 = 0.2304\ \Omega$/phase

The X_{trfr} is given on its own base, so it does not need any adjustment. However, the generator X_d'' customarily given on its own MVA base gets proportionately adjusted to 1 MVA base selected for the calculations here, that is, $X_d'' = (1 \div 10) \times 20\% = 2\%$, and the cable's 2 mΩ/phase is on its 4180 V side, which has $Z_{base} = 1000 \times 4.18^2 \div 1000 = 17.4724$ Ω, making $X_{\text{cable}} = (0.002 \div 17.47) \times 100 = 0.01145\%$ on 1 MVA, 480 V base.

$$\therefore \%\ X_{\text{Total}} = X''_{\text{d.gen}} + X_{\text{cable}} + X_{\text{trfr}} = 2 + 0.01145 + 5 = 7.01145\%.$$

With full 100% voltage before the fault,

$$I_{\text{fault}} = 100\% \div 7.0145\% = 14.256 \text{ pu or}$$
$$14.256 \times 1202.8 = 17{,}147 \text{ A/ph symmetrical rms}$$

(b) We now repeat the calculations using the ohm values looking from the 480 V base.

$X_{\text{trfr}} = 0.05 \times 0.2304 = 0.01152\ \Omega$ $\quad X_d'' = 0.02 \times 0.2304 = 0.004608\ \Omega$

$X_{\text{cable}} = 2$ mΩ gets adjusted by the voltage ratio square to become
$X_{\text{cable}} = 0.002 \times (480 \div 4180)^2 = 0.000026\ \Omega$/ph.
$\therefore X_{\text{total}} = 0.004608 + 0.000026 + 0.01152 = 0.0161544\ \Omega$/ph
$I_{\text{fault}} = (480 / \sqrt{3})$ volts $\div\ 0.0161544\ \Omega = 17{,}155$ A/ph (checks within rounding errors)

Further, since the resistance in this fault loop is zero, the first peak will get fully offset by a factor of $2 \times \sqrt{2}$, that is,

Fault current at first peak, $I_{\text{asym·offset.peak}} = 2 \times \sqrt{2} \times 17{,}155 = 48{,}523\ \text{A}_{\text{peak}}$

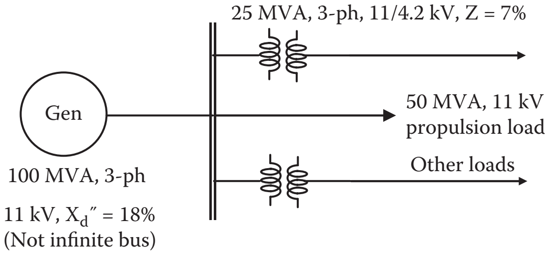
The symmetrical fault current from the generator terminals can be simply calculated by the flip of the voltage ratio, that is, $I_{\text{fault.gen}} = 17{,}155 \times (480 \div 4180) = 1970$ A/ph symmetrical rms, and the first peak will be $2 \times \sqrt{2} \times 1970 = 5572$ A.

Example 9.5

For a 3-phase fault at the spark location in the figure below with all equipment 3-phase and generator of finite size, determine the symmetrical rms fault current. Ignore the series resistance (but not the reactance) of the transformer and the generator. Assume all cable short with negligible *R* and *X*, and follow the following steps:

(a) As is commonly done by power engineers, convert the system on percent values based on the kVA and voltage ratings of the transformer feeding the fault. Then, determine the fault current in amperes at the fault location and also at the generator terminals using the voltage ratio.

(b) Repeat the calculation by converting the system on percent Z values based on the kVA and voltage ratings of the generator, and then determine the fault current in amperes at the generator and the transformer terminals, and compare it with (a).



SOLUTION

The generator in this example is not large enough to be treated as an infinite bus; hence, it will have an effect on the fault current. The problem statement does not say how the transformer is connected. The generator is always Y-connected. We can assume the transformer is also Y-connected and make the calculations per phase. If the transformer were actually Δ-connected, the line current would still be the same as in Y-connection. Only the phase current and voltage in the transformer coils will be different. The actual transformer connection does not matter in calculating the line current. Assuming it is in Y gives the same line current (which is the phase current in Y).

We recall that the percentage of impedance value prorates with the kVA base, but does not change with the voltage base. Therefore,

(a) *On 25 MVA transformer secondary base:* The generator X_d'' of 18% is prorated on the transformer ratings as $Z_{\text{gen}} = 18(\frac{25}{100}) = 4.5\%$.

Total impedance per phase from generator to fault location = $Z_{\text{gen}} + Z_{\text{trfr}} = 4.5 + 7 = 11.5\%$

$$\therefore I_{\text{fault}} = \frac{100\%\,voltage}{11.5\%\,Z_{\text{Total}}} = 8.696\,pu$$

$\therefore$

Transformer base current on 4.2kV side $= \frac{25{,}000/3}{4.2kV/\sqrt{3}} = 3437\,A$ per phase

$\therefore$ Fault current coming out of transformer LV lines = 8.696 × 3437 = 29,900 A/ph

Using voltage ratio, fault current coming out of generator = 29,900 × (4.2 ÷ 11) = 11,412 A /ph

(b) *On the generator base:* The transformer Z of 7% prorated on the generator kVA rating = 7 × (100 ÷ 25) = 28%. Total impedance = $Z_{\text{gen}} + Z_{\text{trfr}} = 18 + 28 = 46\%$.

$\therefore$ Fault current on generator base = 100% voltage ÷ 46% Z = 2.174 pu.

Generator base current $= \frac{100{,}000/3}{11kV/\sqrt{3}} = 5249\,A$

∴ Fault current coming out of the generator = 2.174 × 5249 = 11,412 A /ph

Again using the voltage ratio, fault current coming out of the transformer LV line = 11.412 × (11 ÷ 4.2) = 29,900 A /ph. Theses values are the same as those derived in (a).

9.5.4 GENERATOR TERMINAL FAULT CURRENT

In view of the complexities in transient fault current analysis, the IEEE Standard-45 on shipboard electrical power systems allows using the following rms fault currents pending details of the equipment and system parameters for exact calculations.

The rms value of the average of first peaks in all three phases,

$$I_{\text{avg asymmetrical rms}} = 8.5 \times \text{generator rated rms current} \tag{9.19}$$

The rms value of the first fully asymmetrical peak in the worst phase,

$$I_{\text{fully asymmetrical rms}} = 10.0 \times \text{generator rated rms current} \tag{9.20}$$

The first peak of fully offset current in the worst phase = √2 × rms value from Equation (9.20) must be used in the mechanical force calculations for bracing the structure to avoid mechanical damage under the worst-case short circuit fault.

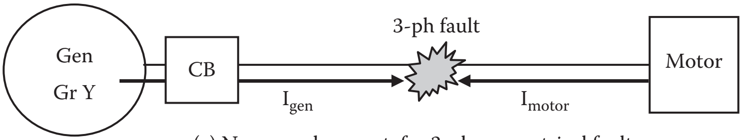
9.5.5 TRANSFORMER TERMINAL FAULT CURRENT

When the component impedances are not known, one can make the most conservative estimate of the fault current using only the impedance of the transformer feeding the fault. This essentially treats the transformer primary side source as the *infinite bus*, defined as the bus of infinite kVA capacity with zero internal source impedance. This is fairly accurate when the transformer rating is small compared to the generator rating. It essentially ignores the generator and cable impedances up to the transformer primary side bus. Such simplification leads to the most conservative fault current estimate of symmetrical rms fault current with full (1.0 pu or 100%) transformer voltage before the fault,

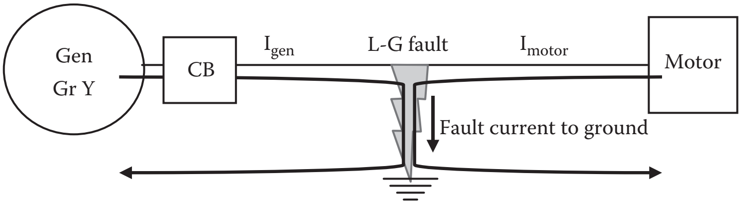
$$\textit{Worst-case fault current } I_{\text{sym.rms}} = \frac{100}{\%\, Z_{\text{trfr}}} \times \textit{Transformer } \text{seco}\textit{ndary rated amps} \tag{9.21}$$

The first peak of worst-case fully asymmetrical fault current in a transformer connected to an infinitely large power source is

$$I_{\text{first.asym.peak}} = \frac{K \cdot MVA_{3ph} 10^6}{\sqrt{3}\, V_{LL}\, Z_{pu}} \qquad \text{A/phase} \tag{9.22}$$



(a) No ground current for 3-ph symmetrical fault



(b) Ground current flow in L-G fault

FIGURE 9.8 Motor contribution to the fault current.

where V_{LL} = rated line voltage in volts, Z_{pu} = transformer series impedance in per unit (pu), K = offset factor from Table 9.1 for transformer X/R ratio ($K = 2.33$ for $X/R = 7$, which is typically found in large power transformers).

9.6 MOTOR CONTRIBUTION TO FAULT CURRENT

If the generator is powering a motor via cable as shown in Figure 9.8, and the cable gets shorted between the generator and the motor, the fault current not only comes from the generator, but also comes from the motor as well. The kinetic and magnetic energy of the motor rotor feeds current back to the fault. It is complex to calculate, but the IEEE-45 standard suggests an approximate estimate of the symmetrical rms fault current from the motor equal to

$$I_{\text{fault.motor.sym.rms}} = M \times \text{motor rated current}$$

where $M = 2$ for motor voltage < 240 V, $M = 3$ for motor voltage between 240 V and 600 V, and $M = 4$ for motor voltage > 600 V. The total current going into the ground fault (not in the cables though) is then equal to that coming from the transformer plus that coming from the motor. In terms of the symmetrical rms value, they are

$$I_{\text{fault.total}} = I_{\text{fault.gen}} + I_{\text{fault.motor}} \tag{9.23}$$

where

$I_{\text{fault.total}}$ = total fault current to L-L-L short
$I_{\text{fault.gen}}$ = fault current from generator side calculated with all known impedances
$I_{\text{fault.motor}}$ = fault current from motor side = 2 to 4 × rated motor current

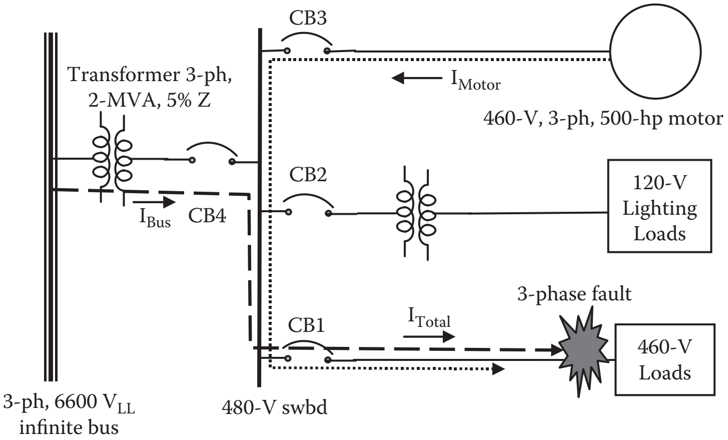
The motor contribution in the fault current can be explained this way. Before the fault, there was a back voltage in the rotor opposing the applied line voltage that was pushing the current into the motor. However, the motor line voltage collapses to zero after the fault, and the motor back voltage becomes the only voltage from the motor to the fault. This results in current flowing from the motor to the fault until the motor electromagnetic and kinetic energies get depleted.

The mechanical analogy of the above is a pump driving an air compressor. The pump has the pressure, and the compressor has the back pressure that is somewhat less than the pump pressure. If the air pipe between the pump and the compressor gets a fault (puncture) in the middle, the air rushes out from the hole to the atmosphere from both sides, from the pump as well as from the compressor.

Since the motor current comes from the load side of the circuit, the generator side cable and circuit breaker do not see this current. As for the motor side cable, the fault current from the motor side is generally less than the fault current from the generator side and for a much shorter duration (only several cycles). For these reasons, the risk that the motor side current poses to the cable or the circuit breaker appears irrelevant when compared to the generator side fault current. One can then wonder why include the motor contribution in the fault current estimate? It is for one reason, and that is the fault current flowing from conductor to conductor through the cable insulation is the sum of fault currents coming from the generator and from the motor. It is this total current that produces heat in the insulation in very small local spots, posing risks of extreme overheating, explosion, and even fire.

Example 9.6

A 3-phase, 6600 V main bus feeds a 3-phase, 2 MVA, 6600/480 V transformer with 5% impedance. The transformer powers multiple loads, including a 3-phase, 500 hp motor as shown below. Determine the symmetrical rms fault current through CB1, CB2, CB3, and CB4 under a 3-phase fault at the spark location assuming (i) all cable impedances are negligible, (ii) the bus has multiple generators of large capacity (i.e., infinite bus), and (iii) the motor-delivering rated load before the fault with 92% efficiency and 90% pf lagging.



SOLUTION

From the infinite main bus to the fault, with all cable impedances negligible, the only impedance is that of the transformer, which is 5%. Therefore, the bus contribution to the fault current, $I_{\text{fault.bus}}$ = 100% ÷ 5% = 20 pu (note that this is pu, not percent)

The rated current on the transformer secondary side = 2000,000 ÷ √3 × 480 = 2406 A/ph

This and all currents below are symmetrical rms values.

∴ CB4 current = generator contributions through 2 MVA transformer secondary = 20 × 2406 = 48,120 A

$$\text{Motor rated current} = \frac{500 \times 746\,watts}{\sqrt{3}x460 \times 0.92x0.90} = 565A$$

Using multiple M = 3 for 460 V motor as per Section 9.6, CB3 current = motor contribution to the fault = 3 × 565 = 1695 A

CB1 current = CB4 current from bus + CB3 current from motor = 48,120 + 1695 = 9,815 A

CB2 current = 0, as it is not involved in this fault.

However, the CB2 rating must be determined by a similar fault current calculations involving CB2 branch. That is why the power engineer makes short circuit fault current calculations at numerous possible fault locations (often using commercially available software) to size all CBs in the system.

9.7 CURRENT LIMITING SERIES REACTOR

Protecting the system components from high fault current that can cause mechanical or thermal damages may require limiting the fault current by adding a series reactance in line. Series resistance is seldom used due to its high I^2R power loss, and iron-core rector would saturate and offer low reactance under high fault current. Air-core reactor with low resistance is, therefore, sometimes used for limiting the fault current. Such a reactor is carefully designed to minimize eddy currents due to leakage flux reaching the neighboring metal parts. The required current limiting reactance value can be determined from the fault current analyses outlined in this chapter. Since a series reactance in power line results in lower fault current, but higher steady-state voltage drop and some I^2R power loss, its design must be optimized with the overall system requirements. This is an example of another trade-off made by the system design engineer. A superconducting coil with zero resistance can be an ideal reactor for such application.

9.8 UNSYMMETRICAL FAULTS

For unsymmetrical faults (L-G, L-L, and L-L-G), the fault current analysis becomes even more complex. It involves the theory of *symmetrical components*, a brief summary of which is given in Appendix A. The IEEE-45 standard gives working formulas for such faults based on the analysis using symmetrical components for unsymmetrical faults. Such analysis includes resolving unsymmetrical (unbalanced) three-phase voltages into three sets of symmetrical (balanced) voltages, one with phase difference of +120° (positive sequence), one with phase difference of –120° (negative sequence), and one with no phase difference at all (zero sequence). The resulting three symmetrical component currents are then determined using the following,

$$\textit{symmetrical component current} = \frac{\textit{symmetrical component voltage}}{\textit{symmetrical component impedance}} \tag{9.24}$$

$$\textit{Unsymmetrical phase currents} = \textit{Phasor sum of three symmetrical component currents} \tag{9.25}$$

We give below the final results for the fault currents in the most frequent unsymmetrical fault (i.e., L-G fault), using the symmetrical components analysis presented in Appendix A. For line A to ground fault, we know that $I_b = I_c = 0$ and the neutral current $I_n = -I_a$. The line A current, however, will be

$$I_a = \frac{V_a}{\frac{1}{3}(Z_1 + Z_2 + Z_0) + Z_f + Z_g + Z_n} \tag{9.26}$$

where V_a = prefault voltage on line A; Z_1, Z_2, and Z_o = total positive, negative, and zero sequence impedance, respectively, in the fault loop; and Z_f, Z_g, Z_n = arcing fault impedance, actual earth impedance, and intentionally placed impedance in the neutral, respectively.

For the special case of dead fault with solidly grounded neutral, $Z_f = Z_g = Z_n = 0$ in Equation (9.26).

For L-L, and L-L-G faults, see Appendix A.

9.9 CIRCUIT BREAKER SELECTION SIMPLIFIED

For determining the worst-case fault current to size the current interrupting capability of the circuit breaker, the fault current analysis can be simplified as follows, where all quantities are per phase and in per unit on a common base with the maximum generator capacity connected. First consider the 3-phase fault current, which is given by $I_{\text{sym.rms}} = E_{\text{gen}}/X_1$. Then consider the 1-phase L-G fault current with solid ground, which is given by $I_{LG.\text{rms}} = 3\ E_{\text{gen}}/(2\ X_1 + X_0)$, where E_{gen} = prefault generator-induced voltage per phase (also known as E_f), X_1 = total positive sequence reactance from the generator to the fault location, and X_0 = total zero sequence reactance from the generator to the fault location. In these totals, we must use the subtransient reactance for the synchronous generator. For selecting the circuit breaker's interrupting capability, we use the greater of the two fault current values, that is,

$$\textit{Circuit breaker rating} = I_{\text{sym.rms}} \text{ or } I_{LG.\text{rms.}}, \textit{whichever is greater} \tag{9.27}$$

PROBLEMS

CHAPTER 9 PROBLEMS—FAULT ANALYSIS

Problem 9.1: Determine the base current and base impedance in a 3-phase power system on the base of 50 $\text{MVA}_{3\text{-ph}}$ and 11 kV_{LL}, first using 1-phase formulas, and then using 3-phase formulas, recognizing that both 1-phase and 3-phase formulas give the same base current and base impedance values *per phase*.

Problem 9.2*:* A 3-phase 6.6 kV distribution center has the source impedance of 0.1 + *j* 0.5 Ω/phase. Determine the symmetrical rms value and the worst-case asymmetrical first peak of the fault current in a balance 3-phase fault at the distribution centre bus.

Problem 9.3*:* A 3-phase, 600 V generator feeds a load via 1-phase feeder and 50 kVA, 1-phase 600/120 V step-down transformer with series impedances of 2 + *j* 6% as shown in Figure P9.3. The 1-phase feeder on the generator side has 0.003 + *j* 0.025 Ω impedance. The generator rating is much greater than 50 kVA, making it an infinitely stiff bus with zero internal impedance. Determine the symmetrical rms current rating of the transformer circuit breaker on the load side, assuming the fault location close to the breaker LV terminals.

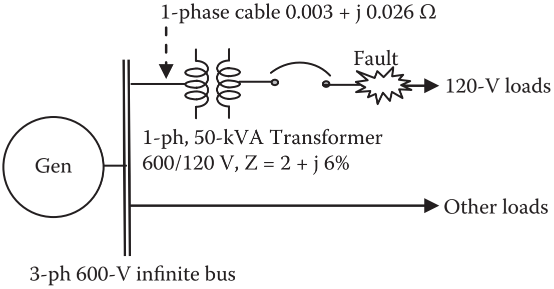


FIGURE P9.3

Problem 9.4: A 15 MVA, 3-phase, 6.6 kV generator with subtransient reactance $X_d'' = 15\%$ feeds a 3-phase, 1 MVA, 6.6 kV/480 V transformer with $X_{\text{trfr}} = 7\%$ via cable that has the leakage reactance $X_{\text{cable}} = 2.2$ mΩ/phase as shown in Figure P9.4. Neglecting all resistances, determine the symmetrical rms value and the worst-case first offset peak of the fault current for a 3-phase fault at the transformer output terminals using (a) all reactance values in percentages, and (b) all reactance values in ohms.

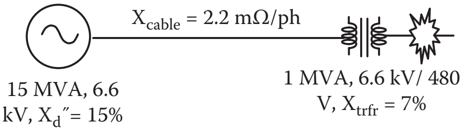


FIGURE P9.4

Problem 9.5: For a 3-phase fault at the spark location in Figure P9.5 with all equipment 3-phase and the generator of finite size, determine the symmetrical rms fault current (i) in 4.2 kV line coming from the transformer, and (ii) in 11 kV line coming from the generator. Ignore the series resistances of the transformer and generator, and assume all cable short with negligible *R* and *X*.

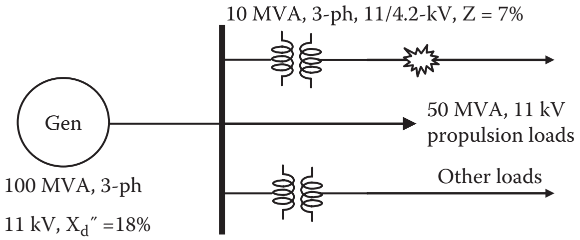


FIGURE P9.5

Problem 9.6: A 3-phase, 4.16 kV main bus feeds a 3-phase, 5 MVA, 4.16 kV/480 V transformer with a 6% impedance. The transformer powers multiple loads, including a 3-phase, 1000 hp motor as shown in Figure P9.6. Determine the symmetrical rms fault current through CB1, CB2, CB3, and CB4 under a 3-phase fault at the spark location, assuming (i) all cable impedance are negligible, (ii) the bus has multiple generators of large capacity to make it an infinite bus, and (iii) the motor delivering rated load before the fault with 90% efficiency and 0.85 pf lagging.

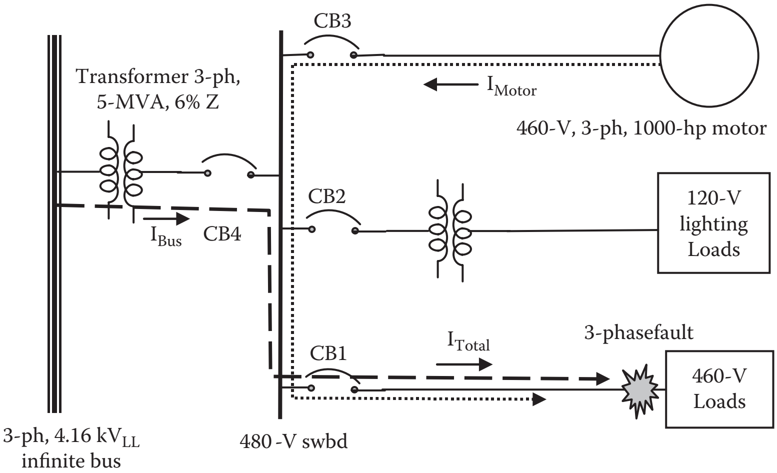


FIGURE P9.6

Problem 9.7: A 3-phase, 1000 kVA, 4.16 kV Δ /480 *V* grounded-*Y* transformer with 5% impedance feeds a 3-phase, 100 hp, 460 V motor. If a 3-phase symmetrical fault occurs in the feeder, determine the fault current, including the motor contribution. Ignore the generator and cable impedances.

Problem 9.8: A 1500 kVA, 3-phase, 480 *V*, *Y*-connected generator with neutral solidly grounded has $R = 2\%$ and subtransient $X_d'' = 14\%$. It supplies two 750 kVA, 3-phase, 480 *V*, *Y*-connected feeders, one of which gets a 3-phase short circuit fault. The transformer in each feeder has $R = 2\%$ and $X = 6\%$ on the nameplate. Ignoring the short cable impedance, determine the symmetrical rms fault current with full generator voltage prior to the fault.

Problem 9.9: Estimate the peak asymmetrical and the average asymmetrical rms fault currents coming out of a 20 MVA, 3-phase, 4160 V, GrY generator terminals if a symmetrical fault develops somewhere in the distribution system. You have no other information.

Problem 9.10: A 3-phase distribution cable gets 3-phase fault 150 ft away from the 460 V, Y-connected generator with neutral grounded. The subtransient impedance of the generator is $0.1 + j\,0.4$ Ω/ph, and the cable impedance is $0.6 + j\,0.03$ Ω/ph per 1000 feet. Determine the symmetrical rms fault current and the first offset peak, ignoring the motor contribution.

Problem 9.11: Determine the symmetrical rms fault current at the generator terminals for a 3-phase fault at the spark location shown in Figure P9.11. Ignore all cable impedances.

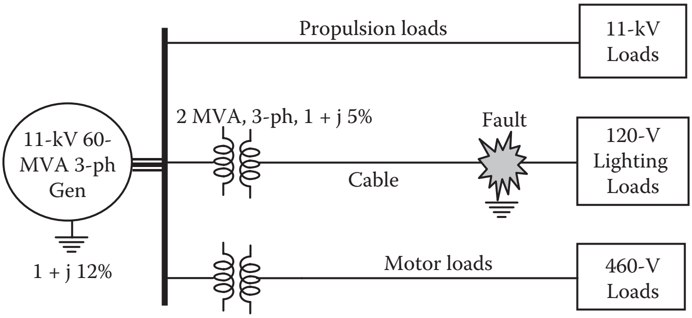


FIGURE P9.11

Problem 9.12: In the distribution systems shown in Figure P9.12, the generator impedance is not known. Determine the symmetrical rms fault current for a 3-phase fault at the spark location. The cable impedance $1 + j\,2\%$ is based on the 200 kVA transformer ratings. Make conservative assumptions where needed.

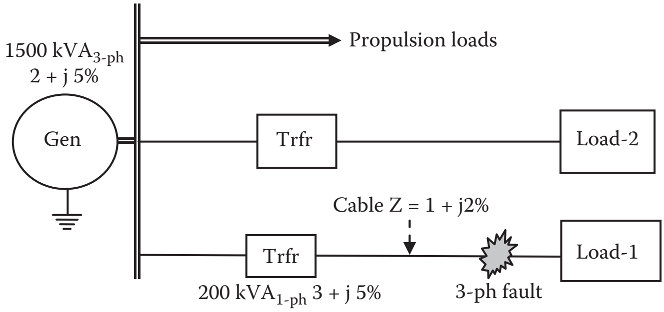


FIGURE P9.12

QUESTIONS

Question 9.1 Explain how the most frequent unsymmetrical L-G fault soon leads to 3-phase symmetrical L-L-L-G fault.

Question 9.2 What system parameters do you need in order to calculate the fault current magnitude?

Question 9.3 By hydraulic analogy, explain why the first peak of transient fault current gets much greater than the symmetrical peak.

Question 9.4 Explain the R-L-C circuit physics leading to the worst-case asymmetrical fault current peak almost twice in the worst case than the symmetrical peak.

Question 9.5 Explain the term *d*-axis and *q*-axis in the synchronous generator and motor.

Question 9.6 Explain the subtransient and transient reactance of the synchronous generator.

Question 9.7 Between the two rough estimates of fault current as per the IEEE-45 standard, which multiplier with the generator rated current, 8.5 or 10, should you use in the structural design of the bus bars and all electrical machines involved in series with the fault?

Question 9.8 Transformer A has a 5% impedance and Transformer B has a 7% impedance. Which one will see greater fault current at the LV terminals?

Question 9.9 Describe in your own words how the motor load contributes to the fault current.

Question 9.10 What trade-offs are involved in using and selecting the current limiting series reactor?

Question 9.11 Which mathematical method is used to analyze unsymmetrical (L-G, L-L, and L-L-G) faults?

Question 9.12 Summarize in a paragraph the simple determination of the circuit breaker's symmetrical fault current interruption rating.

FURTHER READING

Kersting, W.H. 2007. *Distribution Systems Modeling and Analysis.* Boca Raton, FL: CRC Press.

Bergen, A.R. and V. Vittal. 2000. *Power Systems Analysis.* Upper Saddle River: Prentice Hall.

10 System Protection

The power system design includes protection from all credible fault currents and overvoltages that may occur in normal and abnormal operations. The fault (short circuit) occurs by accident or when the insulation breaks down due to aging and vibrations, or due to overvoltages exceeding the strength of the insulation. Giving special emphasis on the ship environment (humidity, high temperature, and vibrations) is important in selecting the insulation type for shipboard electrical equipments.

The heat generation rate and the mechanical force in conductors, being the square function of the current, are the highest at the first peak of the asymmetrical fault current. The current then decays to the symmetrical value in several cycles and steady state value in about 1 second, as the generator reactance rises from low subtransient to transient to steady state synchronous reactance value. The mechanical damage is avoided by bracing the structural parts to withstand the first peak of the mechanical force without exceeding the allowable stress or deflection limits. If that is not possible or practical, the protection system must incorporate a suitable fault current limiting fuse or circuit breaker, or insert a series reactor in the line, to limit the current. The thermal damage is avoided by interrupting the fault, typically in several cycles, before it can overheat the equipment and burn the insulation.

A good protection starts with fast fault detection. The voltage, current, and temperature measurements generally detect faults in the system. For example, when the voltmeter or a sensor detects zero or a very low bus voltage, it is a sure signal of a ground fault on the bus.

The following tasks are part of the system protection design to minimize the extent of outage following a fault:

- Fault current analysis to determine the symmetrical rms and asymmetrical peak fault currents at various locations in the system where a circuit breaker or fuse is needed.
- Selecting each circuit breaker and fuse with required continuous and ampere interruption (AIC) ratings with proper margin
- Strategically placing the fault detection sensors at key locations in the system
- Protection coordination along the power flow line to assure selective tripping among all fuses and circuit breakers with recommended pick up and delay settings
- Arc flash analysis and assessment of the risk and hazard levels arising from fault current

The fuse and circuit breaker are major devices that protect the power system from fault currents. The general design practice in using either one has these features:

- All loads are individually protected to prevent a fault in any one load from damaging the feeder or the power source that is common (community property) for many loads.
- A protective device is generally placed near the source to provide protection against faults in the cable from the source to the load.
- Large fault currents in the system must be interrupted before the generator could lose the transient stability. Relatively lower fault currents must be interrupted before the transient temperature of the conductor reaches the maximum allowable transient limit. For copper conductor, this limit is typically 325°C—about half of its melting temperature—when copper starts softening.

10.1 FUSE

The fuse provides protection by melting away a thin metal link in the faulted circuit. The metal link may be of silver, copper, or nickel, silver being more common for long-term performance stability. The fuse body is generally filled with a sand-type filler (Figure 10.1) to suppress sparks when the fuse link melts and interrupts the inductive energy in the load and wire loops of lead and return conductors. The fuse life is determined primarily by aging of the metal link under thermoelastic cycles of load on and off.

The ground insulation requirements on a fuse vary with the voltage rating. For example, the 120 V fuse may typically require:

- Dielectric withstand capability >500 V measured by megger.
- Ground leakage current at rated circuit voltage <1 mA.
- Withstanding full recovery voltage equal to the open circuit voltage in steady state and up to twice the circuit voltage during transient that appears across the fuse terminals after clearing the fault.
- Resistance after fuse clearing >1 MΩ to 10 MΩ, depending on the rated current and the maximum fault current it can interrupt without exploding.
- After fuse clearing, the body and terminal surface temperatures must remain below 50°C for fuses up to 100 A. For larger fuses up to 500 A, the body temperature must remain below 50°C, but the terminal temperature may rise up to 75°C.

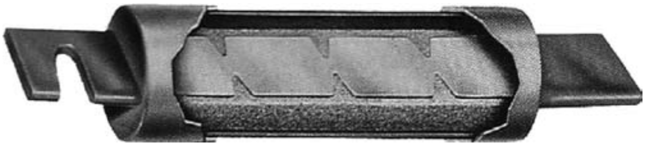


FIGURE 10.1 Typical fuse construction with metal link and sand filler.

10.1.1 FUSE SELECTION

Three key ratings to consider in the fuse selection are: (1) rated amperes it must carry continuously, (2) rated circuit voltage it must support, and (3) the maximum current it must interrupt without exploding, which must be greater than the prospective fault current at the fuse location.

The fuse clears (melts or blows) at higher than rated current as per its fault current versus clearing time characteristic (known as *i-t* curve). Due to inherently large manufacturing variations, the *i-t* curve is typically given by a band as shown in Figure 10.2. For example, at 5 times the rated current, the clearing time may be from 0.01 to 1.0 sec. This must be accounted for in the system design to properly protect the circuit so that it clears when required and does not clear when not required.

The fuse ampere rating must be carefully selected because both a conservatively or liberally sized fuse is bad for the circuit protection. General criteria for selecting the current rating of a fuse are as follows:

- Carry 110% of rated current for at least 4 h without clearing
- Clear within one hour at 135% of rated current
- Clear within 2 minutes at 200% of rated current
- Clear within 1 ms at 1000% (10×) rated current
- Voltage drop below 200 mV at rated current

The selected fuse rating is typically the circuit voltage and 1.2 to 1.3 times the rated current in the load circuit it protects, rounded upward to the next standard available rating.

When the fuse clears, the break in the circuit current causes the full rated voltage to appear across the blown fuse. If the load is inductive, the transient voltage across the fuse may be substantially higher—up to twice the rated line voltage—due to

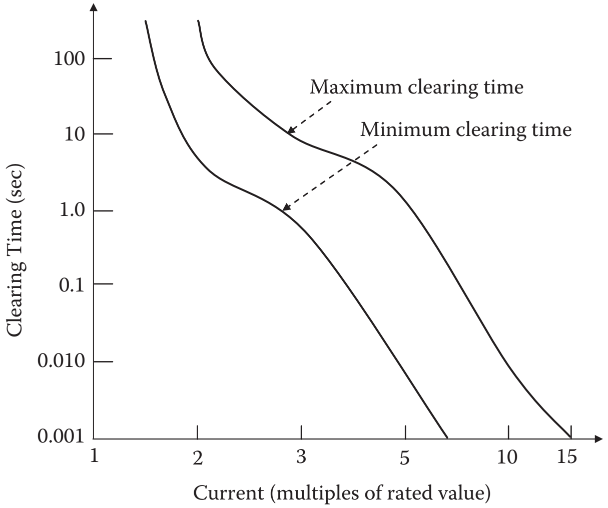


FIGURE 10.2 Fault current versus clearing time band of standard general papoose fuse (normalized with rated current).

the inductive energy kick (analogous to *water hammer* in water pipes). Under this overvoltage, a destructive arc may be formed across the fused element, and may continue to grow. The resulting heat and pressure may cause the fuse to explode in the worst case. The voltage rating of the fuse is therefore selected such that the fuse can interrupt a dead short without shattering or emitting flame or expelling molten metal.

In cargo ships hauling combustible materials, where explosive vapors may be present, the general purpose nonsealed fuse poses a safety concern due to possible arcing when the fuse clear. The hermetically sealed fuse may be used in such applications. It has been developed for safe operation in explosive mixtures of chemical vapors classified by the National Electrical Code as Class 1 environment (hazardous). The hermetically sealed fuse eliminates the explosion possibility by containing the arcing inside. Moreover, it is fast and more predictable in clearing time, since there is no arcing in open air and the associated plasma, which can linger on for a long time.

10.1.2 TYPES OF FUSE

Three types of fuses are available in the industry for different applications:

Standard (single-element) fuse, which is a general purpose fuse used in lighting and small power circuits. It has one element that melts when accumulated heat brings it to the melting temperature. Its i-t characteristic has a rather wide band as was shown in Figure 10.2.

Time-delay (slow-blow or dual-element) fuse, which is designed to ride-through the starting inrush current drawn by certain load equipment—such as motor, transformer, capacitor, heater, etc.—for a short time immediately after turn-on. This fuse has two elements in series, one heavy bead that takes time to heat up under moderate currents under overloads, and the other thin strip with a large dissipation area which melts only when the current rises rapidly to a very high value under faults. The i-t characteristic of such a dual-element fuse also has a large manufacturing tolerance band.

Current limiting (fast-acting) fuse, which clears before a large prospective fault current builds up to the first peak. Its fusible link has brief arcing and melting time such that it clears the fault in about a quarter cycle, resulting in the let-through current much less than the prospective peak fault current that a normal fuse would see. Such fuse is used in delicate heat-sensitive power electronics circuits with diodes, thyristors, transistors, etc. Figure 10.3 shows a typical clearing time characteristic of a 200 A, 600 V current-limiting fuse in a branch circuit with the prospective symmetrical fault current of 10,000 A_{rms} (25,000 A first peak). It will interrupt the fault at a peak let-through value of 8000 A (instead of 25,000 A prospective first peak). Many current ratings for such fuses are available under trade name AmpTrap™.

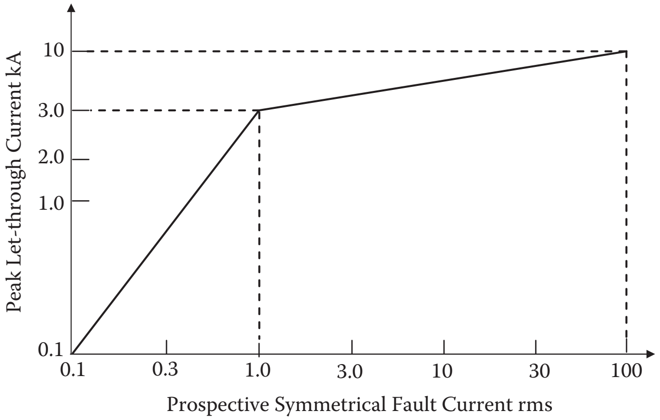


FIGURE 10.3 Prospective symmetrical fault current versus peak let-through current in current-limiting fuse.

Table 10.1 lists typical applications of the three fuse types, and Figure 10.4 compares the current versus average clearing time characteristics. For each type, significant manufacturing variations exist around the average clearing time, as was seen in Figure 10.2 for the standard general purpose fuse.

The fuse rating is based on factory tests conducted at room temperature. The ambient air temperature in actual operation has an influence on the actual rating of the fuse. The fuse rating must be lowered if the ambient air temperature is higher than standard room temperature, and vice versa. Tests suggest that the fuse must be derated or up-rated with the ambient temperature as shown in Figure 10.5. As a rule of thumb, the nominal current rating of fuse operating in high ambient air temperature must be derated by:

0.5% per °C ambient above 25°C for time-delay fuse
0.2% per °C ambient above 25°C for standard general purpose fuse

TABLE 10.1
Fuse Types and Their Typical Applications

Fuse Type	Blow Time at 2 × Rated Current	Typical Applications
Fast-acting (current-limiting) fuse	<1 sec	Power electronics circuits and meter protection
Standard (general purpose) single-element fuse	<10 sec	Most general purpose standard circuits (lighting, small power circuits)
Time-delay dual-element (slow-blow) fuse	>10 sec	Circuits drawing high inrush current on switch-on to avoid nuisance blowing. (Motors, transformers, heaters, capacitor charging, etc.)

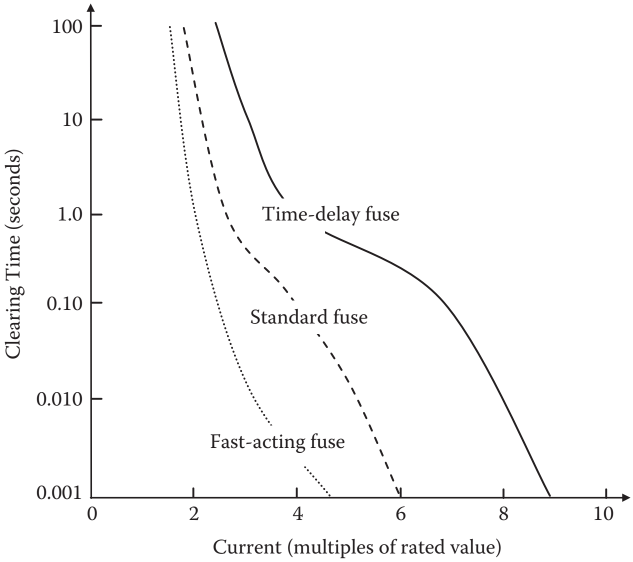


FIGURE 10.4 Fault current vs. clearing time for three types of fuse (average values).

10.2 OVERLOAD PROTECTION

The fuse and circuit breaker clears fault only if the overcurrent is greater than 200% of its own rated current. That leaves a protection gap between 115% overload allowed in most equipments for an hour or two and 200% overload. This gap is bridged by the overload protection. In its simplest and often used form shown in Figure 10.6, it consists of a pair of bimetallic links that remains closed in normal operation, but deforms differentially to open the contacts when overheated.

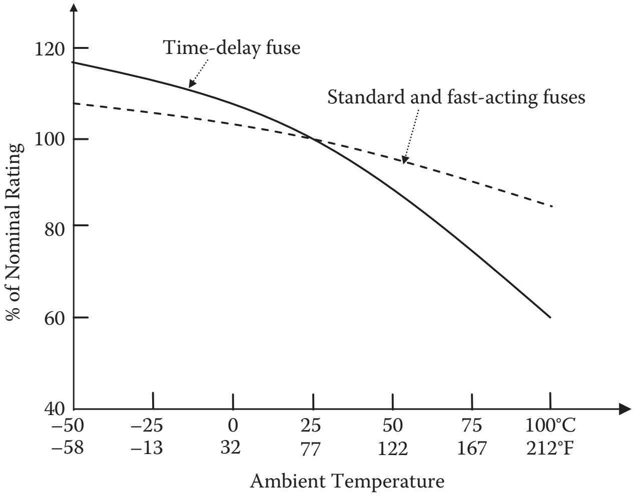


FIGURE 10.5 Derating and uprating factors for fuse vs. operating ambient temperature.

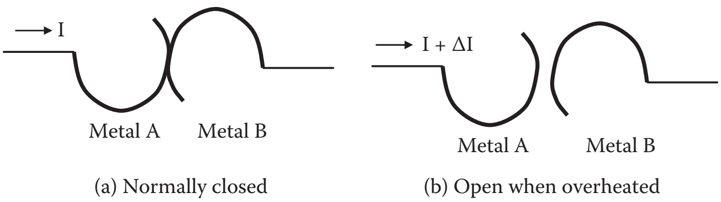


FIGURE 10.6 Overload protection by bimetallic link with normally closed contacts.

Example 10.1

Determine the general-purpose fuse ampere rating for a 1-phase, 120 V, 2 kW load circuit in the boiler room uptake, which has exhaust air temperature of 65°C. Assume the load equipment efficiency of 90% and power factor 85%.

SOLUTION

Recalling that the equipment rating is the output, we have

$$\text{Load current} = \frac{2x1000}{120x\ 0.90x0.85} = 21.8\ A$$

We design the load branch circuit for 30% overload.

∴ Design current = 1.3 × 21.8 = 28.3 A

Using Figure 10.5, the fuse rating for the boiler room uptake with 65°C ambient air should be derated to about 0.90 (hard to read scale).

Alternatively, using the rule of thumb given in Section 10.1.2,

Derating of the general purpose fuse = 0.2% per °C above 25°C

$$= 0.002 \times (65 - 25) = 0.08, \text{ or the derating factor} = 0.92$$

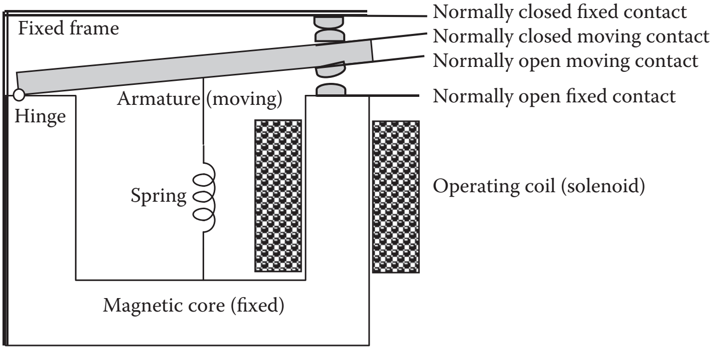
∴ Fuse rating = 28.3 ÷ 0.92 = 30.8 A.

We choose 30 A fuse, as we have 30% margin in the current.

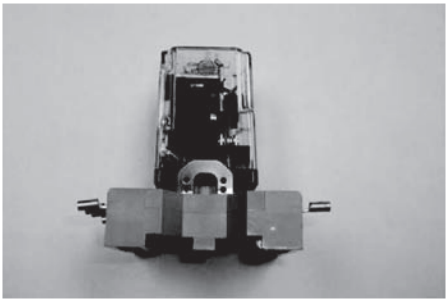
10.3 ELECTROMECHANICAL RELAY

Although not the protective device by itself, the electromechanical relay is an important part of the circuit breaker and various protective schemes in the electrical power system. The electromechanical relay detects the fault current and energizes the operating coil that opens the circuit breaker contacts. Relays are also used in many control circuits. They come in two types, (1) electromechanical relay for high-power circuits, and (2) power electronics (solid-state transistor) relay for low-power circuits.

The electromechanical relay has metal contacts that open or close in response to a signal current in the operating coil as shown in Figure 10.7. The coil (called the *solenoid*) wound around a fixed iron core attracts a movable iron (called the *armature*) when current passes through the coil. The spring force reverses this action when the coil current is removed. The relay is designed to offer an inverse *i-t* characteristic just like the fuse, that is, shorter opening time at higher current. Compared to the power



(a) Interior components



(b) Exterior view

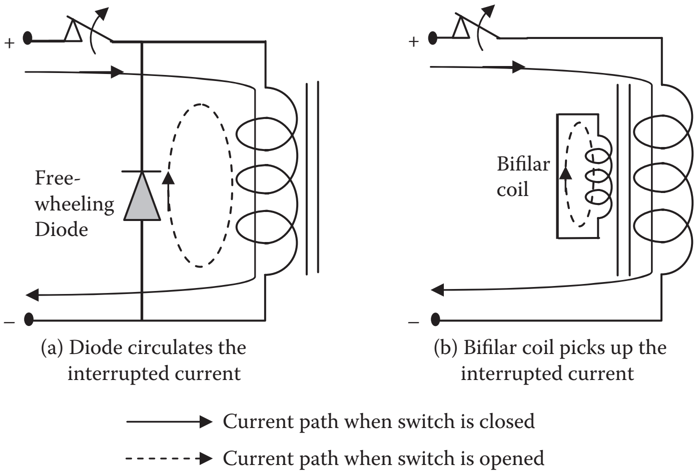
FIGURE 10.7 Electromechanical relay construction and exterior view with connectors.

electronics relay, the electromechanical relay offers high power handling capability and negligible power loss. In this book, we will consider only the electromechanical relays generally used in high power circuits.

Small relays used in control circuits come in holding or latching variety, and are typically rated for 24, 28, 35, etc., volts at various current ratings up to 30 A_{rms}. Contacts are designed such that the voltage drop at rated current is 0.1 to 0.2 V across good contacts at rated current. Large relays used in power circuits come in much higher voltage and current ratings.

When the relay contacts open, the inductive energy of the coil causes arcing until the energy is depleted. This results in much shorter contact life, electromagnetic interference, and fire hazard if a combustible vapor surrounds the relay. For safety, it is important to avoid arcing by absorbing and dissipating the inductive energy of the relay coil. Two schemes generally used for this purpose are described below.

Free-wheeling (arc-suppression) diode: In this scheme shown in Figure 10.8(a), a diode is connected in the reverse voltage bias in parallel with the coil. When the relay contacts are closed, only the relay coil carries the current along the solid line. But when the contacts are opened, dc current in the coil finds a continuing circulation path through the diode along the dotted ellipse. The current then decays



(a) Diode circulates the interrupted current

(b) Bifilar coil picks up the interrupted current

FIGURE 10.8 Relay contact arc-suppression schemes to absorb inductive energy after current interruption.

exponentially in five time constants of the R-L circuit formed by the coil inductance and resistance, which is 5 × L/R. The diode is called a *free-wheeling diode* as it carries the current only to dissipate the stored energy in the relay coil with no other purpose except to prevent arcing when the relay contacts open.

Bifilar (fly-back) shorted coil: In this scheme shown in Figure 10.8(b), the relay coil is wound with two identical wire filaments in parallel, one energizing the relay contacts, and the other shorted on itself. It is like a transformer with two coils, one energized with dc and one coil shorted. Under normal dc current in the relay coil, the shorted coil carries no current, as the dc flux cannot induce any current in it. However, when the relay coil contacts open, the sudden change in its falling current induces the current in the shorted coil equal to the inversed turn ratio, which is 1.0 for the bifilar coils. Therefore, the relay current essentially gets kicked-back or a fly-back to the shorted coil, and the total magnetic energy of the system is preserved. The shorted coil current eventually decays in five time constants of its own R-L circuit. This serves the same purpose as the free-wheeling diode in the above scheme, that is, to prevent arcing when the relay contacts open.

10.4 CIRCUIT BREAKER

The circuit breaker opens and closes electrical contacts automatically in response to an overcurrent, or manually when needed. It is made of various relays with operating coils and heavy electromechanical contacts that open under fault current. The main power contacts of HV circuit breaker, relay, and disconnect switch are usually silver-plated for stable performance over the service life. The circuit breaker current versus opening time characteristic is similar to that of the fuse, that is, higher currents get interrupted in shorter times in the inverse *i-t* relation.

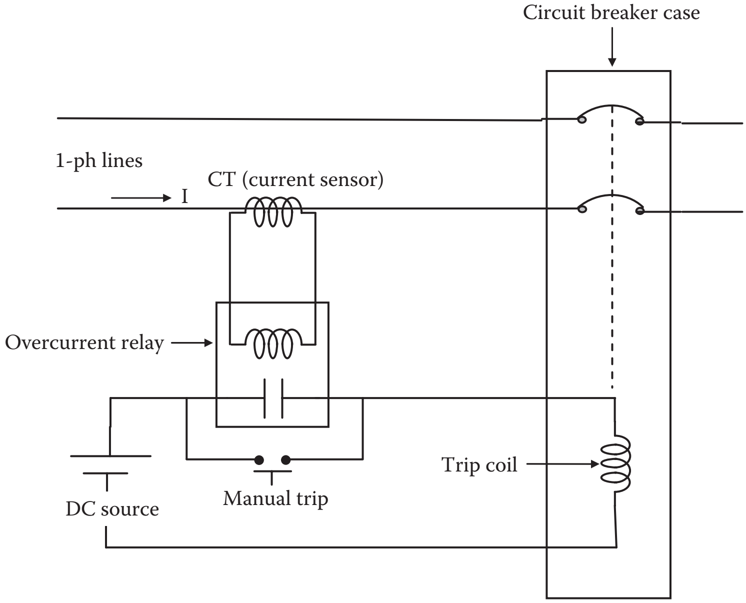


FIGURE 10.9 Circuit breaker scheme with overcurrent relay to open poles (contacts).

The fault is removed automatically by tripping the circuit breaker when the fault is detected by various current and voltage sensors (usually CTs and VTs) located at various locations strategically selected in the system. The abnormal current under a short circuit fault causes the protective relay to operate, activating the tripping circuit in the circuit breaker, eventually opening the circuit breaker contacts and clearing the fault. Figure 10.9 depicts a typical overcurrent protection scheme using a current transformer (sensor), overcurrent relay, and a circuit breaker. In case of a fault, the resulting high line current I is sensed by the CT, the output of which flows through the operating coil of the overcurrent protection relay in its loop, which in turn closes the normally open contacts in the relay. If the relay is a plunger type, it opens instantaneously; otherwise, opening occurs after an intentional time delay that can be adjusted by the system design engineer. Closing the relay contacts energizes the circuit breaker trip coil, which in turn opens the circuit breaker poles (contacts). The arc formed during the circuit breaker pole opening is blown away by blast air or transverse magnetic field between the poles. The arc extinguishes when the arc current comes to its natural zero on the sinusoidal cycle and the insulating property of the interpole gap is resorted after the ionized medium is blown away.

The manual trip push-button when pressed bypasses the overcurrent protection relay contacts and directly energizes the circuit breaker trip coil, opening the circuit breaker poles, even when there is no fault.

The desired time delay characteristic of the overcurrent relay is obtained in two ways: (1) by adjusting the relay spring force and/or the contact positions, and (2) by adjusting the relay coil taps. Various combinations of the two settings give a family

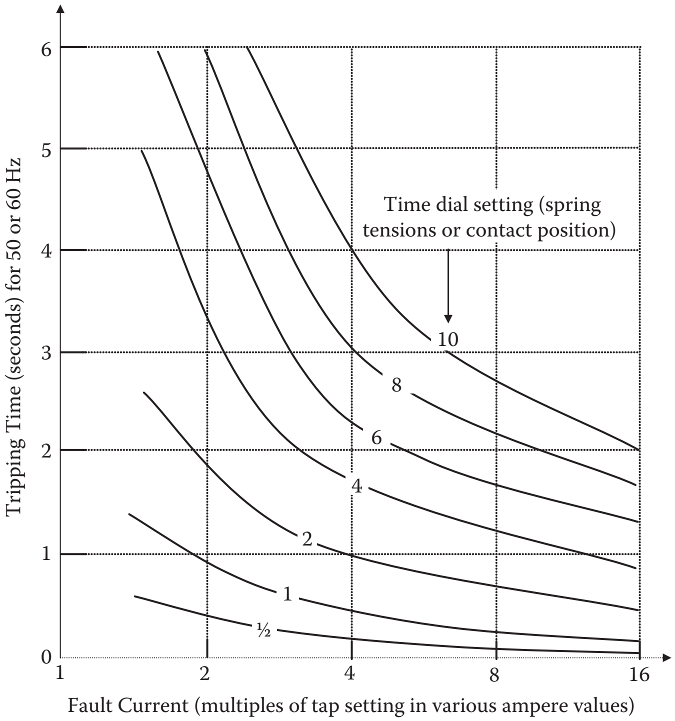


FIGURE 10.10 Two time-delay settings in circuit breaker gives a family if i-t curves to choose from.

of the time-delay characteristic shown in Figure 10.10. The trip settings of various circuit breakers in the system are coordinated such that the circuit breaker closest to the fault is tripped first by a relay circuit, keeping the rest of the system operating normally. In a chain of circuit breakers, the generator breaker should have the longest trip time setting for a given fault current.

10.4.1 Types of Circuit Breaker

Molded case dual-element circuit breaker: This type is generally used for small LV circuits. It is made of two tripping elements and contacts enclosed in a common molded plastic case (Figure 10.11). Its trip is initiated by the thermal element in time inversely proportional to I^2, or by the electromagnetic force, which trips the contact almost instantaneously. Thus, the overload current is tripped by the thermal element, and the fault current is tripped by the magnetic force. Therefore, it is also known as the dual-element thermal-magnetic circuit breaker. The thermal element trips when dissimilar bimetallic contacts are sufficiently heated to deform in a manner that opens the contacts. The magnetic element trips when the operating relay coil (solenoid) produces high electromechanical force on the actuator to open the contacts. A typical molded case circuit breaker has the current versus trip time characteristic (the inverse

FIGURE 10.11 Dual-element molded case circuit breaker.

i-t curve) shown in Figure 10.12. As in the fuse, many manufacturing variations are accounted for by giving a band of clearing time for a given fault current. The engineer must assure that the minimum and maximum values of trip time band meets the system requirements. Table 10.2 lists a few selected NEMA standard continuous current ratings of available molded case circuit breakers.

Air circuit breaker and air-blast circuit breaker: This type is suitable for medium voltages and is generally used indoors, where the fire risk associated with an oil-filled circuit breaker is not acceptable. The insulation and cooling medium between live conductors is air, which makes it economical but bulkier than other types. In the air-blast type circuit breaker, the arc at contacts during the current interruption is quickly blown away by a blast of air or magnetic flux.

Vacuum circuit breaker or pressurized gas-filled circuit breaker: The insulating medium between live conductors in this type of the circuit breaker is vacuum or pressurized gas (SF6 or nitrogen), which requires less spacing, making it compact. It is commonly used indoors or outdoors in high voltage (>35 kV) and high power distribution systems where space is at premium.

Oil-filled circuit breaker: It is widely used outdoors for high power distribution in land-based power grid and industrial plants in voltage ratings in hundreds of kV (Figure 10.13). Oil is one of the best insulation and cooling mediums, widely used in high voltage equipment. It is generally not used indoors to avoid a potential oil spill and fire hazard in case the oil tank ruptures following a fault.

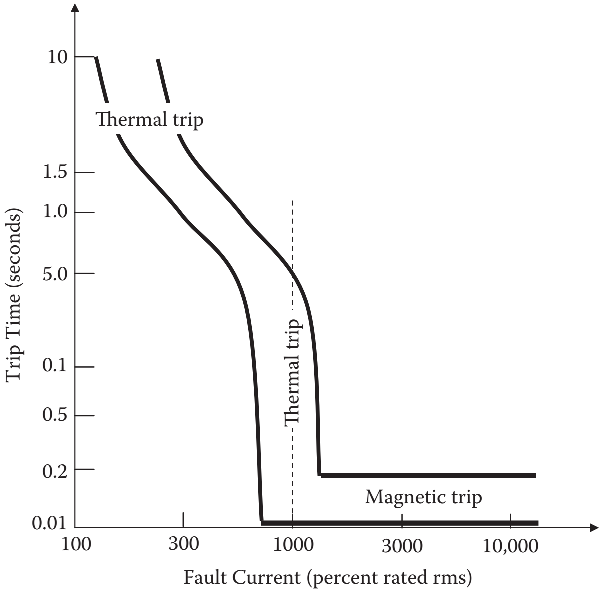


FIGURE 10.12 Current versus clearing time of dual-element molded case circuit breaker.

All of the above circuit breakers use mechanical contacts that arc and wear when opened to interrupt fault current. Power *electronics* circuit breakers with no mechanical contacts or moving parts are being developed for high power applications, and may become available in the near future.

The system with a high *X*/*R* ratio is hard on the circuit breaker operation, since it results in a high asymmetrical peak for the same symmetrical rms value of the fault current. When the current carrying contacts open, the magnetic energy stored in the system's leakage inductance keeps the current flowing until all the magnetic energy is diverted and/or dissipated in some form. The contacts continue arcing until the magnetic energy is depleted, or the current naturally comes to zero in its natural sinusoidal cycle in the ac system. The dc current is difficult to break because

TABLE 10.2
NEMA Standard Frame Sizes for Continuous Ampere Ratings of Molded Case Circuit Breakers

Frame Size (Partial List)[a]				
Frame 100	**Frame 200**	**Frame 400**	**Frame 800**	**Frame 1200**
15 A	125 A	200 A	300 A	700 A
30 A	175 A	250 A	400 A	1000 A
40 A	200 A	300 A	500 A	1200 A

[a] Many other ratings are available but not listed here.

FIGURE 10.13 HV oil circuit breaker in outdoor substation yard.

there is no natural sinusoidal zero in dc. The arcing in ac or dc must be minimized and diverted away from the contacts by blowing air at it, or by magnetic force, or by absorbing the arc energy in an insulating fluid such as oil or pressurized gas. After the current is interrupted, the voltage between the contacts rises to the full system voltage after some time, but momentarily to an even higher value up to twice the system voltage rating (the water-hammer effect) as shown in Figure 10.14. This is called the *recovery voltage*. At the peak of the recovery voltage, if the intercontact space is still ionized due to the just-extinguished arc, the contact may strike back and reestablish the fault. Adequate space between open contacts in the circuit breaker design is provided to avoid the arc striking back and reinitiating the fault.

The standard low voltage (<2 kV) circuit breaker specifications in various frames are listed in columns in Table 10.3. For example, the second column gives various specifications of the 4000 A circuit breaker that can be placed in line where the continuous operating current is 4000 A_{rms}, which can be set for momentary over-current from 4000 to 16,000 A for protection coordination. It can be used for rated system voltage from 800 to 1600 V line-to-line rms, with a corresponding decrease in the short circuit current ratings (both asymmetrical

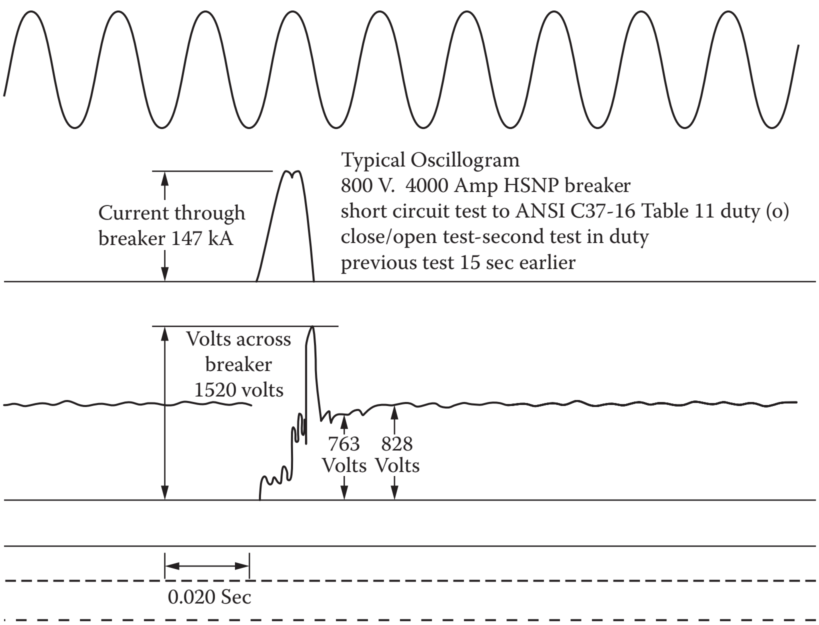


FIGURE 10.14 Voltage recovery and restriking voltage in circuit breaker following current interruption.

peak and sustained rms kA ratings). That is, the short circuit current interruption capability is higher at lower line voltage, and vice versa, as indicated by slashes in rows 3 through 6.

The standard ratings of indoor circuit breakers in the 4.76 kV to 38 kV range are shown in Table 10.4. These circuit breakers have some limited flexibility in use at somewhat higher voltage at a proportionally reduced current within their own *K*-factor (not to be confused with the transformer *K*-rating). For example, if the line voltage is higher by a factor of $K \leq 1.3$, then these two kA ratings must be lowered by a factor $1/K$.

TABLE 10.3
Typically Standard Circuit Breaker Specification for <2 kV Lines

Continuous Current Rating Maximum (A_{rms})	4000	6000	8000	10,000	12,000
Overload current setting range (kA)	4–16 kA	6–24 kA	8–32 kA	10–40	12.48
Voltage rating (V_{rms})	800/1600	800/1600	800/1600	800/–	800/–
Short circuit rating peak (kA prospective)	200/100	200/100	200/100	160/–	135/–
Sustained (kA)	120/60	120/60	120/60	100/60	120/60
Short time withstand current ratings—250 ms	132/132	132/132	159/159	159/–	159/–
Operating coils (close and trip coils) control voltage (V_{dc})	125	125	125	125	125
Close coil current average at 125 V_{dc} (A)	38	38	38	38	38
Trip coil current average at 125 V_{dc} (A)	2	2	2	2	2

TABLE 10.4
Standard Ratings of Indoor Circuit Breaker in 4.76 kV–38 kV Range

Rated System Voltage kV_{LLrms}	Rated Continuous Current kA_{rms}	Rated Symmetrical Fault Current Interrupt kA_{rms}	Rated Symmetrical Fault Current Interrupt MVA_{rms} (within K Factor)[a]
4.76	1.2	8.8	72.5
4.76	1.2, 2.0	29	240
4.76	1.2, 2.0, 3.0	41	340
8.25	1.2, 2.0	33	470
15	1.2, 2.0	18	470
15	1.2, 2.0	28	730
15	1.2, 2.0, 3.0	37	960
38	1.2, 2.0, 3.0	21	1380

[a] This column equals $\sqrt{3}\ kV_{LLrms} \times kA_{sym.fault.rms}$ using column 1 and 3 data. The *K*-factor of the circuit breaker, typically 1.3, gives the range of applying lower voltage up to factor $1/K$ and increase the fault interrupt capacity by factor *K*, such that the fault MVA capacity remains the same as listed in this column.

10.4.2 CIRCUIT BREAKER SELECTION

The circuit breaker has several ratings, such as

- A rated continuous current that it carries within an allowable temperature rise,
- A rated line voltage that it supports with adequate insulation to the ground,
- A symmetrical short circuit MVA or fault current that it can interrupt without thermal damage,
- An interrupting (tripping) time, which is typically between 5 and 15 cycles, and can be adjusted for coordination with other downstream and upstream circuit breakers,
- A switching overvoltage withstand capability without restriking the contacts after tripping,
- A lightening impulse voltage withstand capability,
- The first asymmetrical peak current withstand capability without mechanical damage to itself.

The engineer must consider all ratings and factors listed here to select the proper circuit breaker for the application at hand. However, the key considerations in selecting the circuit breaker are:

- The continuous current rating must be greater than the maximum load current it must carry.
- The current interrupt capability must be greater than the maximum fault current it could see (as calculated in Chapter 9).
- Circuit breaker voltage rating must be greater than the operating line voltage.

The required current interrupting capability of the circuit breakers can be determined as follows, where all quantities are per phase and in per unit (pu) on a common MVA and kV base with the maximum generator capacity connected. First, consider a 3-phase fault current, which is given by $I_{sym.rms} = E_{gen}/X_1$. Then, consider a 1-phase *L*-*G* fault current with solid ground, which is given by $I_{LG.rms} = 3\ E_{gen}/(2\ X_1 + X_0)$, where E_{gen} = prefault generator-induced voltage per phase (also known as E_f), X_1 = total positive sequence reactance from the generator to the fault location, and X_0 = total zero sequence reactance from the generator to the fault location. In these totals, use the subtransient reactance for synchronous generator. For selecting the circuit breakers interrupting capability, use the greater of the two fault current values, $I_{sym.rms}$ or $I_{LG.rms.}$

There are different settings on the circuit breaker that the engineer can effectively use for proper protection. Following is an example of choosing and setting various circuit breaker ratings for protecting a ship service generator with rated current of 1600 A:

- Continuous rated current 1600 A
- Interruption capability 60 kA symmetrical rms
- Long time delay: 10–30 sec at 225% of rated current (3600 A). This is adjusted for protection from an overload that is not high enough to trip the circuit breaker.
- Short time delay: 2–20 cycles in inverse relation with current at 4–10 kA. This is higher than the overload range but not high enough to cause an instantaneous trip.
- Instantaneous trip: At 48–60 kA. This is to give protection from a dead short at the generator terminals or close to it in the systems.

Example 10.2

The 8.25 kV circuit breaker in Table 10.4 can be used for rated fault current interruption of 33 kA at rated 8.25 kV_{LL} system voltage. Determine the range of system voltage and interruption current it can be used for.

SOLUTION

The circuit breaker selection is usually facilitated by rating the circuit breaker by its MVA capability that is $\sqrt{3} \times 33\ kA \times 8.25\ kV = 470$ MVA for this circuit breaker. It is called the *short circuit MVA capability of the circuit breaker*, as listed in the last column of Table 10.4. The MVA capability can be split between the current and voltage within the factor of 1.3 (not any higher). Therefore, this circuit breaker can be used for a maximum of 33 × 1.3 = 43 kA at system voltage of 8.25 ÷ 1.3 = 6.35 kV, giving the fault current interruption range between 33 kA and 43 kA and the corresponding system voltage range between 8.25 kV and 6.35 kV, respectively, such that the product of kV and kA remains the same 470 MVA.

∴ This circuit breaker can be used for 33 kA interruption at 8.25 kV, or 43 kA interruption at 6.35 kV, and other current and voltage ratings in-between.

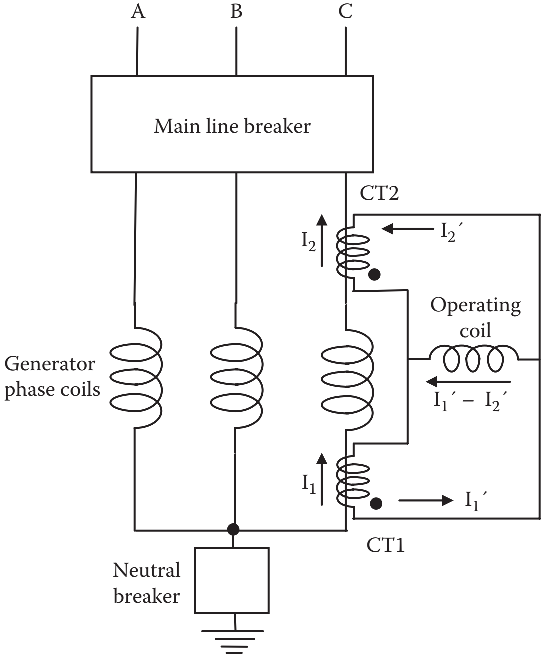


FIGURE 10.15 Differential protection scheme for internal fault in generator stator winding.

10.5 DIFFERENTIAL PROTECTION OF GENERATOR

The generator is protected from external faults by a circuit breaker at its output terminals. A fault internal to the generator may not change the external line current to trip the circuit breaker, but can damage the machine due to internal heating. A widely used scheme for protecting the generator from internal faults—applicable to transformer bank and bus bars, also—is depicted in Figure 10.15. It shows the protection scheme for phase C only, but the same scheme is applied on phases A and B as well. With no internal fault, the currents I_1 and I_2 are equal in magnitude and in phase; the differential current $I_1' - I_2'$ in the relay operating coil is zero, and so is the trip coil current. However, when there is an internal fault involving phase C, $I_1 \neq I_2$ and the differential current $I_1' - I_2'$ flows in the operating coil. This energizes the trip coil, leading to opening the circuit breaker poles. The CTs must be identical for this scheme to work satisfactorily, or else the minimum pick up current that will close the relay contacts must be set as a percentage of the generator rated current so that a small imbalance in the CTs will not cause nuisance tripping.

10.6 DIFFERENTIAL PROTECTION OF BUS AND FEEDERS

The differential protection scheme for protecting the bus and feeders is depicted in a one-line diagram of Figure 10.16. In a 3-phase implementation, three differential relays are required, one in each phase. All CTs used in such scheme must be

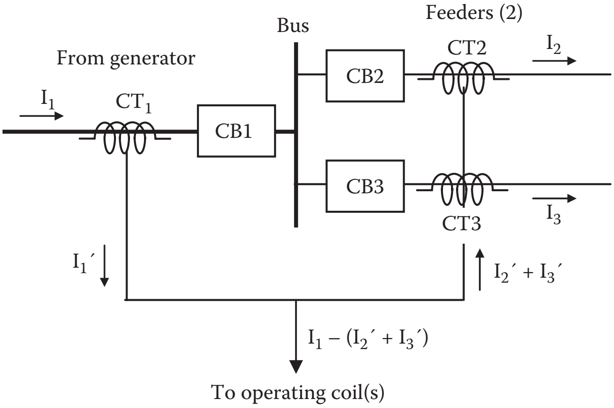


FIGURE 10.16 Differential protection scheme for bus and feeders.

identical. The CT2 and CT3 are in parallel, adding their output currents. If there is no fault on the bus, the feeder currents $I_2 + I_3 = I_1$, and so does the outputs of the CTs, that is, $I_1' = I_2' + I_3'$. The polarities of the CTs windings are selected such that the relay operating coil draws the differential current $I_1' - (I_2' + I_3')$, which is zero, and the differential relay does not operate when there is no bus fault. In case of a bus fault, the operating coil would see the differential current, activating the trip coil and opening the circuit breaker poles. The scheme can be extended to a large numbers of feeders from a common bus, although some care is needed to match the CT outputs to avoid nuisance tripping.

10.7 GROUND FAULT CURRENT INTERRUPTER

Like the previous two schemes, the ground fault current interrupter (GFCI) is also a differential protection scheme. A 1-phase GFCI widely used in power distribution systems is shown in Figure 10.17. The feeder cable goes through a current sensor—a

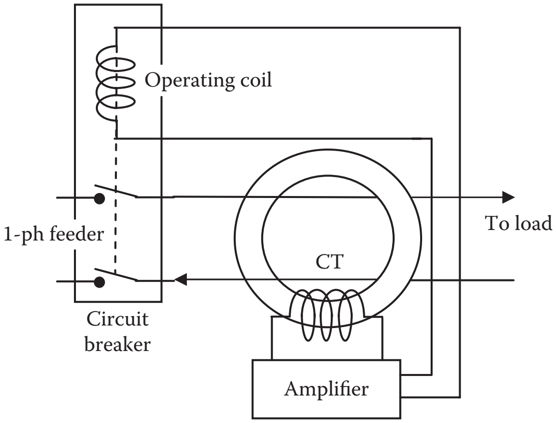


FIGURE 10.17 Ground fault current interrupter (GFCI) working principle.

current transformer (CT) similar to a clamp-on ammeter—that measures the net current difference in two wires going through the eye of the CT sensor. Under healthy operation, the lead and return current are equal, and the sensor output is zero. A short in one line conductor to ground even via a soft fault (high impedance fault) results in higher than rated current in the lead wire with no change in the return current. The net unbalance current is picked up by the sensor, which is usually very small due to high CT turn ratio. It is, therefore, amplified and then sent to the actuator coil of electromagnetic relay to open the circuit breaker.

The NEC requires that all 120 V 1-phase outlets installed outdoors or in bathrooms and kitchens have the GFCI. Industrial power systems must also use such GFCI in 600 V Y-connected system in each distribution panel board.

10.8 TRANSFORMER PROTECTION

The transformer is generally protected on both primary and secondary sides by fuse or circuit breaker as shown in Figure 10.18. The continuous current rating of the breaker is selected as follows.

On the secondary side, general purpose fuse or
CB2 rating = 1.3 × rated secondary current.

On the primary side, because we must allow for the magnetizing inrush current on the switch-on, a time-delay fuse or circuit breaker may be used. Often, the time-delay circuit breaker is not available in small sizes. Then a larger general purpose circuit breaker or fuse must be used. Thus, the two options are,

a time-delay fuse or CB1 rating = 1.3 × the rated primary current, or

a general purpose fuse or CB1 rating = 2 × 1.3 × rated primary current.

Example 10.3

Determine the continuous current rating and the rms fault current interruption rating of circuit breakers or fuses on both sides of a 100 kVA, 480VΔ/208VY, 3-phase transformer that has 5% impedance.

SOLUTION

First, we determine the continuous line currents and then the circuit breaker or fuse ratings on both sides (all current below are continuous current rms values).

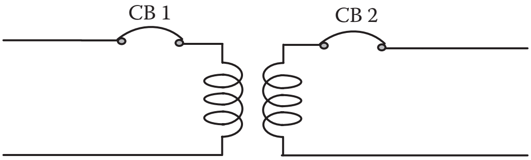


FIGURE 10.18 Circuit breaker protection on both sides of transformer.

Secondary side rated line current = 100 × 1000 ÷ √3 × 208 = 277.6 A
General purpose CB or fuse rating on secondary side = 1.3 × 277.6 = 361 A
Primary side rated line current = 100 × 1000 ÷ √3 × 480 = 120.3 A
General purpose CB or fuse rating on primary side = 2* × 1.3 × 120.3 = 313 A
Or, time-delay CB or fuse rating on primary side = 1.3 × 120.3 = 156.4 A

* Factor 2 in the general purpose circuit breaker or fuse rating on the primary side is to avoid nuisance tripping of the circuit under the magnetizing inrush current when the transformer is first connected to the supply lines. Time-delay circuit breaker or fuse on the secondary side is not required, because it does not see the magnetizing inrush current. The magnetizing current flows only in the primary side lines on the instant the transformer connecting to the supply lines.

Now, the symmetrical fault current on both sides = 100% V ÷ 5% Z = 20 pu
Fault current interruption rating of secondary side CB or fuse = 20 × 277.6 = 5552 A
Fault current interruption rating of primary side CB or fuse = 20 × 120.3 = 2406 A

10.9 MOTOR BRANCH CIRCUIT PROTECTION

The three types of fuse used to protect small motors are illustrated for a 10 hp motor example in Table 10.5. It shows that for a motor with full load current of 30 A, the continuous current rating requires a fuse rating of 90 A for a standard fuse to allow for the starting inrush current, or 35 A for a time-delay or current limiting fuse. If the prospective fault current is 5000 A_{rms} symmetrical, the standard and time-delay fuses will let through 5000 A peak before interrupting, whereas the current-limiting fuse would let through only 2500 A (one-half as much), thus reducing the mechanical force and heating by a factor of four.

The motor branch circuit is protected by a fuse or a circuit breaker of proper rating in relation to its starting kVA code letter to avoid tripping under starting inrush current. Typical maximum rating of these devices are shown in Table 10.6. A motor with the starting kVA code A requires a fuse rating of 300% of the full load current (FLA) with standard fuse, but only 150% of FLA if a time-delay fuse is used. If a molded case circuit breaker is used, then its instantaneous magnetic trip should be set to 700% of FLA, and the inverse time thermal trip should be set at 150% of FLA. The table is applicable to all sizes of motors, as the current rating is stated in terms of the percentage of the motor full load ampere rating.

TABLE 10.5
Fuse Type Used to Protect 10 hp Motor with Full Load Current of 30 A

Fuse Type	Fuse Rating Needed	Let-Through Current in Circuit with Prospective Fault Current (rms Symmetrical)		
		3000 A	**4000 A**	**5000 A**
Standard design	90 A to allow for starting inrush amps	3000 A	4000 A	5000 A
Time-delay	35 A	3000 A	4000 A	5000 A
Current-limiting	35 A	1700 A	2000 A	2500 A

TABLE 10.6
Maximum Rating or Setting of Fuse or Circuit Breaker in 1-phase or 3-phase Induction or Synchronous Motors Motor Branch Circuit with Full Voltage or Reduced Voltage Starting

Motor Starting kVA Code Letter for AC Motors	Percent of Full Load Amperes			
	Fuse rating		Circuit breaker setting	
	Normal type	Time-delay type	Instantaneous trip	Inverse time trip
Code A motors	300	150	700	150
Code B to F	300	250	700	200
Codes F to V	300	250	700	250
Dc motors	200	150	250	150

10.10 LIGHTNING AND SWITCHING VOLTAGE PROTECTION

The lightning voltage can enter the ship via shore connection or outdoor cables running on the deck. Most insulations fail during one of the following events:

Switching overvoltage due to the inductive and capacitive energy of the system overshooting the system voltage that can be as high as 2 × rated voltage. This is analogous to overpressure due to *water hammer* when a water pipe is suddenly shut off (switched-off).

Lightning overvoltage of magnitude $I \times Z_o$, where I is a fast-rising lightning current wave entering the system via cable with impedance Z_o, known by three names—the characteristic impedance, wave impedance, or the surge impedance of the cable. The lightning voltage entering the equipment can vary from 15 kV to 1500 kV and may last for about 100 μs.

When a switching or lightning overvoltage occurs in the system that exceeds the insulation strength, the insulation between two live conductors or between one line and the ground can break down in three ways (Figure 10.19):

Puncture through solid insulation, which can otherwise withstand the voltage stress of 300–500 V/mil between the conductors (1 mil = 1/1000 in. = 25.4 μm).

Strike through air, oil, or gas insulating medium, which can otherwise withstand the voltage stress of 100 to 200 V/mil. Striking an arc through air between HV conductor and grounded metal enclosure is an example of such failure.

Creepage (flashover) along the insulation surface, which can start at about 20 V_{rms}/mil for 60 Hz voltage along a clean surface in air, and at about 10 V_{rms}/mil along a dusty or greasy surface.

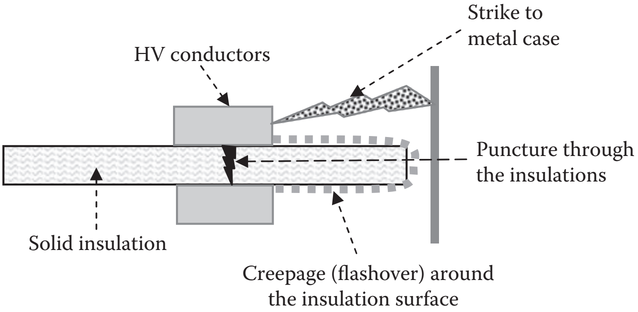


FIGURE 10.19 Three mechanisms of electrical insulation breakdown.

Since the creepage strength of the insulation surface is the least, special insulator stand-offs are used to increase the creepage length between the HV conductor and the ground. Figure 10.20 shows such curvy-shaped porcelain insulator stand-offs between HV lines and grounded metal parts (structure or enclosure). Strings of similar porcelain insulator disks are used to hang HV lines on from the transmission towers we often see along highways.

Under normal operating voltage, corona degrades the insulation. Solid void-free insulation typically used in dry type electrical equipments can withstand about 80 V_{rms}/mil (2 kV/mm) at 60 Hz for 20–30 years. However, corona (partial discharge) may start at 25 V_{rms}/mil in voids left in the insulation manufacturing process. For this

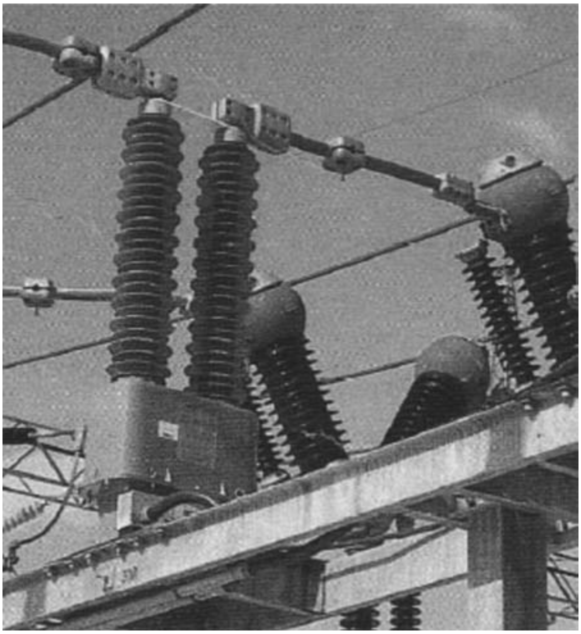
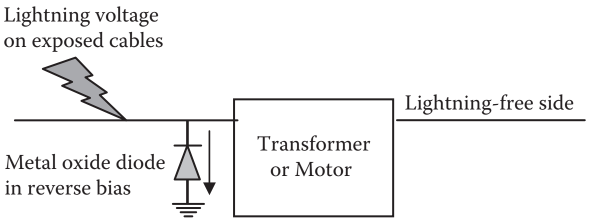


FIGURE 10.20 Insulator stand-offs provide longer creepage distance from HV lines to ground.

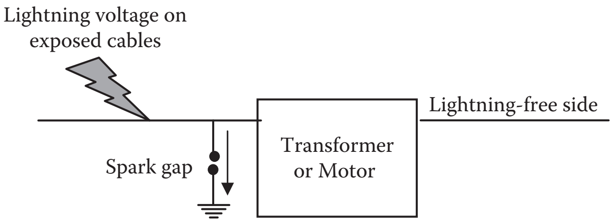
reason, 20 V_{rms}/mil is a good working design limit in most solid insulations with the dialectic constant between 3 and 5.

The lightning current can enter the equipment via incoming line, resulting in overvoltage that can cause damage, particularly at the line connection end of the equipment. The International Standard IEC-1022.1 covers the requirements for protecting equipments against lightning. The generator, transformer, and motor are designed with a certain minimum basic impulse level (BIL) that the equipment can withstand under lightning and switching type transient voltages consistent with the lightning risk. The risk is proportional to the number of thunderstorms per year in the area. Electrical equipment is further protected from lightning damage by placing a lightning arrester in any one of the three alternative configurations shown in Figure 10.21.

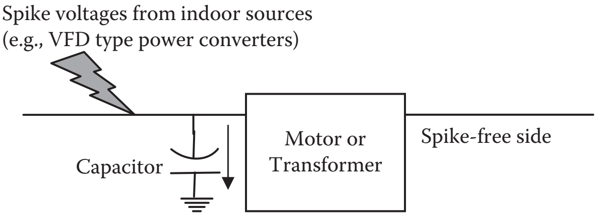
a. *Metal oxide*: Functionally, the arrester works like a diode in reverse, which breaks down at lightning voltages. They are made from lead peroxide or Thyrite® hockey-puck shape pellets stacked with series gaps. It consists of a



(a) Metal oxide diode at reverse breakdown diverts lightning current to ground



(b) Spark gap diverts lightning current to ground and limits the voltage



(c) Capacitor provides low impedance path to ground for high frequency spike voltages

FIGURE 10.21 Lightning arrestor alternatives for protecting electrical equipment from lightning.

column of diode type lead peroxide pellets with small series gaps assembled in a porcelain tube placed before the equipment as shown in Figure 10.21(a). The semiconducting pellet material has highly nonlinear resistance that behaves like a diode in the reverse bias. The series gap isolates the diode elements from the line voltage until sparked over by a lightning impulse that breaks down the reverse biased diodes. Arresters use a zinc oxide and petticoat insulator standoff. The arrester material does not get damaged after the breakdown. It recovers its normal voltage blocking property once the lightning voltage is diverted to the ground. Therefore, it does not need to be replaced after every lightning strike. It does, however, wear out after a certain number of lightning hits, and may need to be replaced periodically after several years depending on the type of the material used. The arrester must have proper voltage rating. Too high a rating will not protect the equipment, and too low a rating will cause damage to the arrester itself.

b. *Spark-gap:* A spark-gap before the equipment as shown in (b) is another way of providing lightning protection. The air in the spark gap under normal operation keeps the ground current zero, but it breaks down under high lightning voltage, diverting the lightning current to the ground. The air gap recovers its blocking property once it cools down after a strike, but may need replacement after numerous lightning strikes causing surface erosion.
c. *Capacitor:* The best protection for electrical equipment can be achieved by a combination of the lightning arrester and the capacitor placed as close as possible to the equipment as shown in Figure 10.21(c). Such a protection scheme is particularly used for motors, where the pf improvements capacitors are also normally used. The lightning voltage that passes onward beyond the lightning arrester reaches the motor and is absorbed by the capacitor. The low impedance of the capacitor for virtually high frequency lightning voltage smoothes out the steep front part of the voltage surge as it enters the stator winding. The capacitor's approximate ratings found satisfactory in most cases are: 1 μF/phase for system voltage 600 V or less, 0.5 μF/phase for $600 < V < 6.6$ kV, and 0.25 μF/phase for 6.6 kV to 13 kV systems.

In addition to the lightning protection outlined above, a tall metal rod is used to create a lightning protection zone around the area to be protected, including on ship. Such a protection zone has a conical volume (Figure 10.22), starting from the tip of the rod, and extending to the floor radius that depends on the rod height and the polarity of lightning strike.

10.11 SURGE PROTECTION FOR SMALL SENSITIVE LOADS

The surge protector (suppressor) protects small sensitive loads from damaging voltage surges (spikes). The voltage spikes can come from the utility power lines or generated from local sources. For example, a large fuse-blow event in the neighborhood can generate large voltage spikes along the power lines, damaging the equipment that is plugged into nearby ac outlets. The surge suppressor protects the connected

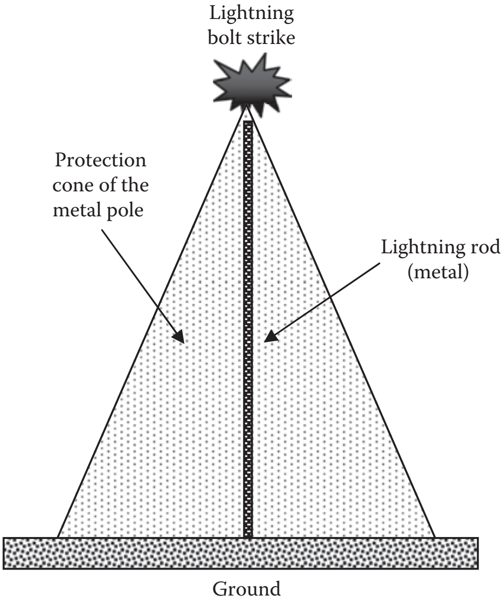


FIGURE 10.22 Protection cone of lightning rod.

loads by diverting the voltage spikes to ground through the outlet's ground wire. For this reason, running a surge suppressor from an ungrounded outlet prevents the surge suppressor from providing such protection.

The surge suppressor is connected in front of the load. It typically contains a diode-type material in the reverse bias to the ground. It blocks the ground current under rated line voltage. However, if the line voltage exceeds the diode breakdown voltage, it becomes conductive and diverts the voltage spike to the ground. The highest voltage the load equipment would, therefore, see is the reverse break down voltage. The diode is designed to absorb the generated heat in joules without thermal failure. Once cooled down, its heals itself, that is, it recovers the reverse voltage blocking capability. In commercially available surge suppressors, the key components used to reduce or limit the high voltage spikes may include one or more of the following:

Metal oxide varistor (MOV) is the most common protection component, which is typically granular zinc oxide that conducts current only under a voltage above its rated breakdown voltage. It typically limits (clamps) voltage to about three to four times the normal circuit voltage. It degrades in performance when exposed to repeated spikes, hence has a finite life. MOVs may be connected in parallel to increase current capability and life expectancy, providing they are matched sets (MOVs have a tolerance of approximately 20% on voltage ratings). MOVs usually are thermal-fused to avoid short circuits and other fire hazards. A failure light indicates a blown thermal fuse on some models. However, even an adequately sized MOV protection eventually degrades beyond acceptable limits without a failure light indication.

The Zener diode, also known as an avalanche diode, is another clamping semiconductor similar to the MOV. It provides the best limiting action of protective components, but has a lower current capability. Spike voltage can be limited to less than two times the normal operation voltage. Its life expectancy is long if current impulses remain within the device rating. The zener diode may fail short if the rating is exceeded. Since it does not degrade with use, it is often used where spikes are more frequent.

Selenium voltage suppressor is used to limit voltage spikes mostly in high-energy dc circuits such as in the exciter field of an ac generator. It can absorb high energy, but does not clamp well. However, it has a longer life than MOV.

There are many rating systems that measure the surge protection:

Joule rating: It is a good parameter for gauging the surge suppressor's ability to absorb surge energy. Since surges commonly occur for microseconds or shorter durations, the energy is typically under 100 J. A well designed surge protector does not rely on absorbing the surge energy, but more on surviving the process of redirecting it. It fails gracefully in reverse bias to divert most of the surge energy to ground, thus sacrificing itself, to protect the equipment plugged to it. In general, 200 J rating gives basic protection and 400 J surge suppressor provides a good protection to most small computer-type equipment. The ampere rating is not a measure of the surge suppressor ability to protect the load.

Clamping voltage: It specifies the voltage at which the metal oxide varistor inside the suppressor breaks down and diverts the surge to the ground. A lower clamping voltage, also known as the *let-through voltage*, indicates better protection, but a shorter life expectancy. The lowest three levels of protection defined in the UL rating are 330 V (suitable for most 120 V ac loads), 400 V, and 500 V.

Noise filter: Most surge suppressors also include high frequency special EMI suppressor (electrical noise filter). The EMI suppression is stated as decibel level (Db) at a specific frequency (kHz or MHz). The higher Db rating provides greater attenuation of the incoming noise.

Response time: A short time delay of several nanoseconds exists before the MOV breaks down in reverse. However, surge voltages usually take a few microseconds to ramp up and reach their peak values. The surge suppressor with a nanosecond delay responds fast enough to suppress the most damaging portion of the spike.

ABS-approved surge suppressors: For shipboard applications, the ABS rules specifically recommend the surge suppressors typically made by Brooks Power Systems, HFS Inc., DSK Inc., and EFI Electronics Corp. All shipboard surge protectors must interrupt both line leads to prevent equipment damage during a surge.

10.12 PROTECTION COORDINATION

All protection devices (circuit breakers, fuses, etc.) must be properly coordinated. It means that, under a fault downstream, an upstream protection device with higher rated load current must interrupt later in time than all downstream devices.

Such coordination avoids an upstream feeder from losing power before the downstream feeder. It can be achieved by choosing properly rated fuses and circuit breakers with increasing current versus trip times as we move upstream of the power flow. Recall that the circuit breaker trip time for a given fault current can be adjusted by the user within its design range. Another system design coordination applies to the ring bus or tie sections. Since the power can come from two sides of the fault, the fault must be islanded (isolated or sectionalized) from both sides from where power can flow into the fault.

10.13 HEALTH MONITORING

Health monitoring involves detecting abnormal system operation, which is not a short circuit or other fault that requires tripping the circuit, but degrades the system performance. When such an abnormal condition is detected, an alarm is raised—rather than the circuit interrupted. For example, a heath monitoring system for a large capacitor bank installed for power factor improvement is shown in Figure 10.23, where each phase of the grounded Y may use multiple capacitor units in parallel. Each capacitor unit is usually fused to clear an internal fault. With all capacitors healthy, the current in all capacitor phases will be balanced with equal magnitudes and 120° out of phase, with their phasor sum equal to zero, and the ground current CT sensor will register zero. However, when one capacitor fuse melts, the current drawn by that phase capacitor becomes zero and the other two phase currents add to the phasor sum that will have a nonzero value in the neutral. This current is sensed by the CT and fed to a light and/or an alarm signal, while the motor continues working normally with less improved pf.

In an ungrounded system, the Y-connected capacitors will have neutral floating. With all capacitors healthy, the insulated neutral to ground voltage will be zero because of the balanced operation. However, with one capacitor fuse blown, the

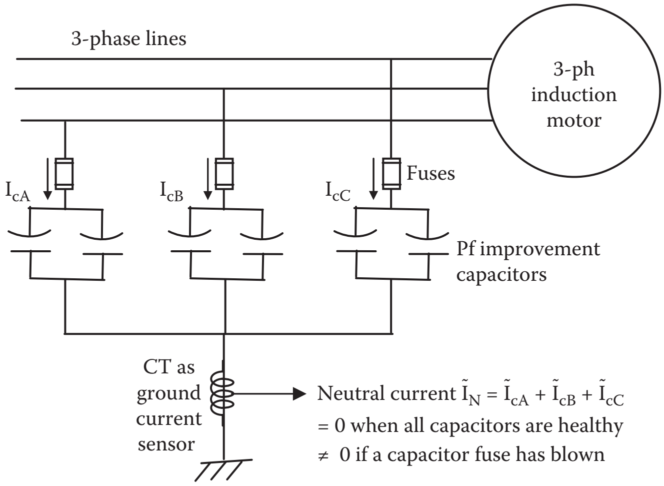


FIGURE 10.23 Health monitoring by neutral ground current sensor.

unbalanced voltages will cause the neutral-to-ground voltage to be nonzero, which is then detected by VT and fed to the alarm circuit.

Other protections that are generally implemented in most systems are:

- Thermal protection against blockage of cooling air that is done by temperature sensors embedded in the stator slots
- Under voltage in case of a brown-out or single-phasing of the lines
- Reverse power protection to keep parallel generators from motoring and overloading other generators

10.14 ARC FLASH ANALYSIS

The electrical power system engineer designs a system to protect itself. However, not much attention has been paid in the past for personnel safety, such as arc flash protection. With medical costs at over a million dollar per person injured by arc flash burn, the emerging industry standards now reflect the increasing importance of conducting the arc flash analysis, such as IEEE-1584 for performing arc flash hazard calculations, NFPA-70(NEC) that governs electrical installations, NFPA-70B(NEC) that governs the maintenance of electrical equipment, etc.

The system protection analysis and the circuit breaker and fuse selection are based on the bolted fault (hard faults with ground fault impedance equal to zero). However, most faults start as soft L-N faults with arcing, which turn into 3-phase faults. The arcing generates heat that radiates outside and may injure personnel in the proximity of the faulted equipment. The risk of injury is proportional to the energy released by arc. Arc flash analysis predicts the amount of the thermal energy in calories per cm^2 area following the worst-case faults, leading to selections of the protective equipment and flash protection boundaries for personnel working around the equipment as listed in Table 10.7.

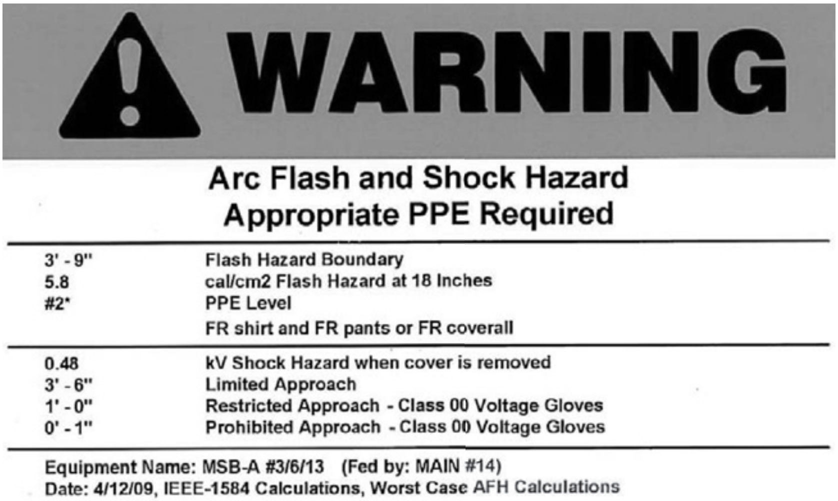
Arc flash hazard comes from many factors, such as short circuits due to creepage along the built-up conductive dust or corrosion, accidental shorts due to working

TABLE 10.7
Personal Protection Requirement for Various Arc Flash Energy Density in cal/cm^2 (Adapted from NFPA-70E)

NFPA-70E 2004 Equipment Requirements (Proposed)

Category	Energy level	Typical personal protective equipment required (NFPA-70E)
0	2 cal/cm^2	Non-melting flammable materials
1	4 cal/cm^2	Fire Resistant (FR) shirt and FR pants
2	8 cal/cm^2	FR shirt, FR pants, cotton underwear
3	25 cal/cm^2	Two layers FR clothing, cotton underwear
4	40 cal/cm^2	FR shirt, FR pants, multilayer flash suit, cotton underwear

Note: Other equipment includes face protection face shield and/or safety glasses; hand protection leather over rubber for arc flash protection; and leather work boots above 4 cal/cm^2.



WARNING

Arc Flash and Shock Hazard
Appropriate PPE Required

3' - 9"	Flash Hazard Boundary
5.8	cal/cm2 Flash Hazard at 18 Inches
#2*	PPE Level
	FR shirt and FR pants or FR coverall

0.48	kV Shock Hazard when cover is removed
3' - 6"	Limited Approach
1' - 0"	Restricted Approach - Class 00 Voltage Gloves
0' - 1"	Prohibited Approach - Class 00 Voltage Gloves

Equipment Name: MSB-A #3/6/13 (Fed by: MAIN #14)
Date: 4/12/09, IEEE-1584 Calculations, Worst Case AFH Calculations

FIGURE 10.24 Typical arc flash and hazard warning label (with permission from EasyPower®, ESA).

with a screw driver or dropping a tool, and improper work procedures. Although live work is discouraged even by qualified workers, work is often done on live equipment to minimize the down-time cost. For this reason, it is mandated by NFPA that arc flash studies be conducted and warning labels be placed on equipment with flash hazard boundary for carrying out unavoidable work around the energized equipment. Such studies must account for the worst cases in all possible system configurations and conditions. The system modifications and addition since the equipment was first installed must be taken into account. The arc flash studies are often carried out using commercially available software, such as Easy Power®, which quantify the following on the arc flash warning labels (Figure 10.24):

- Incident energy in calories per cm^2 at various working distances
- Flash hazard boundaries for limited approach, restricted approach, and prohibited approach
- Personal protection equipment (PPE) class for workers to wear doing live work, such as wearing voltage rated gloves, cotton underwear with fire resistant (FR) shirt and FR pants, and using properly selected tools
- Shock hazard in kV when the cover is removed for live work

No standards or recommendations exist for arc flash study in dc systems, which is difficult and not well understood at present. However, the dc in today's power system is generally derived by ac-dc converter (rectifier), such as in dc motor drives and steel mills. In such cases, the arc flash hazard is analyzed on the ac side of the rectifier. It is then assumed that the dc side risk is the same as that on the ac side, as the rectifier merely processes the power without bucking or boosting the system energy. This assumption of the equal hazard on the ac and dc sides may not be accurate, and may require more than usual margin in implementing the results.

PROBLEMS

Problem 10.1: Determine the time-delay fuse rating for a 3-phase, 208 V, 5-hp motor branch circuit in the engine room, which has an ambient air temperature of 55°C. Assume the motor efficiency of 92% and power factor 80%.

Problem 10.2: The 4.76 kV circuit breaker in Table 10.4 can be used for a rated fault current interruption of 41 kA at rated 4.76 kV_{LL} system voltage. Determine the range of system voltage and interruption current it can be used for.

Problem 10.3: Determine the continuous current rating and the rms fault current interruption rating of circuit breaker or fuse on both sides of a 300 kVA, 600VΔ/208VY, 3-phase transformer that has 6% impedance.

Problem 10.4: A 3-phase, 1000 kVA, 480 V, Y-connected generator with ungrounded neutral has armature resistance R_a = 1% and subtransient reactance X_d'' = 10%. It feeds (Figure P10.4) a 3-phase, 75 kVA, 480/208 V transformer with R = 1% and X = 5%. The cable has R = 0.08 Ω and X = 0.026 Ω per phase per 1000 ft. For a L-L-L fault with no ground involved, 150 ft away from the transformer terminals, determine the symmetrical rms value and the first offset peak of the fault current in 150 ft cable. Repeat the calculations to determine the circuit breaker fault current rating on the 75 kVA transformer secondary terminals.

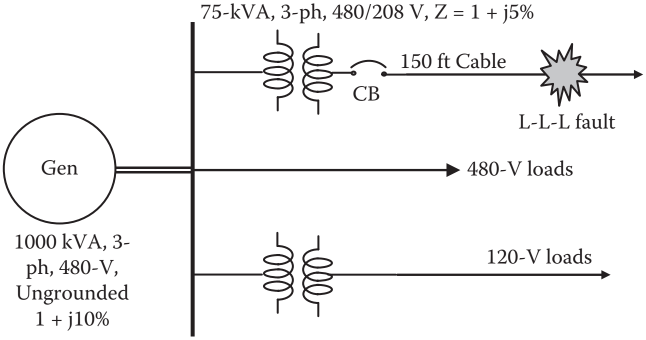


FIGURE P10.4

Problem 10.5: In a health monitoring scheme for power factor correction capacitors shown in Figure 10.23, the three capacitor banks in phase A, B, and C draw 12 Arms each when healthy. Determine the neutral current magnitude (i) when all capacitors are healthy, and (ii) when one capacitor bank gets an internal short causing its fuse to melt.

QUESTIONS

Question 10.1 Identify equipment and reasons that may require a time-delay fuse and a fast-acting fuse.

Question 10.2 Why must each load branch circuit have an overload protection, in addition to the fuse?

Question 10.3 How is arcing prevented at the relay contacts while breaking the current?

Question 10.4 Why do the circuit breaker contacts see arcing when the current is interrupted, and how it is blown away in circuit breakers?

Question 10.5 What is the differential relay and how does it work?

Question 10.6 Describe alternative ways of protecting major electrical equipments from lightning overvoltages.

Question 10.7 What does the protection coordination mean and how it is achieved?

Question 10.8 Briefly describe the arc flash analysis and its end product.

Question 10.9 Identify three alternative ways insulation between HV and LV conductors can break down, leading to flashover.

FURTHER READING

Sleva, A. 2009. *Protective Relay Principles*. Boca Raton, FL: CRC Press.
Kimbark, E. 1950. *Power System Stability.* New York: John Wiley & Sons.
IEEE Standard-1584. 2008. Guide for Performing Arc Flash Hazard Calculations.
National Fire Protection Association. 2009. NFPA-70E Arc Flash Analysis. Boston.

11 Economic Use of Power

The economical use of power in this chapter means doing the same required work with a minimum expense of energy. In land-based or shipboard power systems, major opportunities in energy savings come from:

- Using a high efficiency motor that runs several hours every day over the year
- Improving the power factor of the equipment or the entire plant to bring it close to unity
- Using a variable-frequency drive to match the motor speed with the load requirement
- Storing energy when it is available at a lower cost and using it later when it has higher value
- Converting the kinetic energy of a moving mass into electricity when braking (regenerative braking)
- Selecting the correct rating of the equipment (not oversizing and then using at light loads)

An energy saving project often requires an initial capital investment, which may be recovered in a few years by a continuing stream of savings in monthly energy bills. We first cover the analytical tools to determine the profitability of such projects.

11.1 ECONOMIC ANALYSIS

Making a decision to invest in energy-saving equipment requires an economic analysis of the initial capital investment and the monthly savings that can be realized over the life of the proposed equipment. We review here the basics of such analysis and learn how to arrive at an economic decision, whether or not the proposed investment is profitable. Two alternative ways of financing the project are covered next.

11.1.1 Cash Flow with Borrowed Capital

If the initial capital cost is borrowed to finance the project, then the principal and interest must be paid back to the bank or company coffers over a period of time on a monthly basis. In return, we receive the benefit of lower utility bills in an industrial plant or reduced fuel consumption on a ship every month over the equipment life. If the project is financed by initial principal P borrowed for M months at an interest rate of i per month, we must make monthly (principal + interest) payments of P_{month} to the lender for M months, which is given by

$$P_{\text{month}} = \frac{P \cdot i}{\left[1 - \dfrac{1}{(1+i)^M}\right]} \ \$/\text{month} \tag{11.1}$$

In return, we see savings every month on our energy bill for the typically 20 to 30 years life of the equipment. If the monthly energy savings is E_{month} kWh/month, and the electrical energy cost is C_{energy} \$/kWh, then the monthly savings in energy bill is $S_{\text{month}} = C_{\text{energy}} \times E_{\text{month}}$ \$/month. If the monthly savings is greater than the monthly payment, then we have a net positive cash flow of $\text{Net}_{\text{month}} = S_{\text{month}} - P_{\text{month}}$ every month for M months until the borrowed capital is paid off. After M months, the monthly payments stop, and the net positive cash flow jumps to $\text{Net}_{\text{month}} = S_{\text{month}}$.

The net cash flow of $\text{Net}_{\text{month}} = S_{\text{month}} - P_{\text{month}}$, positive or negative, in the initial months does not impact the profitability of the project. It merely indicates the actual monthly cash flow. One can avoid monthly negative cash flow by borrowing for a sufficiently long time such that the monthly payments to the bank are equal to or less than the energy savings per month, that is, having $P_{\text{month}} < S_{\text{month}}$.

It is noteworthy that Equation (11.1) is also valid for a home mortgage or car loan. For example, if \$200,000 is borrowed to buy a home with a 30-year mortgage at 6% annual interest rate, then $i = 0.06/12 = 0.005$ and $M = 12 \times 30$, which will require principal plus interest payments of

$$P_{\text{month}} = \frac{200{,}000 \times 0.005}{\left[1 - \dfrac{1}{(1+0.005)^{12\times 30}}\right]} = \frac{1000}{\left[1 - \dfrac{1}{6.022575}\right]} = \$\ 1199.10 \text{ per month}$$

11.1.2 PAYBACK OF SELF-FINANCED CAPITAL

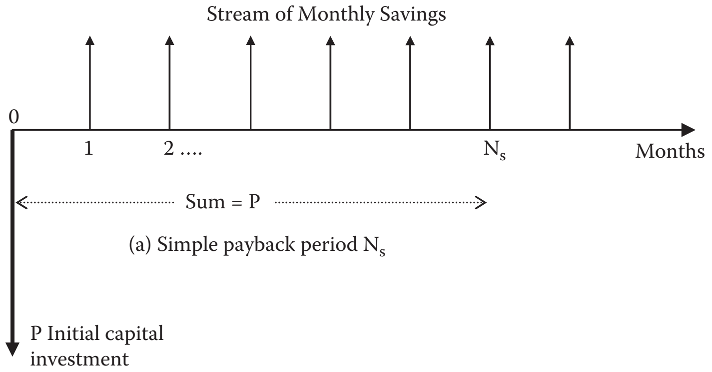
When the initial capital investment is self-financed, the net monthly savings is all positive cash flow from month one. In essence, the company paid a big chunk of money on day one to get a series of monthly savings in the future. To be fair with our economic analysis in such a case, the future benefits are discounted at a rate equal to the prevailing interest rate to compensate for the lost opportunity of earning interest if the money were deposited in the bank instead. As an example, if the prevailing interest rate is 1% per month (i = 0.01 pu), \$2000 coming at the end of 12 months has the present worth of $2000/(1 + 0.01)^{12} = \$\ 2000 \times 0.8875 = \$1775$. If this amount is invested in the bank at 1% per month interest rate, compounded monthly, it will grow to \$2000 at the end of the 12th month. The factor $1/(1 + i)^{12} = 0.8875$ here is called the *discount factor* for the 12-month waiting period at the discount rate of i per month. Thus, the discount rate shrinks the future fund into its present worth, whereas the interest rate grows the present fund into its future worth. The two rates may be different, but most investors use the same number for both and interchangeably call it the interest rate.

Discounting the future income at the prevailing interest rate i makes saving at the end of nth month worth less than that at present by the discount factor $1/(1+i)^n$. Therefore, we convert each future monthly saving of S_{month} at the end of the nth month into its equivalent present worth $\text{PW}_n = S_{\text{month}}/(1+i)^n$. The present worth of total savings PW_{Total} during N months is the sum total of all PW_n from $n = 1$ to N,

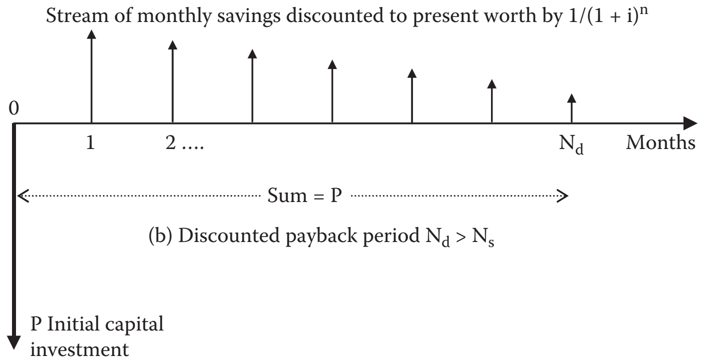
$$PW_{\text{Total}} = \sum_{n=1}^{N} \frac{S_{\text{month}}}{(1+i)^n} = \frac{S_{\text{month}}}{i}\left[1 - \frac{1}{(1+i)^N}\right] \tag{11.2}$$

If the present worth of all monthly savings exceeds the initial capital cost, that is, if $PW_{\text{Total}} > P$, then obviously the project is profitable. On the other hand, $PW_{\text{Total}} = P$ results in a financially break-even project, and $PW_{\text{Total}} < P$ results in financial loss.

Many managers and investors think in terms of the payback period, that is, how many months it takes to get the initially invested principal P back from the monthly savings. Figure 11.1 in (a) depicts the initial principal P paid (negative) at day zero,



(a) Simple payback period N_s



(b) Discounted payback period $N_d > N_s$

FIGURE 11.1 Monthly savings following an initial capital investment.

and then a continuing string of monthly savings (positive) of equal amount. The present worth of monthly savings, discounted to their present worth, are shown in (b), gradually shrinking more and more as we move farther into future months. The simple payback period (without discounting the future savings by the interest rate) is given from (a),

$$\text{Simple payback period } N_{\text{simple}} = P/S_{\text{month}} \text{ months} \tag{11.3}$$

If we discount the future savings by the prevailing interest rate—as in (b)—the payback period will get a little longer. The discounted payback period, say, N_{disc} months, is obtained by equating P with PW_{Total} given by Equation (11.2) over N_{disc} months. Thus, discounted payback period is obtained by solving the following equation for N_{disc}:

$$P = \frac{S_{\text{month}}}{i}\left[1 - \frac{1}{(1+i)^{N_{\text{disc}}}}\right] \tag{11.4}$$

Alternatively, it can be determined by setting up an EXCEL spread sheet for adding the discounted monthly savings until the accumulated savings adds up to the initial capital investment. Table 11.1 is an example of such a spreadsheet for \$100 per month energy savings and 1% monthly discount (interest) rate. If the energy saving equipment has the initial capital cost of \$2000, then the discounted payback period is 22.5 months. If the equipment cost is \$3000, then it will pay back in 36 months.

Typical payback periods for engineering projects are 36–60 months, with 36 months considered attractive by most managers. Projects with a payback period longer than 60 months (5 years) rarely get financed in the industry, except for other considerations such as environmental or public relation reasons.

11.2 POWER LOSS CAPITALIZATION

Utility power engineers often think in terms of the monetary present worth of saving 1 kW power loss in equipment like the transformer or generator, which generally stays connected all the time, 24 h a day and 365 days a year, regardless of whether or not it is delivering power at the output terminals. For example, if the transformer has 5% power loss at rated load, it would have approximately 2% fixed continuous power loss in the magnetic core at all loads (even at no-load), and the remaining 3% loss in resistance of the coils, which would vary with the square of actual load current at any given time. The energy cost of 2% fixed continuous loss adds up to a large sum over 25 to 30 years of the equipment life. If the energy cost is C_{energy} \$/kWh and the interest (discount) rate is i per month, every kW reduction in power loss saves 1 kW × 24 h/day × 365 days/year = 8760 kWh per year, which amounts to 8760 × C_{energy} \$/year or 730 × C_{energy} \$/month. Over a 30-year equipment life, the present worth of this annual saving is called the *power loss capitalization rate* (PLCR), given by

$$PLCR = \frac{730\ C_{energy}}{i}\left[1 - \frac{1}{(1+i)^{30\times12}}\right] \quad \$/\text{kW} \tag{11.5}$$

TABLE 11.1
EXCEL Spreadsheet for Determining Discounted Payback Period for an Investment

Month (n)	Energy Cost Saving $	Discount Factor = $1/1.01^n$	Discounted Savings $	Cumulative Savings $
1	100	0.9901	99.01	99.01
2	100	0.9803	98.03	197.04
3	100	0.9706	97.06	294.10
4	100	0.9610	96.10	390.20
5	100	0.9515	95.15	485.34
6	100	0.9420	94.20	579.55
7	100	0.9327	93.27	672.82
8	100	0.9235	92.35	765.17
9	100	0.9143	91.43	856.60
10	100	0.9053	90.53	947.13
11	100	0.8963	89.63	1036.76
12	100	0.8874	88.74	1125.51
13	100	0.8787	87.87	1213.37
14	100	0.8700	87.00	1300.37
15	100	0.8613	86.13	1386.51
16	100	0.8528	85.28	1471.79
17	100	0.8444	84.44	1556.23
18	100	0.8360	83.60	1639.83
19	100	0.8277	82.77	1722.60
20	100	0.8195	81.95	1804.56
21	100	0.8114	81.14	1885.70
22	100	0.8034	80.34	1966.04
23	100	0.7954	79.54	2045.58
24	100	0.7876	78.76	2124.34
25	100	0.7798	77.98	2202.32
26	100	0.7720	77.20	2279.52
27	100	0.7644	76.44	2355.96
28	100	0.7568	75.68	2431.64
29	100	0.7493	74.93	2506.58
30	100	0.7419	74.19	2580.77
31	100	0.7346	73.46	2654.23
32	100	0.7273	72.73	2726.96
33	100	0.7201	72.01	2798.97
34	100	0.7130	71.30	2870.27
35	100	0.7059	70.59	2940.86
36	100	0.6989	69.89	3010.75
37	100	0.6920	69.20	3079.95

If the equipment saves S_{kW} in power loss and costs ΔP more, it will save money over 30 years only if $\Delta P < S_{kW} \times \text{PLCR}$. A case where $\Delta P = S_{kW} \times \text{PLCR}$ will result in breakeven. However, 30 years is too long for most managers, who would be generally interested in 1 kW loss saving advantage only if the additional capital cost $\Delta P < \frac{1}{3} S_{kW} \times \text{PLCR}$.

Example 11.1

A 1000 kVA transformer is open for procurement. Vendor A offers 97% efficiency at $90,000 price and vendor B offers 98% efficiency at $100,000 price. Determine the profitability of buying the high-efficiency transformer. Assume that (i) quoted efficiency is at rated load at 90% pf lagging, (ii) core loss = one third total loss at a rated load, (iii) energy cost = 0.15 $/kWh, and (iv) interest (discount) rate = 9% per year. Frame your answer in terms of the power loss capitalization rate of saving 1 kW power loss in the magnetic core, which remains connected all year around for 30 years.

SOLUTION

The additional cost of transformer B over A is $10,000. The power loss capitalization rate per kW loss for 30 years (360 months) is given by Equation (11.5) with i = 9/12 = 0.75% per month,

$$PLCR = \frac{730 C_{\text{energy}}}{i}\left[1 - \frac{1}{(1+i)^{360}}\right] = \frac{730 \times 0.15}{0.0075}\left[1 - \frac{1}{(1+0.0075)^{360}}\right] = 13{,}618 \text{ \$/kW}$$

The output power at 0.90 pf = 1000 kVA × 0.90 = 900 kW.

The total power loss at 97% efficiency = 900 ÷ [1/0.97 – 1] = 27.8 kW, of which ⅓ × 27.8 = 9.27 kW is in the fixed core loss.

At 98% efficiency, total power loss is 900 ÷ [1/0.98 – 1] = 18.37 kW, of which ⅓ × 18.37 = 6.12 kW is in fixed core loss.

Difference in fixed loss that stays all year around = 9.27 – 6.12 = 3.15 kW.

At the above power loss capitalization rate, 3.15 kW means 3.15 × 13,618 = $ 42,897 present worth. Since the high efficiency transformer costs $10,000 more, the simple payback period is about one quarter of the full 30 years life, or roughly 7.5 years.

Although many engineers and managers may not be interested in any investment opportunity with longer than a few years' payback period, the utility managers with long-term views may be interested in buying a high efficiency unit if they have some free capital available for such low risk investment.

11.3 HIGH EFFICIENCY MOTOR

All motors combined consume about 58% of total electrical power generated. Large induction motors of standard design have efficiency in the range of 90% to 95%. Since large 3-phase induction motors consume most of the electrical energy, the motor industry offers high efficiency motors under the class *E* design at an additional cost to support the national energy conservation plan. However, the economics of using high efficiency motor depends on the up-front additional capital cost, energy cost that can be saved every month, and the discount rate for the future savings. Since power loss = input power – output power, and efficiency η = output power/input power, we can write kW power loss in terms of the efficiency and horsepower output of the motor as follows:

$$kW_{\text{loss}} = 0.746\,HP\left(\frac{1}{\eta} - 1\right) \tag{11.6}$$

Power saving by using a motor of higher efficiency instead of a lower efficiency is

$$kW_{\text{saving}} = 0.746\,HP\left(\frac{1}{\eta_{low}} - \frac{1}{\eta_{high}}\right) \tag{11.7}$$

$$\text{Monthly } \$_{\text{monthly.saving}} = kW_{\text{saving}} \times \text{hours/month used} \times \text{\$/kWh energy tariff} \tag{11.8}$$

Buying a high efficiency motor is justified if the cumulative present worth of the monthly savings in the utility bill or the ship's fuel consumption over the service life of the motor exceeds the additional capital cost. In applications where a large motor runs at full load for 3000 hours or more per year (250 hours per month on average), the added capital cost may be generally recovered (paid back), typically within a few years. After that, it is all net savings every month over the service life of the motor that is typically 25 to 30 years.

In equipment where the motor runs only few hours every day at light loads, the energy saving may not add up to the added capital cost of buying a high efficiency motor in a reasonable number of years. In such lightly used systems, buying the standard efficiency motor may be economical. However, with large custom-ordered power equipment (motor, transformer, generator, motor drive, etc.) that does not work at full load all the times, we still can save some energy based on Equation (3.9) developed in Section 3.5. It shows that the equipment efficiency varies with load, and is the maximum at load point where the fixed loss equals the variable loss. The engineer in charge of buying large custom-ordered equipment may have an option of specifying the percentage of load point where the equipment should be designed to have the maximum efficiency. If the buying engineer specifies the maximum efficiency load point equal to the percentage of load at which the equipment would be operating most of the time, the energy savings can be realized even in standard efficiency equipment. In any power equipment design—standard or high efficiency—the maximum efficiency is never at full load, or at light load either; it is typically between 75% to 85% of the full rated load. The engineer can specify the maximum efficiency load point at a lower load if it is used lightly for most of the time.

Since the efficiency is lower at light load, buying a motor or transformer much larger than the actual load it would deliver most of the time results in waste of energy. For example, if the motor is required to deliver 50 hp or less most of the time, and occasionally 65 hp for a couple of hours at a time, then buying any size much greater than 50/0.80 = 62.5 hp is not energy efficient. Other considerations may prevail in some cases, but they must be carefully weighed in before making the decision.

Example 11.2

In a large motor drive application under consideration, either a 5000 hp induction motor or a 5000 hp synchronous motor can be used. The estimated system efficiency of the synchronous motor with LCI drive is 96% and that of the induction motor with CSI drive is 94%. The motor is projected to run 600 hours per month. Determine the total energy cost saving over 30-year expected life of the system using the synchronous motor if the energy cost is 10 cents per kWh. Also determine the present worth of life-time savings discounted at 6% annual rate (0.5% per month).

a. *Approximate life-time saving:* Approximate difference in input power = Difference in % efficiency = 96 – 94 = 2%, and Δ Energy cost = 0.02 × 5000 hp × 0.746 kW/hp × 600 h/month × 0.10 $/kWh = $4,476 per month × 360 months = $1,611,360 over 30-year life.
b. *Exact life-time saving:* Exact difference in input power ={1/0.94 – 1/0.96} × 100 = 2.2163%, and Δ Energy cost = 0.022163 × 5000 hp × 0.746 kW/hp × 600 h/month × 360 months × 0.10 $/kWh = $ 1,785,600 over 30-year life.

   The difference between the exact and approximate power savings is about 10% (2.2163 versus 2%), and the difference in dollar saving of 1,785,600 – 1,611,360 = $174,240 is also about 10% higher than the approximate saving. The difference is not negligible, suggesting us to avoid the approximate approach taken in (a).

c. Discounted present worth of exact savings = $1,785,600 ÷ $(1 + 0.005)^{360}$ = $296,485.

The synchronous motor system would be economical if its additional cost were less than $296,485 over the induction motor system cost.

Example 11.3

A new 10 hp motor is required to replace an old one. It will run 500 hours per month and the energy cost is $0.15 per kWh. You have two options available (i) standard motor with 90% efficiency at $1500 cost, or (ii) high efficiency motor with 92% efficiency at $2000 cost. Determine the simple and discounted payback periods of investing in the high efficiency motor at a discount rate of 6% per annum.

SOLUTION

Power loss saving by using a high efficiency motor versus a standard motor =

$$10 \times 0.746 \left[\frac{1}{0.90} - \frac{1}{0.92} \right] = 0.18 \text{kW}$$

Monthly saving = 0.18 × 500 h × 0.15 \$/kWh = \$13.51
Simple payback period = (2000 – 1500) ÷ 13.51 = 37 months
Discounted payback period at 0.5% discount rate per month from Equation (11.4) is

$$500 = \frac{13.51}{0.005}\left[1 - \frac{1}{(1+0.005)^{N_{\text{disc}}}}\right], \text{ which gives } N_{\text{disc}} = 41 \text{ months.}$$

As expected, this is longer than the simple payback period of 37 months.

Alternatively, one can use an EXCEL spreadsheet similar to Table 11.1, using discount factor = 1 ÷ $(1 + 0.005)^n$ for savings coming after n months. The discounted payback period is when the accumulated presents worth of the monthly savings exceed the additional upfront capital cost. The student is encouraged to set up such a spreadsheet and verify that it also gives the discounted payback period of 41 months.

If the standard motor were damaged and needed repairs that would cost \$750, or it could be scrapped with a salvage value of \$150, one could trade the repair versus buying a new high efficiency motor at an additional cost of \$2000—750 – 150 = 1100. Then, the simple payback period would be 1100 ÷ 13.51 = 81.5 months. Although this is relatively long, it may still be attractive to scrap the damaged motor and replace it with a new high efficiency motor.

11.4 POWER FACTOR IMPROVEMENT

In practical power circuits, current $\tilde{I}$ drawn by the load lags the source voltage $\tilde{V}$ by an angle θ in the 30° to 40° range. The real work is done only by the current component $I \times \cos\theta$, which is in phase with the voltage. The quadrature component $I \times \sin\theta$ is orthogonal to the voltage, and does no real work. It merely charges the inductor and capacitor in the load circuit for ½ cycle, and discharges for the next ½ cycle that follows, contributing zero average work during every cycle. The quadrature component of the current that goes in this charging and discharging effectively leaves less conductor area available for the in-phase component, thus reducing the real power transfer capability of the conductor. In this sense, $I \times \sin\theta$ is like the bad cholesterol in our blood veins, effectively narrowing the blood arteries, not a healthy situation. Reducing the quadrature component of the current is, therefore, important for the power system health.

With power factor pf = cosθ, where θ = phase difference between the source voltage and the load current, the average real power in ac load is given by Watts = $V_{rms} \times I_{rms} \times \cos\theta$, reactive power VAR = $V_{rms} \times I_{rms} \times \sin\theta$, and apparent power VA = $V_{rms} \times I_{rms}$. The power distribution switchboard usually displays kW and kVA, and sometimes pf of the aggregate load. The power triangle trigonometry we covered in Section 1.6 gives the following relations:

$$\text{kVA}^2 = \text{kW}^2 + \text{kVAR}^2, \quad \text{kW} = \text{kVA} \times \text{pf, and kVAR} = \text{kVA} \times \sqrt{(1 - \text{pf}^2)} \qquad (11.9)$$

The pf has the maximum value of 1.0 (best) and the minimum value of zero (worst, delivering no average real power, even while drawing full current at full voltage). Most practical plants draw power at a lagging pf between 0.8 and 0.9. Poor pf results in higher monthly cost to the plant for the following reason. The utility company typically levies two charges every month to the user:

Energy charge per kWh consumed to recover the fuel cost, as the amount of fuel used by the generator prime mover depends on the kWh (real energy) used, and

Demand charge per maximum kVA drawn over the billing period to recover the fixed capital cost of all the equipments (generators, transformers, distribution lines, and cables), which depends on the kVA ratings.

$$\therefore \text{ Monthly utility bill } \$_{\text{month}} = C_{\text{energy}} \times \text{kWh} + C_{\text{demand}} \times \text{kVA} \tag{11.10}$$

where C_{energy} = energy charge rate \$/kWh and C_{demand} = demand charge rate \$/kVA. Average utility tariff rates in the United States in 2011 are about \$0.12 per kWh and \$12 per peak kVA drawn over the month (the demand meter is reset every month) for most power users. If the pf is poor, the plant draws more kVA for the same power. By improving the pf, we draw the same real power while drawing less kVA, thus reducing the monthly demand charge, although the energy charge would remain the same.

Example 11.4

The electrical power usage in a factory is shown in the following table. The utility contract calls for energy charge of \$0.15 per kWh and demand charge of \$15 per peak kVA over any 15 min period during the month. Determine (i) the utility bill in a 30-day month, and (ii) if the pf during the first shift is improved to unity, determine the new monthly bill.

Shift	Hours Used	Average kW	Peak Demand kVA
1	8	1000	1400
2	8	500	700
3	8	100	150

SOLUTION

a. The monthly kWh energy used = 1000 × 8 + 500 × 8 + 100 × 8 = 12,800 kWh/day

Peak kVA demand during the 1st shift = 1400 kVA

$$\therefore \text{ Monthly bill} = 12{,}800 \times 30 \times 0.15 \text{ energy charge} + 1400 \times 15 \text{ demand charge} = \$78{,}600$$

b. With unity pf in the 1st shift, the kVA demand will be equal to the kW, that is, 1000 kVA, while the energy consumption will remain the same.

$\therefore$ New monthly utility bill = 12,800 × 30 × 0.15 + 1000 × 15 = \$72,600, which is \$6000 per month less than before.

Example 11.5

A daily power consumption profile of a customer is shown in the table below, where the kW loads are average over the period listed and kVA demand meter readings are peak over 15 min intervals during the period. This pattern repeats for all days in a 30-day month. The utility tariff to this consumer is 12 cents per kWh plus $15 per peak kVA demand over the month. Determine the monthly utility bill (a) with present usage pattern, and (b) with pf improved to unity. Also determine the daily load factor of this consumer as seen by the utility company.

Time Period	Power Usage kW	kVA Demand
12 a.m.–4 a.m.	100	135
4 a.m.–8 a.m.	150	190
8 a.m.–12 a.m.	250	330
12 p.m.–3 p.m.	500	725
3 p.m.–8 p.m.	700	900
8 p.m.–12 p.m.	125	150

SOLUTION

a. From the first two columns of the table, we derive daily energy usage = 100 × 4 + 150 × 4 + 250 × 4 + 500 × 3 + 700 × 5 + 125 × 4 = 7500 kWh day or 7500 × 30 = 225,000 kWh per month. The peak kVA demand for the day is 900 kVA, which occurs over a 15 min period between 3 p.m. and 8 p.m. It does not matter if this happens every day or not, so long as it occurs at least once in the month. The consumer will be charged for this kVA demand for the entire month to recover the capital cost of the equipment capacity installed by the utility company.

   Monthly bill = 225,000 × 0.12 energy charge + 900 × 15 demand charge = $40,500.

b. If this consumer maintains unity power factor over every 15 minute interval during 3 p.m. to 8 p.m. at least, if not the entire day, the peak kVA demand will be equal to peak kW demand, which will be 700 kVA, instead of 900 kVA. That can save (900 − 700) × 15 = $3000 per month.

Many consumers in such tariff would indeed install capacitors to maintain 0.95 power factor, at least from noon to 8 p.m.

We see that the peak power load is 700 kW, at which rate the total daily consumption would be 700 × 24 = 16,800 kWh per day, whereas the actual energy usage is only 7500 kWh per day.

Daily load factor of this consumer for the utility company = 7500 kWh ÷ (700 kW × 24 h) = 0.4464 or 44.64%.

In ships, the demand charge does not really apply, so the benefits of pf improvement on ships get modified as follows:

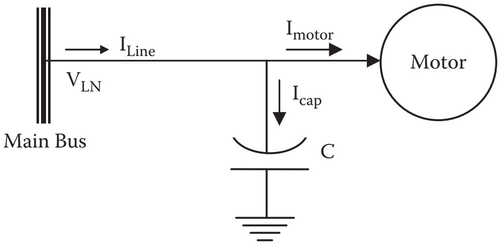
1. Lower shipboard generator and transformer kVA ratings for delivering the same real power (saves in the equipment capital cost and volume)
2. Lower cable voltage drop as per Equation (3.15)

3. Lower I^2R power loss by reducing the line current required for the same real power
4. Lower fuel consumption rate due to (3)

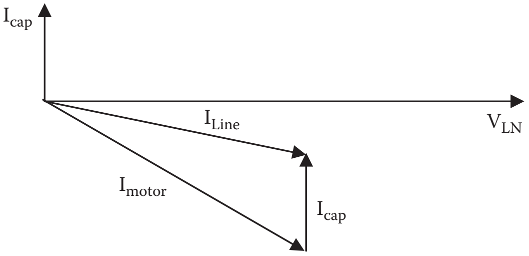
For improving load pf that is lagging, we first recall that poor pf is caused by large inductance in the load circuit. Therefore, the pf can be improved by compensating the inductance by connecting a capacitor bank in parallel with the load as shown by the 1-line diagram in Figure 11.2(a). The capacitor is always connected in parallel with the load for better reliability. If the parallel-connected capacitor fails, the load would still run at the old poor pf doing the same work as before, only drawing more kVA until the failed capacitor is replaced. A failure in a series-connected capacitor, on the other hand, would cause complete loss of power to the load.

The current drawn by capacitor, $I_{cap} = V/X_{cap} = C\omega V$, which leads the voltage by 90°. It is added with the motor current in the phasor diagram shown in Figure 11.2(b). We note that the resulting line current now is lagging the voltage by a smaller angle than the motor current, and its magnitude is lower, which gives added secondary benefit of lower I^2R loss in the line cable.

The line current drawn from the source is now $\tilde{I}_{Line} = \tilde{I}_{cap} + \tilde{I}_{motor}$, the phasor sum, which is always less than $\tilde{I}_{motor}$ due to the compensating effect of leading $\tilde{I}_{cap}$ with lagging motor current. Thus, the motor now produces the same real power with less current drawn from the source, reducing the kVA demand from the line and the monthly demand charge in the utility bill. The energy charge, however, remains the same, except a small reduction in I^2R loss in the line cable.



(a) Capacitor connected in parallel with motor



(b) Phasor diagram of the voltage and current per phase

FIGURE 11.2 Power factor improvement capacitor and resulting line current phasor diagram.

11.4.1 CAPACITOR SIZE DETERMINATION

The following analysis determines the capacitor size in farads for improving the power factor angle from θ_1 (old) to θ_2 (new). From the power triangles shown in Figure 11.3 with old and new pf, denoted by suffix 1 and 2, respectively, we have the following values per phase:

Real power in both cases = kW (we get the same work done in both cases)
Reactive power $kVAR_1 = \text{kW} \tan\theta_1$ and $kVAR_2 = \text{kW} \tan\theta_2$

The difference between the two kVARs comes from the capacitor, that is,

$$kVAR_{\text{cap}} = kVAR_1 - kVAR_2 \tag{11.11}$$

The above formula can be rearranged in the following easier form,

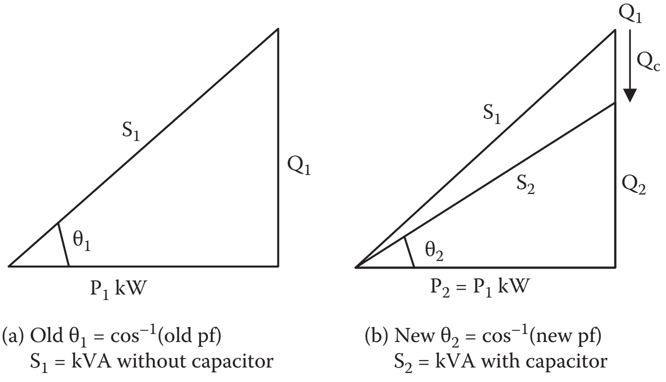
$$kVAR_{\text{cap}} = kW_{\text{Load}}\left(\sqrt{\frac{1}{PF_{\text{old}}^2}-1} - \sqrt{\frac{1}{PF_{\text{new}}^2}-1}\right) \tag{11.12}$$

The reactive power VAR in a reactance (inductive or capacitive), $Q = I^2X = V^2/X$.

For capacitor, $X_{\text{cap}} = 1/\omega C$ $\qquad \therefore\ VAR_{\text{cap}} = \dfrac{V_{LN}^2}{1/\omega C} = \omega C V_{LN}^2$

$$\therefore\ kVAR_{\text{cap/ph}} = \omega C V_{LN}^2/1000 \quad \text{and} \quad C = \frac{1000\ \text{kVAR}_{\text{cap/ph}}}{\omega\ \text{V}_{LN}^2}\ \text{F/ph} \tag{11.13}$$

In 3-phase systems, three capacitors are generally connected in Y as shown in Figure 11.4, or in Δ connection if necessary for system considerations.



(a) Old $\theta_1 = \cos^{-1}$(old pf)
S_1 = kVA without capacitor

(b) New $\theta_2 = \cos^{-1}$(new pf)
S_2 = kVA with capacitor

FIGURE 11.3 Power triangles with old and new power factors.

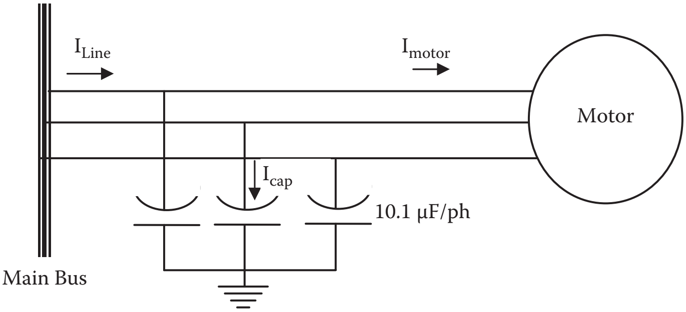


FIGURE 11.4 Three-phase capacitors connected in Y in parallel with load.

Example 11.6

Determine the capacitor rating in kVAR and also in farad to improve the load pf from 0.70 lagging (i.e., $\theta_1 = 45.6°$) to 0.90 lagging (i.e., $\theta_2 = 25.8°$) at a 3-phase, 60 Hz, 460 V distribution point delivering 1500 kW real power. Also determine the simple pay back period if the motor runs for 300 hours per month, the utility demand charge is \$10 per kVA, energy charge is \$0.15 per kWh, capacitor cost is \$60 per kVAR installed, and cable power loss is 2% per phase.

SOLUTION

We will make all calculations per phase.

Power delivered = ⅓1500 = 500 kW, and power loss in cable = 0.02 × 500 = 10 kW.

With old pf, $kVAR_{old}$ = 500 tan 45.6° = 511 kVAR. This must be reduced to new $kVAR_{new}$ = 500 tan 25.8° = 242 kVAR by capacitor connected in parallel with the motor.

$$\therefore kVAR_{cap} = 511 - 242 = 269 \text{ kVAR/ph}$$

Phase voltage $V_{LN} = 460\ V_{LL} \div \sqrt{3} = 265.6$ volts/ph, and $\omega = 2\pi \cdot 60 = 377$ rad/sec.

Equation (11.13) gives $C = 1000 \times 269 \div (377 \times 265.6^2) = 10.1 \times 10^{-6}$ F/ph = 10.1 μF/ph

The Y-connected capacitor bank will have 10.1 μF in each phase of the Y. If the capacitors are connected in Δ for some reason, then X_C in ohms in each Δ-leg must be 3× that in Y-phase, or C in μF in Δ-leg must be 1/3 of that in Y-phase, that is, ⅓ × 10.1 = 3.367 μF . However, the voltage rating of C in Δ is 460 V instead of 265.6 V in Y, resulting in the same 269 kVAR/ph rating of the capacitor both ways, in Y or in Δ.

With this capacitor added in parallel, kVA_{old} = kW/pf = 500 ÷ 0.70 = 714.28 and kVA_{new} = 500 ÷ 0.90 = 555.56. The differences in kVA demand = 714.28 − 555.56 = 158.72 kVA per phase. Improving the pf from 0.70 to 0.90 reduces the line current to 0.70 ÷ 0.90 = 0.7778 without reducing the real power output of the equipment. This will reduce I^2R loss in the cable to $(0.7778)^2 = 0.605$ pu, saving 0.395 pu or 39.5% from the initial cable power loss of 10 kW per phase, which amounts to 3.95 kW/phase.

Now, for 3-phase, KVA saving = 3 × 158.72 = 476.16 kVA, and
kW saving in cable loss = 3 × 3.95 = 11.85 kW.

Cost savings = 10 × 476.16 kVA + 0.15 × 11.85 kW × (300 hours/month) = 4762 + 533 = \$ 5295 per month

Total 3-phase capacitor cost = \$60 per kVAR × 3 × 269 kVAR/ph = \$ 48,420

The capacitor cost will be recovered (paid back) in 48,420 ÷ 5295 = 9.14 months, which certainly is a good investment. This is an extreme example of poor pf to start with, which gives such a short payback period. For a typical plant with a reasonable pf to start with, the payback period may run in a few years.

Since the reactive power supplied by 1-phase capacitance C farads/ph is given by $VAR_{1ph} = V_{LN}^2/Xc$, where $Xc = 1/(2\pi f \times C)$, or $VAR_{1ph} = V_{LN}^2 \times 2\pi f \times C$. For 3-phase capacitors, $VAR_{3ph} = 3 \times V_{LN}^2 \times 2\pi f \times C = V_{LL}^2 \times 2\pi f \times C$. Thus, the capacitor kVAR depends on both voltage and frequency. The capacitor can be safely operated at voltage and frequency somewhat different than the nameplate rating within a certain range. However, since capacitance C remains constant for a given construction, we can write kVAR relations in the ratio of new to old kVARs at different frequency and voltage,

$$\frac{kVAR_{3ph}^{new}}{kVAR_{3ph}^{old}} = \left(\frac{V_{LL}^{new}}{V_{LL}^{old}}\right)^2 \cdot \left(\frac{f^{new}}{f^{old}}\right) \tag{11.14}$$

Equation (11.14) is used to determine the modified kVAR rating of the capacitor at frequency and voltage different than the rated values.

Example 11.7

A 200 kVAR, 3300 V, 60 Hz, 3-phase, American-made capacitor bank is operating at 2400 V, 50 Hz in Europe. Determine its new kVAR capacity in the European power system.

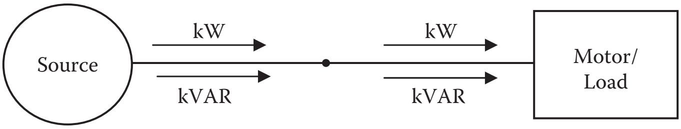
SOLUTION

Using Equation (11.14), we have $\frac{kVAR_{3ph}^{new}}{200} = \left(\frac{2400}{3300}\right)^2 \cdot \left(\frac{50}{60}\right) = 0.4408$

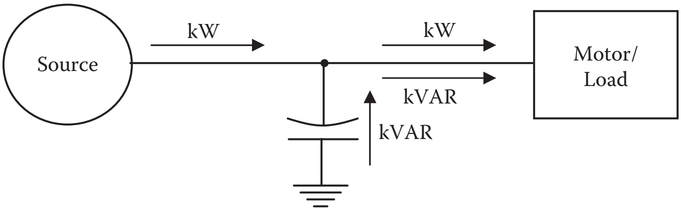
$$\therefore\ kVAR_{3ph}^{new} = 0.4408 \times 200 = 88.16$$

The kVAR capacity is reduced to 0.4408 pu or 44.08%, primarily due to significant reduction in the voltage.

The capacitor improves the pf as explained in Figure 11.5. Without the capacitor in (a), the load kW and kVAR both are supplied by the source. With capacitor in (b), the capacitor kVAR leads the voltage by 90°, whereas inductive load kVARs lag the voltage by 90°. Thus, they have opposite polarity; when one is going away from the junction node *J*, the other is going toward the node J. The net result is that the inductive kVAR is supplied by the capacitor kVAR, and the supply lines are relieved from the kVAR load all together, reducing the line current.



(a) Without capacitor



(b) With capacitor

FIGURE 11.5 The kW and kVAR flow with and without capacitors.

11.4.2 Parallel Resonance with Source

A concern in placing the pf improvement capacitor C_{pf} is the parallel resonance with the Thevenin source inductance L_{source} at natural undamped resonance frequency

$$f_r = \frac{1}{2\pi\sqrt{L_{source}C_{pf}}}\ \text{Hz} \tag{11.15}$$

On a bus with a power electronics load drawing harmonic currents, this concern is amplified if any of the current harmonic frequencies match with this resonance frequency. For example, the 6-pulse power electronics converter draws harmonic currents of the order 5, 7, 11, 13 …, which have frequencies 300, 420, 660, 780 … Hz, respectively, in 60 Hz systems. If the resonance frequency $f_r = 300$ Hz, the total 5th harmonic source impedance will be very high, resulting in high 5th harmonic voltage drop internal to bus. This drop, in turn, results in a high bus voltage distortion that will impact all other loads (linear and nonlinear) at the bus. While placing the pf improvement capacitor, such a resonance condition should be avoided by all means to maintain the quality of power at the bus.

11.4.3 Safety with Capacitors

The medium voltage capacitor ratings generally available for pf improvements are listed in Table 11.2. They are made with two parallel plates enclosed in an insulating liquid, and an internal discharge resistor across the terminals to bleed the stored energy after turning the power off. For personnel safety, the NEC code requires sizing the bleed resistor to bring the voltage below 50 V in LV capacitors within 1 minute, and within 5 minutes in HV capacitors.

Overcorrecting the pf by placing too many capacitors can make the pf less than unity in the leading direction, which is as bad as the lagging pf. There is additional

TABLE 11.2
Standard Rating for 1-phase, Low and Medium Voltage Capacitors for pf Improvement

Voltage Range	kVAR
240 V, 480 V, and 600 V	Numerous kVAR ratings available
2400–12,470 V	50
2700–13,280 V	100
4160–13,800 V	150
4800–14,400 v	200
6640–15,125 V	300
7200–19,920 V	400
7620–20,800 V	500

danger in overcorrecting the pf. The motor working at the leading pf may work as the self-excited induction generator when tripped off the lines, and generate high terminal voltage and shaft torque that can damage the motor. The NEMA Safety Standards MG2, therefore, recommend that the pf capacitor value should not exceed that required to correct the no-load pf to unity. Moreover, switching capacitors at the motor terminals when needed is not recommended with the elevator motor, multi-speed motor, motor in jogging or dynamic braking application, and motor used with open position Y-Δ starter or reduced-voltage auto transformer. In such cases, the motor manufacturer should be consulted.

11.4.4 DIFFERENCE BETWEEN PF AND EFFICIENCY

Improving the pf and improving the efficiency—although both save operating cost—are two different things. They address two different aspects of the power utilization economics. Assuming the same real power output of the equipment at a fixed line voltage and frequency, their differences are as follows:

i. Pf improvement reduces the power plant capital cost that is based on the kVA load, whereas the efficiency improvement reduces the fuel cost.
ii. Pf improvement is done by the plant owner by installing capacitors to lower the kVA demand. The upfront cost of the capacitors is offset by monthly savings in the kVA demand charge.
iii. Efficiency can be improved only by the equipment manufacturer by

using larger conductors to reduce I^2R power loss in the conductors
using larger core to reduce magnetic (hysteresis and eddy) losses
using better bearings to reduce friction, etc.

The upfront additional cost of high efficiency equipment is offset by monthly savings in the kWh energy charge.

Table 11.3 lists the reasons, effects, and remedies of poor pf and poor efficiency.

TABLE 11.3
Reasons and Remedies for Poor pf and Poor Efficiency

	Poor Power Factor	Poor Efficiency
V and I phase relations (visible on oscilloscope)	V and I out of phase (less productive for average real power)	No effect on phase of V and I
Reason	High leakage inductance in equipment and cables	High unwanted resistance in equipment and cables
Effect 1	Draws more current and kVA from the lines	Draws more real power from the source
Effect 2	High losses in the cables	High losses in the equipment
Effect 3	Moderate temperature increase	High temperature increase
Cost	High kVA demand charge in monthly utility bill, or high generator, transformer, and cable ratings required in ships	High kWh energy charge in utility bill, or high fuel consumption rate in ships
Remedy	Use capacitors to neutralize inductance	Use heavier conductors and larger core in transformer and motor
For best performance	Have $\omega L = 1/\omega C$, which gives $X = 0$ with V and I in phase and pf = 1.0 (unity)	[a] Fixed losses in core = variable losses in conductors

[a] In buying large custom-ordered equipment, specify this condition at load point where the equipment is operated for most of the time.

11.5 ENERGY STORAGE DURING NIGHT

Electrical energy has many benefits over other forms of energy, but also has a severe shortcoming. It cannot be easily stored in large quantities, and must be used as produced in real time. However, if it could be stored, a new dimension in economic use of power can be realized. For a large power plant as an example, the incremental cost of producing electrical power at night is low when the demand is low. If the electrical power produced at night can be stored, it can be used to meet the peak demand during daytime when the value of electrical power is high. Such load leveling for the utility power plant results in a lower peak rating of the equipment and lower equipment cost.

Major energy storage technologies—some fully developed and some in early stages—are: (i) electrochemical battery, (ii) compressed air, (iii) flywheel, (iv) super-capacitor, (v) superconducting magnet, and (vi) pumped hydro plant. Among these, the battery has been used for small- and medium-scale energy storage for decades, and is the most developed and matured technology. The flywheel is also now commercially available for UPS suitable for many industrial and commercial applications. In ships, flywheel energy for starting gas turbines and meeting the pulse power demand for weapons in combat ships has been investigated by the U.S. Navy. The battery is covered in Chapter 12.

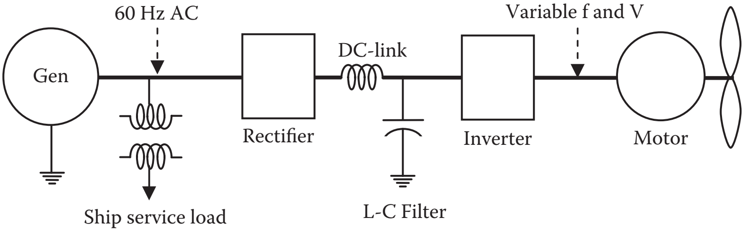


FIGURE 11.6 Variable frequency motor drive converters with dc link.

11.6 VARIABLE SPEED MOTOR DRIVES AC AND DC

We have seen in Chapter 5 that (i) ac motor speed can be changed by changing the frequency and voltage in the same ratio, and (ii) dc motor speed can be changed by changing the terminal voltage and field resistance in the same ratio. The ac motor speed control is achieved by the power electronics converter that converts fixed-frequency, fixed-voltage power into a variable-frequency, variable-voltage power source. Such converter is called the *variable frequency drive* (VFD). The power electronics converter used to control dc motor speed converts only the voltage level, and is called the *variable speed drive* (VSD).

The VFD consists of a power electronics rectifier and inverter set as shown Figure 11.6 schematic. It is widely used with large motor driving pumps or ventilating fans running long hours every day with variable fluid pumping requirement. It is desired to maintain the stator flux constant during the motor operation at all speeds in order to produce full rated torque and to avoid magnetic saturation in the core. The ac coil voltage Equation (3.2) gives $V/f = 4.444 \times \text{turns} \times \text{flux}$. Therefore, maintaining a constant rated flux in ac motor requires maintaining the constant rated V/f ratio while reducing the frequency for the entire speed range.

The VFD, particularly in a fluid pumping plant such as an oil refinery, saves energy as follows. The old conventional way of reducing the fluid flow rate was to throttle the valve, while the ac motors kept running at full speed. This puts the system in conflict; the motor running at full speed while inserting a hydraulic resistance in the fluid flow, resulting in inefficient utilizations of the motor power. The energy-efficient way is to reduce the motor speed in response to reduced flow requirement. The power saving using VFD drive and no throttle valve can be 20% to 30%. The VFD has made significant advances over the last two decades in the industrial and shipboard applications.

11.7 REGENERATIVE BRAKING

The regenerative braking (RGB) can significantly save energy in applications where a load with large inertia starts and stops frequently. In regenerative braking, we convert the kinetic energy of moving or rotating mass, or the potential energy of an elevated mass, into electrical energy that is fed back to the source—battery or power grid—from where we again draw the energy to accelerate the system from zero to full speed. At the same time, the use of mechanical brake pads is reduced to about 10%, reducing the harmful dust they spread in the environment. Saving 35% to 50% energy is possible in some applications using regenerative braking.

An ideal candidate for regenerative braking has the following features:

Runs on an electric source (grid or a battery) where the regenerated electrical energy can be fed back
Has frequent starts and stops
Has large inertia or steep grade requiring high energy input to accelerate

Some examples of such candidates are:

City metro trains and buses
Hybrid cars where the braking energy is stored in batteries for later use
Electrical cranes and hoists raising and lowering heavy loads
Elevators in tall buildings
High-speed, high-inertia lathe machines, drills, mills, etc.

Regenerative braking requires the motor to work as the generator during deceleration. With an induction motor, it requires ramping down the supply frequency such that the motor speed becomes and continuously remains supersynchronous. This makes the slip negative, hence the torque also negative by Equation (5.11), forcing the machine to work as the generator. This requires a variable frequency drive. With a dc motor, regenerative braking requires gradually ramping down the motor terminal voltage, such that the machine back voltage becomes and continues to remain above the terminal voltage. This reverses the current direction, making the machine work as the generator, and sending the power back to the supply. The dc RGB system requires a variable voltage converter.

The control system schematic for both ac and dc RGB is shown in Figure 11.7, where the power electronics converter output—frequency in ac and voltage in dc—is controlled by the speed feedback signal.

11.7.1 INDUCTION MOTOR TORQUE VERSUS SPEED CURVE

If the induction motor speed were varied over an extended speed range from synchronous speed backward to twice the synchronous speed forward, the theory we covered in Chapter 5 would give the motor torque versus speed characteristic shown

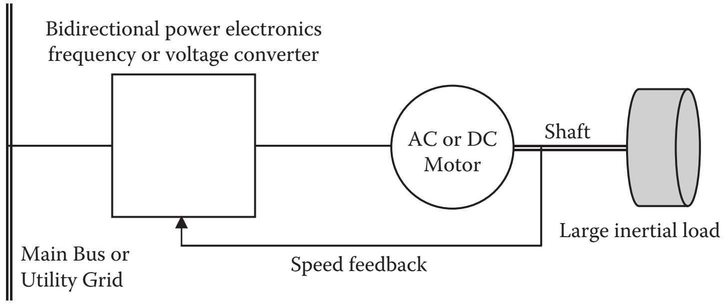


FIGURE 11.7 Control system schematic of regenerative braking.

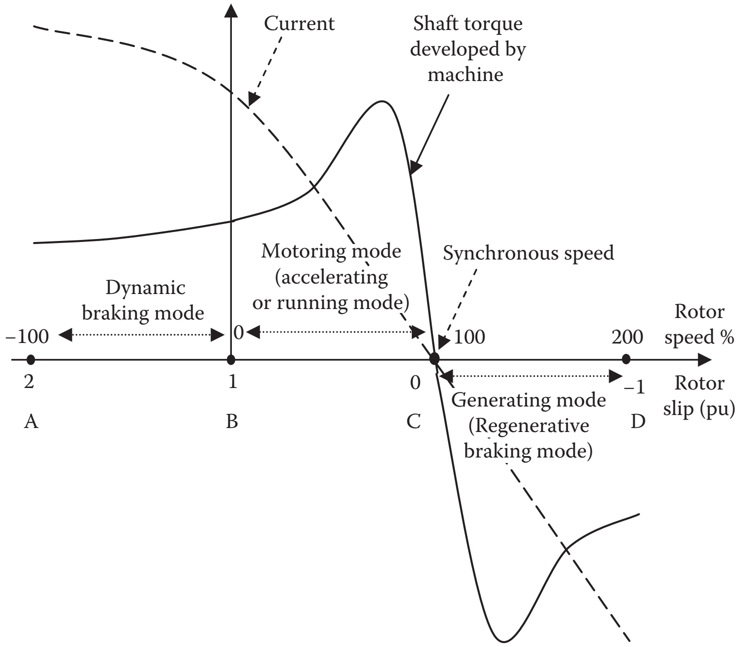


FIGURE 11.8 Induction motor torque and current versus speed over $-n_s$ to $+ 2n_s$ speed range.

in Figure 11.8. It shows that above the synchronous speed, the motor—although running in the same positive direction—develops negative torque, giving negative mechanical power, meaning it absorbs the mechanical power at the shaft instead of delivering. Since the current also becomes negative (flowing backward to the source), this negative power is converted into the electrical power and fed back to the lines, that is, the machine now works as the generator. On the left-hand side of zero speed, at negative speed (i.e., running backward), the motor torque is still positive, again giving negative mechanical power, but still draws the current in the same positive direction from the source. This power, therefore, is still drawn from the source, but is dissipated in I^2R heat in the rotor. The additional power loss in this region comes from the kinetic energy of the rotor, which slows down in speed, as if a brake was applied, hence the name *dynamic braking*. It is commonly used in many diesel–electric locomotives to limit wear on the mechanical brakes. Thus, in dynamic braking, the power flows backward from the shaft, but does not come out as electrical power at the machine terminal for outside use. It is rather dissipated in rotor resistance—often external resistance inserted via slip rings, which are cooled by forced air that can be used to heat the passenger compartments in winter.

Thus, the regenerative braking is not to be confused with dynamic braking. Regenerative braking means that the braking energy is converted into electrical energy that can be reused. Dynamic braking is not really regenerative since it does not reuse the kinetic energy, but instead dissipates it as heat through the resistor. We discuss only the regenerative braking in the section that follows.

Thus, the induction machine operating regions in Figure 11.8 are as follows:

Region A to B: Dynamic braking
Region B to C: Motoring
Region C to D: Generating

11.7.2 INDUCTION MOTOR BRAKING

We illustrate the working principle of regenerative braking in further detail in Figure 11.9. We consider a 4-pole, 60 Hz, 1750 rpm induction motor, which has the synchronous speed $n_s = 120\,f/P = 120 \times 60/4 = 1800$ rpm. If this motor is running at full rated speed of 1750 rpm, and we suddenly lower the frequency to 50 Hz in one step, the new synchronous speed will be $120 \times 50/4 = 1500$ rpm. However, the motor will still keep running near 1750 rpm for a while due to moving or rotating mass inertia. The operating point shifts from O_1 on a 60 Hz curve to O_2 on a 50 Hz curve, generating the machine torque equal to$-T_G$ at that instant. The negative sign here means that the shaft absorbs the torque and converts into electrical power, depleting the kinetic energy and subsequently braking the motion of moving mass attached to the shaft (train, car, etc.). If we want to stop the motor with continued regenerative braking, then we gradually ramp down the frequency from 60 Hz to a low value, simultaneously ramping down the voltage to keep the *V/f* ratio constant.

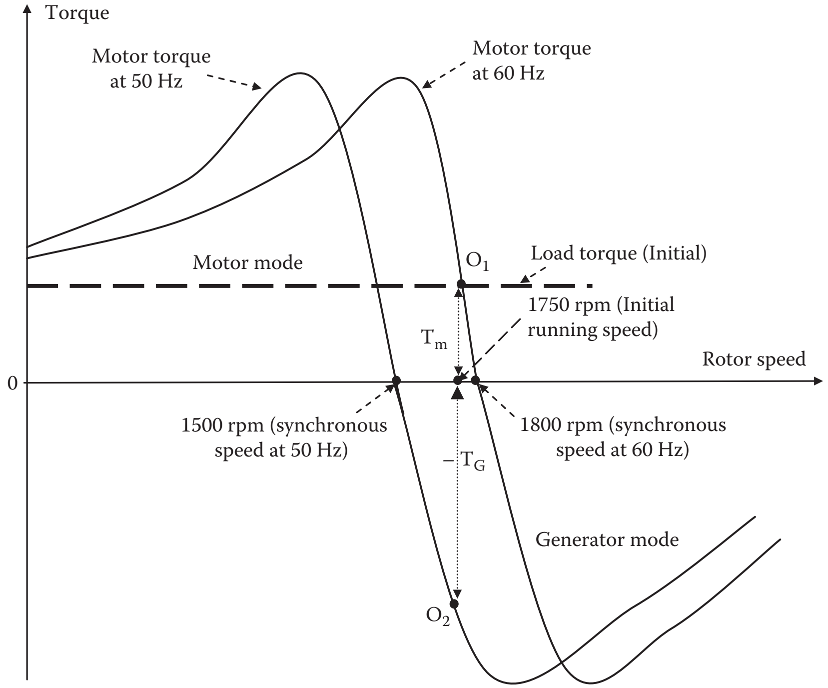


FIGURE 11.9 Regenerative braking of induction motor by changing the supply frequency from 60 Hz to 50 Hz in the first step.

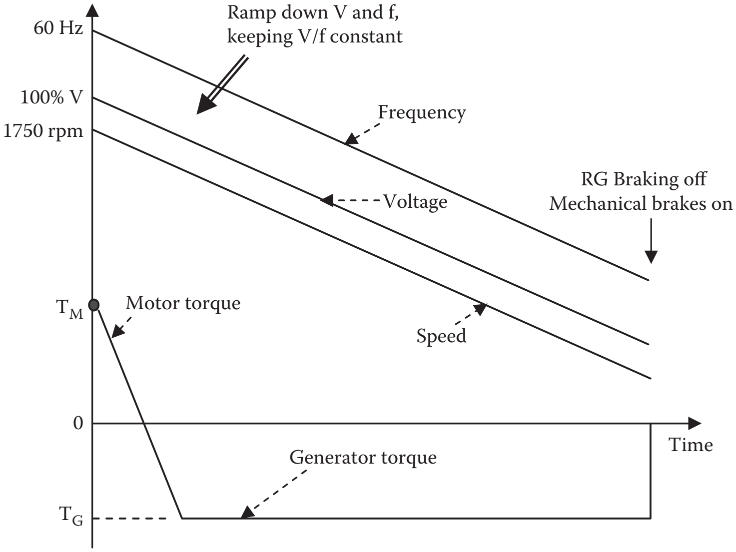


FIGURE 11.10 Frequency and voltage ramped down for induction motor braking from full to near zero speed.

In SI units, power = torque × ω_{mech}, where $\omega_{mech} = 2\pi \times \text{rpm} \div 60$ rad/sec and rpm = actual running speed of machine.

$$\therefore \text{ Regenerative power fed back to source } P_{RG} = \frac{T_G \times 2\pi \times rpm}{60} \qquad (11.16)$$

As the speed comes down, the frequency and voltage are lowered further down, and down, and down … to keep the machine torque negative in order to continuously recapture most of the kinetic energy of the system as shown in Figure 11.10. When negligible kinetic energy is left in the system at a very low speed, the RGB is turned off and the mechanical brakes are applied to bring the vehicle to a full stop. The regenerative power fed back to the source in this ramp down is given by Equation (11.16).

A simpler alternative explanation of the induction motor braking follows. The motor typically runs 3%–5% below the synchronous speed with positive slip = (synchronous speed − rotor speed), and I_{rotor} = constant × slip. If the frequency is suddenly reduced, the synchronous speed becomes lower than the rotor speed, which cannot change suddenly due to inertia. The slip momentarily becomes negative, and the current and torque become negative, converting the kinetic energy of the rotor inertia into the electrical energy and feeding back into the source (grid or battery).

Example 11.8

A 3-phase, 4-pole, 60 Hz induction motor is running at 1746 rpm under full load. Determine the regenerative braking torque at the instant of frequency suddenly reduced to 95% using variable frequency drive while maintaining the *V/f* ratio constant. Develop your answers in percentage of rated values.

SOLUTION

From the motor fundamentals, we know the following, where all *K*s are the proportionality constants.

Electromechanical torque of the motor, $T_{em} = K_1 \cdot$ stator flux $\cdot$ rotor current (a)

With constant *V/f* ratio, stator flux amplitude remains constant (b)

Rotor current = $K_2 \cdot d\varphi/dt = K_3 \cdot$ rotor slip speed (c)

Equations (b) and (c) in (a) give $T_{em} = K_4 \times$ rotor slip speed

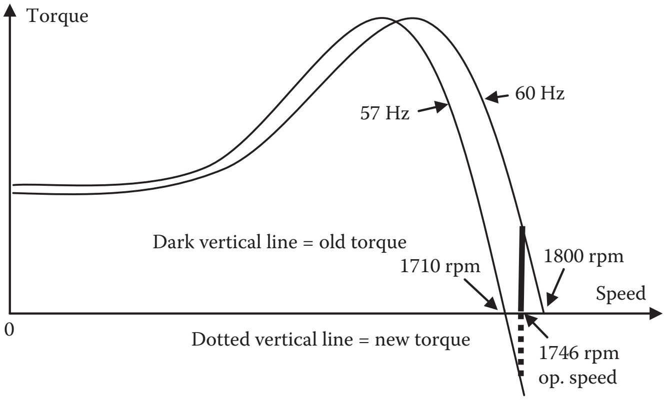
At 60 Hz, torque = rated value, synch speed = 1800 rpm for 4-pole motor, rotor speed = 1746 rpm, and slip speed = 1800 – 1746 = 54 rpm (positive).

At a suddenly reduced frequency to 95%, that is, at a new frequency of 0.95 × 60 = 57 Hz, the new synchronous speed = 1710 rpm, but the motor would still continue to run at 1746 rpm for a while due to inertia, so the new slip speed = 1710 – 1746 = –36 rpm (negative)

New Torque ÷ Old Torque = New slip speed ÷ Old slip speed = – 36 ÷ 54 = – 0.667

New Torque = (– 0.667) × (Old torque) = – 66.7% of old torque

The negative sign signifies the regenerative (not motoring) torque, converting the inertial energy of the rotor into electrical energy and feeding back to the source; simultaneously slowing down the motor speed, hence called the *regenerative braking torque*.



11.7.3 DC MOTOR BRAKING

The regenerative braking scheme with dc motor is shown in Figure 11.11. In the motoring mode, with notations shown in the figure, the armature current going into the machine is given by

$$I_a = \frac{V_T - E_a}{R_a} = \frac{V_T - K\, I_f\, n}{R_a} \tag{11.17}$$

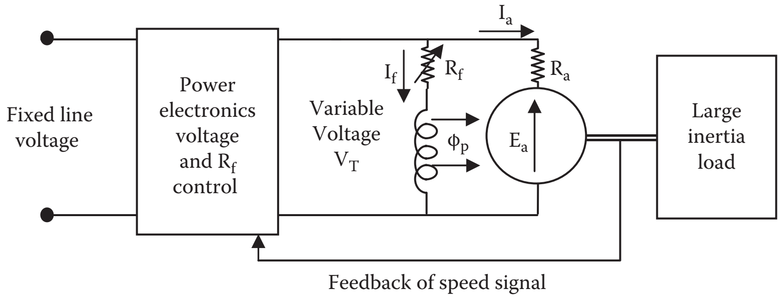


FIGURE 11.11 Variable speed drive configuration for dc motor.

For regenerative braking, we lower the machine terminal voltage V_T by power electronics voltage controller such that V_T is lower than the counter emf, that is, $V_T < K\ I_f\ n$. The armature current I_a—given by Equation (11.17)—now becomes negative, flowing backward from the machine terminals. The machine now works as the generator and the load's kinetic energy gradually depletes as it gets converted into electrical power fed back to the source. To extract most of the kinetic energy of the load, we keep lowering V_T so that I_a remains negative as the speed gradually drops to a low value, when we switch to the mechanical brakes. The power fed back to the source in this process is given by

$$P = E_a \cdot I_a = (K \cdot I_f \cdot n)\left(\frac{V_T - K \cdot I_f \cdot n}{R_a}\right) \tag{11.18}$$

In a simpler explanation, the motor back voltage is always less than the terminal voltage, making the current flowing into the rotor. If the terminal voltage is suddenly reduced before the back emf (function of speed) can adjust, the rotor current reverses and so does the torque and power flow. This depletes the kinetic energy of the moving mass, which slows down. In dc machines, $I_f = V_T/R_f$ ratio is maintained constant while reducing the terminal voltage during the entire regenerative braking operation as shown in Figure 11.12. A constant V_T/R_f ratio essentially keep the flux constant at rated value to maintain the rated torque on the shaft.

11.7.4 NEW YORK AND OSLO METRO TRAINS

For estimating possible energy saving with regenerative braking in metro trains as a study project, we first review the Long Island–New York City metro trains in operation for decades, and the newly installed metro trains (T-Bane) in Oslo, Norway.

The Long Island trains use nominal 600 Vdc power drawn from the third rail running parallel to two traction tracks. Each car is 10 ft wide × 10 ft tall × 51 ft long on 4 axles, sits 44 passengers, and is individually propelled by its own motors. Each car weighs 40 tons empty and 50 tons full. A train may use 4 to 12 cars, depending on

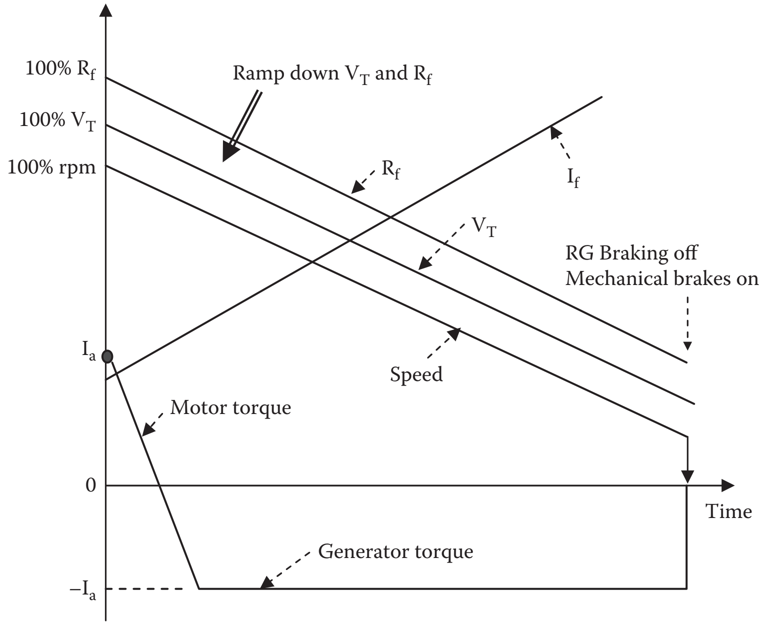


FIGURE 11.12 Voltage and field resistance ramped down for dc motor braking from full to near zero speed.

the route and time of the day. It can travel up to 55 mi/h (mph) on the standard-gage tracks. All cars in a train are controlled from a central control station. Acceleration is controlled at 2.5 mph/sec rate up to full speed of 55 mph. Deceleration is at 2.5 mph/sec rate by regenerative braking from 55 mph down to 10 mph, and then by pneumatic air-brakes to full stop.

The dc motors are 4-pole compound wound with commutating poles, using both series and shunt field coils, which are changed as required during the acceleration and deceleration periods. Each motor is rated 100 hp, 300 V, 280 A, 1175 rpm. Each car has four motors and gear assemblies. All four motors are connected in series for reduced voltage starting to control the initial starting inrush current, and the motors are connected (two in series × two in parallel) during full-speed operation.

The total resistance (drag) R on each rail car is a function of the static and dynamic rolling frictions, and the aerodynamic drag. On a straight track without curvature or grade, it is given by Davis formula,

$$R_1 = 1.3 + (29/w) + b \cdot V + (c \cdot A \cdot V^2/w \cdot n) \quad \text{Lbf per ton of car weight} \qquad (11.19)$$

where

w = car weight on each axle (tons per axle of car)
b = dynamic friction coefficient (0.04)
c = aerodynamic drag coefficient (0.0025 for the first car and 0.00035 for the trailing cars)

A = cross section perpendicular to travel (~100 ft^2)
n = number of axles per car
V = speed of travel (mph)

$$\text{The total resistance to each car, } R_{\text{car}} = R_1 \cdot w \cdot n \tag{11.20}$$

Total resistance R_{Train} to the entire train is then the sum of R_c's of all cars, including the first car, which see greater aerodynamic resistance, that is, $R_{Train} = \Sigma R_{car}$. If η = motor shaft to traction efficiency (~85%), then horsepower required for the train to overcome this drag is

$$\text{HP} = R_{\text{Train}} \cdot V \div (375 \cdot \eta) \quad \text{where } R_{\text{Train}} = \Sigma R_{\text{car}} \tag{11.21}$$

The City of Oslo during the 2008–2010 time period replaced its 65 old T-bane metro trains with new three-car trains from Siemens that are about 35% more energy efficient than the old trains. This is achieved by using the regenerative braking in the new trains that converts up to 45% of the kinetic energy of the moving train into electric energy when braking.

The environment benefits are a major side benefit of the regenerative braking. By one estimate, the average metro train or tram contributes about 25 g of CO_2 to the atmosphere per km traveled. The regenerative braking can reduce it to about 15 g. The T-bane trains made from light weight aluminum require less energy to accelerate in the first place. Using the regenerative braking, it recaptures about 45% of the electrical energy back into the grid. Using the balance of the energy (55%) mainly from hydro-electric power, Oslo has one of the smallest carbon foot-prints for a city of its size in the world.

Two design challenges with the regenerative braking in the metro train system are:

i. The power generation occurring in one decelerating train has to match in time with power demand from another nearby train that is accelerating, so that less net power the metro authority has to draw from the power grid at a given time.
ii. The distribution line voltage drops when the train is drawing power during acceleration, and voltage rises when the train is decelerating with RGB. The voltage drop and rise are equal in magnitude. If the drop is say 7% (relatively high since train at low voltage draws heavy current), the voltage fluctuation is 2 × 7 = 14%, which is not easy to work with in a practical system. For this reason, power distribution stations for the train power where the voltage can be closely regulated need to be closely spaced, so that 7% in this example can be reduced to say 2.5%, giving a manageable 2 × 2.5 = 5% voltage fluctuation. This may not be easy in many large cities, where the right-of-way to build new distribution stations is difficult to obtain. This situation is currently faced by New York City subway trains; many of them are still not using RGB.

PROBLEMS

Problem 11.1: A 2 MVA transformer is open for procurement. Vendor A offers 96.5% efficiency at $160,000 price and vendor B offers 97.8% efficiency at $180,000 price. Determine the present worth of buying the high efficiency transformer. Assume that (i) quoted efficiency is at rated load at 90% pf lagging, (ii) core loss = 1/3 total loss at rated load, (iii) energy cost = 0.12 $/kWh, and (iv) discount rate = 12% per year. Answer in terms of the power loss capitalization rate.

Problem 11.2: A 5000 hp variable frequency motor drive is under consideration, using either an induction motor or a synchronous motor. The estimated system efficiency of the synchronous motor with drive is 92% and that of the induction motor with drive is 88%. The motor is projected to run 600 hours per month on average. Determine the total energy cost saving over 30-year expected life of the system using the synchronous motor drive if the energy cost is 12 cents per kWh. Also determine the present worth of life-time savings discounted at 0.75% per month.

Problem 11.3: A new 25 hp motor is required to replace an old one. It will run 500 hours per month, and the energy cost is $0.15 per kWh. You have two options available (i) standard motor with 90% efficiency at $3000 cost, or (ii) high efficiency motor with 93% efficiency at $4000 cost. Determine the simple and discounted payback periods of the additional cost of high efficiency motor at a discount rate of 9% per annum.

Problem 11.4: The table below shows a factory's electrical power usage pattern. The utility contract calls for energy charge of $0.12 per kWh and demand charge of $14 per peak kVA over any 15 min period during the month. Determine (i) the utility bill in a 30-day month, and (ii) if the pf during the first shift is improved to unity, determine the new monthly bill.

Shift	Hours Used	Average kW	Peak Demand kVA
1	8	1200	1800
2	6	800	1000
3	4	600	800

Problem 11.5: The daily power consumption profile of a customer is tabulated below, where the kW loads are average over the period listed and kVA demand meter readings are peak over 15 minute intervals during the period. The utility tariff to this consumer is 15 cents per kWh plus $18 per peak kVA demand over the month. In a 30-day month, determine the monthly utility bill (a) with present usage pattern, and (b) with pf improved to unity, and (c) the daily load factor of this customer.

Time Period	Power Usage kW	kVA Demand
12 a.m.–3 a.m.	200	270
3 a.m.–8 a.m.	200	240
8 a.m.–10 a.m.	500	700
10 a.m.–3 p.m.	900	1200
3 p.m.–9 p.m.	600	800
9 p.m.–12 p.m.	200	250

Problem 11.6: Determine the capacitor rating in kVAR and also in farad to improve the load pf from 0.75 lagging to 0.95 lagging at a 3-phase, 60 Hz, 480 V distribution point delivering 2400 kW real power. Assume that the motor runs for 400 hours per month, the utility demand charge is $15 per kVA, energy charge is $0.15 per kWh, capacitor cost is $70 per kVAR installed, and cable power loss is 1% per phase.

Problem 11.7: A 200 kVAR, 2400 V, 50 Hz, 3-phase European-made capacitor bank is connected to 3300 V, 60 Hz lines in the United States. Determine its new kVAR capacity in the United States.

Problem 11.8: A 3-phase, 100 hp, 2-pole, 60 Hz induction motor is running at 1740 rpm under full load. Determine the regenerative braking torque at the instant of reducing frequency to 96% using a variable frequency drive while maintaining the *V/f* ratio constant.

Problem 11.9: What is the present worth of $1000 coming at the end of (i) 6th month, (ii) 24th month, and (iii) 30 years at the discount rate of 1% per month?

Problem 11.10: Your plant needs to buy a 1000 hp motor for a new conveyor belt. You have the following information: (i) standard motor having 90% efficiency costs $96,000, (ii) high efficiency motor having 95% efficiency costs $145,000, (iii) the motor will be operating 6 days a week, 16 hours per day, all year around, (iv) the energy cost to your plant is 0.10 $/kWh, (v) cost of money to your company is 1% per month, and (vi) life of the motor is 30 years. As the project manager, you are considering buying a high efficiency motor. Ignoring the savings in the demand charge, first determine the savings per month in the energy cost. At that monthly savings, determine the discounted payback period for the additional cost of high efficiency motor. Would you buy the high efficiency motor and why? Be quantitative in your answer.

Problem 11.11: The plant you are working uses 3-phase, 30 MW, 60 Hz power at 6.6 kV, 0.80 pf lagging. Determine (a) the size of a 3-phase Y-connected capacitor bank that will improve the pf to 0.95 lagging, (b) current drawn from the utility mains with the old pf and the new pf, and (c) simple payback period for recovering the capital cost of the capacitor. Assume: (i) power loss reduction in the cables is negligible, (ii) capital cost of capacitors is 40 $/kVAR installed, (iii) discount rate is 1% per month, and (iv) monthly utility tariff is 10 $/kVA demand + 0.10 $/kWh energy.

QUESTIONS

Question 11.1 Explain the difference between terms the *interest rate* and *discount rate* used in the economic analysis for determining a profitability of investment in a capital project.

Question 11.2 What is the difference between simple payback period and discounted payback period?

Question 11.3 Why is the power loss capitalization rate more important for the transformer and generator than for the motor?

Question 11.4 Explain clearly the difference between poor power factor and poor efficiency, and their causes and remedies.

Question 11.5 Explain the difference between regenerative braking and dynamic braking, and at what speed they take place.

Question 11.6 Identify a few more candidates for RGB (other than listed in the book).

Question 11.7 Why does the RGB of the dc motor require variable voltage and variable field resistance, whereas the ac motor requires variable frequency and variable voltage?

Question 11.8 What is a rough estimate of the percentage of energy that can be saved using RGB in city metro trains with stations only one mile apart?

Question 11.9 Name types of energy storage systems you can use for large-scale energy storage, say, for a wind farm or a solar park.

FURTHER READING

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12 Electrochemical Battery

Electrical power, although a very convenient form of energy to distribute and use, cannot be easily stored on a large scale. Almost all electrical power generated by utility power plants is consumed simultaneously in real time. However, various technologies are presently available to store energy on a relatively small scale in electrical, mechanical, chemical, and magnetic form. The energy storage densities in these alternatives and their typical durations of use are compared in Table 12.1.

The electrochemical battery stores energy in the electrochemical form for a wide variety of applications in consumer products and industrial plants. Its energy conversion efficiency from electrical to chemical or vice versa is about 85%. There are two basic types of electrochemical battery:

Primary battery: It converts chemical energy into electrical energy, in which the electrochemical reaction is nonreversible, and the battery after a full discharge is discarded. It has high energy density, both gravimetric (Wh/kg) and volumetric (Wh/liter). For this reason, it finds applications where high energy density for one-time use is required.

Secondary battery: It is also known as the rechargeable battery. Its electrochemical reaction is reversible. After a discharge, it can be recharged by injecting a direct current from an external source. In the charge mode, it converts the electrical energy into chemical energy. In the discharge mode, it is reversed, converting the chemical energy into electrical energy. In both charge and discharge modes, about 15% of energy is converted into heat each way, which is dissipated to the surrounding medium. Therefore, the round trip energy conversion efficiency is 0.85 × 0.85, that is, 70% to 75%, depending on the electrochemistry.

The rechargeable battery is used in industrial plants and ships for (a) emergency power for essential loads, (b) control circuits, and (c) starting power for generator prime mover. At the utility power grid level, large batteries have applications for: (1) energy storage in wind farms and solar parks, (2) smart grid with communication and cyber security lines, (3) improving transmission system reliability to prevent voltage collapse and blackout and, (4) improving the quality of power in the utility and end-user electrical systems, and (5) programs to reduce emissions, promote more efficient energy use, and improve environment quality.

The internal construction of a typical electrochemical cell, primary or rechargeable, is shown in Figure 12.1. It has positive and negative electrode plates with insulating separators and a chemical electrolyte inbetween. The two groups of electrode plates are connected to two external terminals mounted on the casing. The cell stores electrochemical energy at a low electrical potential, typically 1.2 to 3.6 V, depending

TABLE 12.1
Electrical, Mechanical, and Magnetic Energy Storage Technologies

Energy Storage Technology	Energy form in Storage	Storage Duration in Typical Applications	Relative Energy Density (kWh/kg)
Capacitor	Electrical field	< Seconds	Low
Inductor	Magnetic field	< Seconds	Moderate
Battery	Electrochemical	Days, months	Moderate
Flywheel	Kinetic energy	Days, months	High
Superconducting magnet (coil)	Magnetic field	Days, months	Very high

on the electrochemistry. The cell's charge-holding capacity is denoted by C, which is measured in the ampere-hours (Ah) it can deliver. Using the basic definition of 1 A = 1 coulomb/sec, we have 1 Ah = 1 coulomb/sec × 3600 seconds = 3600 coulombs of charge. A cell of capacity C can deliver C amperes for 1 hour or C/n amperes for n hours. The Ah capacity depends linearly on the electrode plate area. The cell voltage, on the other hand, depends solely on the electrochemistry, and is independent of the plate area.

The battery is made of numerous electrochemical cells connected in a series-parallel combination to obtain the desired battery voltage and current. A higher voltage requires a greater number of cells in series, whereas a higher current requires a greater number of cells in parallel. The overall battery rating of all cells combined is stated in terms of the average voltage during discharge and the Ah capacity it can deliver before the voltage drops below the specified limit. The product of the average voltage and the Ah rating forms the Wh energy rating of the battery that it can deliver to the load from the fully charged condition.

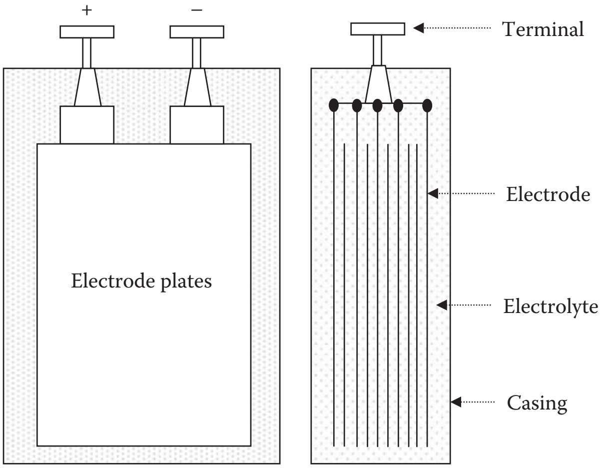


FIGURE 12.1 Electrochemical cell construction.

12.1 MAJOR RECHARGEABLE BATTERIES

Several battery technologies have been fully developed and are in use in the industrial and consumer markets. New electrochemistries are being continuously developed by the United States' advance battery consortium for a variety of applications, such as electric vehicles, spacecraft, utility load leveling and, of course, for wind and solar power systems with inherently intermittent power generation. These electrochemistries are

- Lead-acid (Pb-acid)
- Nickel-cadmium (NiCd)
- Nickel-metal hydride (NiMH)
- Lithium-ion (Li-ion)
- Lithium-polymer (Li-poly)
- Sodium battery

The average voltage during discharge depends on the electrochemistry, as listed in Table 12.2. The energy densities of various batteries, as measured by the Wh capacity per kilogram mass and Wh per liter volume, are compared in Figure 12.2. The selection of the electrochemistry for a given application is a matter of performance, size, and cost optimization. Some construction and operating features of the foregoing electrochemistries are presented in the following text.

12.1.1 LEAD ACID

This is the most common type of rechargeable battery in use today because of its maturity and high performance per unit cost, even though it has the least energy density by weight and volume. In the lead-acid battery under discharge, water and lead sulfate are formed, the water dilutes the sulfuric acid electrolyte, and specific gravity of the electrolyte decreases with the decreasing state of charge. The recharging reverses the reaction, in which the lead and lead dioxide are formed at the negative and positive plates, respectively, restoring the battery to its originally charged state.

TABLE 12.2
Average Cell Voltage During Discharge in Various Rechargeable Batteries

Electrochemistry	Typical Applications	Cell Voltage	Remarks
Lead-acid	Most widely used in industry, cars, boats	2.0	Least-cost technology
Nickel-cadmium	Portable tools	1.2	Exhibits memory effect
Nickel-metal hydride	Automobiles	1.2	Temperature sensitive
Lithium-ion	Computers, cell phones, spacecraft	3.6	Safe, contains no metallic lithium
Lithium-polymer	Computers, cell phones, spacecraft	3.0	Contains metallic lithium

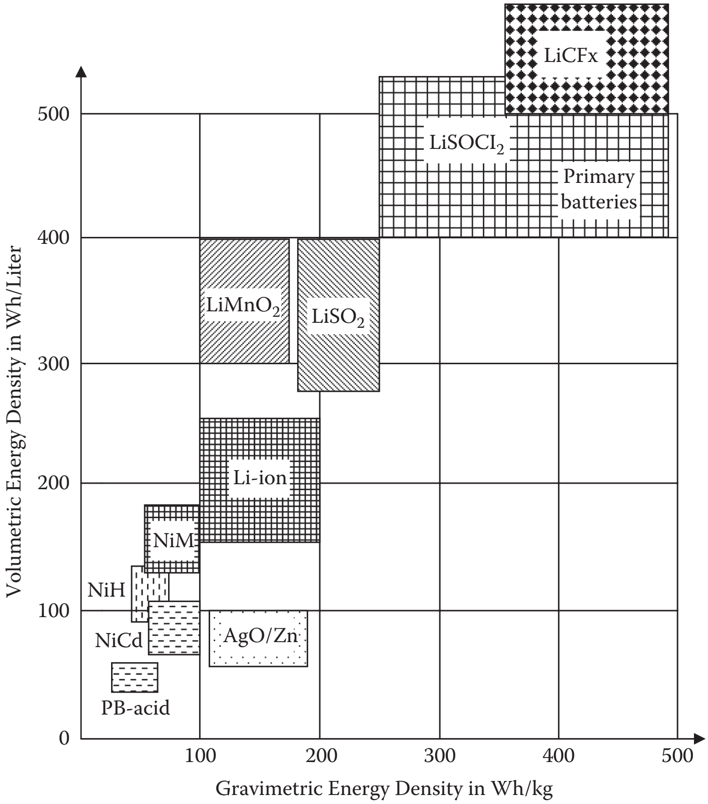


FIGURE 12.2 Energy density comparison between various battery cells.

The lead-acid battery comes in various versions. The shallow-cycle version is used in automobile and shipboard engine starting, where a short burst of energy is drawn from the battery to start the engine. The deep-cycle version, on the other hand, is suitable for repeated full charge and discharge cycles. Most energy storage applications require deep-cycle batteries. The lead-acid battery is also available in sealed *gel-cell* version with additives, which turns the electrolyte into a nonspillable gel. The gel-cell battery, therefore, can be mounted sideways or upside down. The high cost, however, limits its use in military avionics.

12.1.2 NICKEL CADMIUM

NiCd is a matured electrochemistry, in which the positive electrode is made of cadmium and the negative electrode is made of nickel hydroxide. The two electrodes are separated by Nylon™ separators and placed in potassium-hydroxide electrolyte in a stainless steel casing. With a sealed cell and about 50% weight of the conventional lead-acid, the NiCd battery has been widely used in the past to power most rechargeable consumer products. It has a longer deep cycle life, and is more temperature tolerant than the lead-acid battery. However, this electrochemistry has a memory effect (explained in Section 12.3.5), which degrades the capacity if not used for a long time. Moreover, cadmium has recently come under environmental regulatory scrutiny. For these reasons, the NiCd is being replaced by NiMH and Li-ion batteries in laptop computers and other similar high-priced consumer electronics.

12.1.3 Nickel Metal Hydride

NiMH is an extension of NiCd technology, which offers an improvement in energy density and has a negligible memory effect. The major constructional difference is that its anode is made of a metal hydride that eliminates the environmental concerns of cadmium. Compared to NiCd, it has lower peak power delivering capability, higher self-discharge rate, and is susceptible to damage due to overcharging. It is also expensive at present, although the future price is expected to drop significantly. This expectation is based on current development programs targeted for large-scale applications of this technology in electric vehicles.

12.1.4 Lithium Ion

Li-ion technology is a recent development, which offers about three times the energy density over that of lead-acid. Such a large improvement in the energy density comes from lithium's low atomic weight of 6.9 versus 207 for lead. Moreover, lithium-ion has a higher cell voltage, 3.5 V per cell versus 2.0 V for lead-acid and 1.2 V for other electrochemistries. This requires fewer cells in series for a given battery voltage, thus reducing the assembly cost.

On the negative side, the lithium electrode reacts with any liquid electrolyte, creating a sort of passivation film. Every time when the cell is discharged and then charged, the lithium is stripped away, a free metal surface is exposed to the electrolyte, and a new film is formed. This is compensated by using thick electrodes, or else the life would be shortened. For this reason, Li-ion is more expensive than NiCd. Moreover, the lithium-ion electrochemistry is vulnerable to damage from overcharging and other shortcomings in the battery management. Therefore, it requires a more elaborate charging circuitry with adequate protection against overcharging.

New companies in the United States have started to develop new Li-ion batteries for electric cars and to compete with the present Asian suppliers such as Sanyo and Hitachi. Companies like A123 and Sakti3 are focusing on key performance requirements for electric car batteries that (1) provide enough power for acceleration, (2) store enough energy to guarantee the required range, (3) survive numerous charge/discharge cycles over 8 to 10 years for the average user, and (4) keep costs below the level acceptable to electric car buyers. The Li-ion battery has high power and energy density compared to other batteries at present. This results in concentrated heat in a small volume, causing a high temperature rise that can cause the battery to explode, as was experienced recently in some laptop computers. At about $1000 per kWh cost in 2010, the GM Volt's 16 kWh battery alone would cost about $16,000. This is rather high at this time, but is expected to fall to one-half in several years based on the ongoing developments.

12.1.5 LITHIUM POLYMER

This is a lithium battery with a solid polymer electrolyte. It is constructed with a film of metallic lithium bonded to a thin layer of solid polymer electrolyte. The solid polymer enhances the cell's specific energy by acting as both the electrolyte and the separator. Moreover, the metal in solid electrolyte reacts less than it does with a liquid electrolyte. A few companies are developing the lithium polymer battery for cars that may double the range compared to other batteries.

12.1.6 SODIUM BATTERY

The sodium battery is the newest electrochemistry that is entering the market. The General Electric Company makes this advanced battery in a new plant near Albany, New York, for hybrid locomotives, heavy service vehicles, and backup storage in wind and solar energy farms. The sodium electrochemistry is based on a sodium-metal halide technology, which is better suited for the short bursts of intense power required to get a vehicle moving, and also for high power over a long period of time.

12.2 ELECTRICAL CIRCUIT MODEL

The battery—as a first approximation—works like a constant internal voltage source V_i with a small internal resistance R_i as shown in Figure 12.3, both of which vary with Ah discharged as follows:

$$V_i = V_o - K_1 \cdot DoD \qquad \text{and} \qquad R_i = R_o + K_2 \cdot DoD \tag{12.1}$$

where,

$$\text{Depth of discharge, } DoD = \frac{Ah\ drained\ from\ battery}{Rated\ Ah\ \ capacity} \tag{12.2}$$

$$\text{State of charge, } SoC = \frac{Ah\ remaining\ in\ battery}{Rated\ Ah\ capacity} = 1 - DoD \tag{12.3}$$

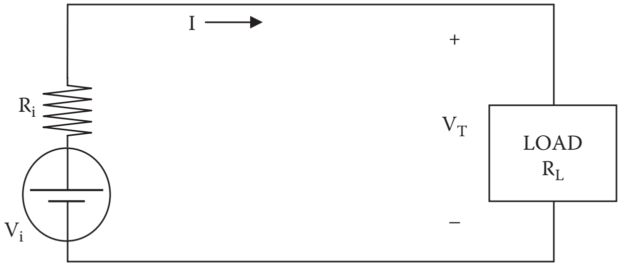


FIGURE 12.3 Electrical model of battery for steady-state performance.

V_i = open-circuit (or electrochemical) voltage that decreases linearly with DoD
R_i = internal resistance that increases linearly with DoD
V_o and R_o = values of V_i and R_i in fully charged state (DoD = 0)
K_1 and K_2 = constants determined by curve fitting through the battery test data

Thus, the value of V_i is lower and R_i is higher in a partially discharged state (i.e., when DoD > 0). The terminal voltage of a partially discharged battery delivering load current I is less than V_i by the internal voltage drop $I \cdot R_i$; that is,

$$V_T = V_i - IR_i = V_o - K_1 DoD - IR_i \tag{12.4}$$

The power delivered to the external load is I^2R_L, and the internal power loss is I^2R_i, which is dissipated as heat. As the battery is discharged, its internal resistance R_i increases, which progressively generates more heat.

Example 12.1

A battery with the internal resistance of 0.01 Ω/cell needs to deliver 10 A current at 120 V at the load terminals. Determine the number of cells in series if the electrochemical voltage is 1.25 V/cell.

SOLUTION

First, we recognize that every cell in series must carry the load current, 10 A in this example. The battery terminal voltage = number of series cells × (internal electrochemical voltage less the voltage drop in the internal resistance per cell).

With N series cells, $120 = N\,(E_i - I \times R_i) = N\,(1.25 - 10 \times 0.01)$

$\therefore\ N = 120 \div 1.15 = 104.35$ cells in series.

We then choose 105 cells to have a slightly higher than 120 V at the load terminals at the beginning of battery life, which would decrease slightly with age.

12.3 PERFORMANCE CHARACTERISTICS

As stated earlier, the cell capacity is measured in ampere-hours (Ah). A cell of capacity C Ah can deliver C amperes for one hour or C/n amperes for n hours. Each cell delivers the rated Ah at the terminal voltage equal to the cell voltage (1.2 to 3.6 V) that depends on the electrochemistry. The battery charge and discharge rates are stated in units of its capacity C. For example, charging a 100 Ah battery at $C/10$ rate means charging at 100/10 = 10 A, at which rate the battery will be fully charged in 10 hours. Discharging a 100 Ah battery at $C/2$ rate means drawing 100/2 = 50 A, at which rate the battery will be fully discharged in 2 hours.

The battery voltage varies over time with the charge level, but the term *battery voltage* in practice refers to the average voltage during discharge. The battery capacity is defined as the Ah charge it can deliver before the voltage drops below a certain limit (typically about 80% of the average voltage). Energy storage capacity is the product of voltage and Ah capacity, that is, V × Ah = (VA) × h = Wh.

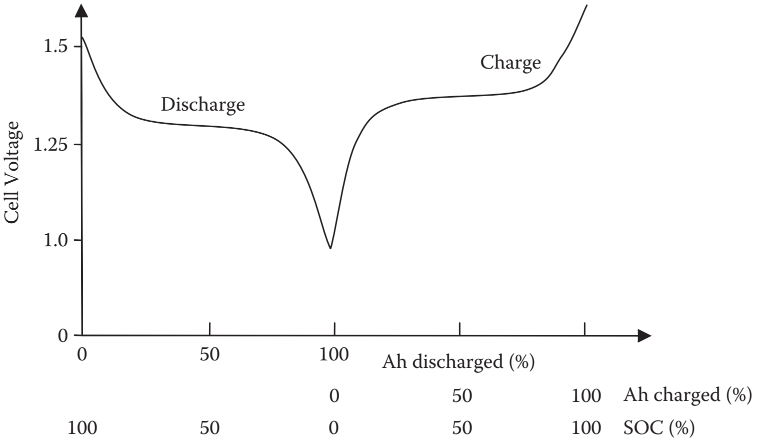


FIGURE 12.4 Discharge and charge voltages of a 1.25 V cell.

The battery performance is characterized in terms of (1) charge/discharge (c/d) voltages, (2) charge/discharge Ah ratio (*c/d* ratio), (3) self-discharge and trickle charge rates, (4) round trip energy efficiency, and (5) life in number of charge/discharge cycles (*c/d* life).

12.3.1 CHARGE/DISCHARGE VOLTAGES

Figure 12.4 shows the charge/discharge voltages of a nominally 1.25 V cell, such as NiCd and NiMH. We notice that the battery voltage has a flat plateau while discharging and another plateau during charging. The average values of these plateaus are known as the discharge voltage and charge voltage, respectively. The *c/d* voltages also depend on temperature and on how fast the battery is charged or discharged.

12.3.2 *C/D* RATIO (CHARGE EFFICIENCY)

It is defined as the ratio *Ah charged/Ah discharged* with no net change in the state of charge. The *c/d* ratio—always greater than unity—depends on the charge and discharge rates and the temperature as shown in Figure 12.5. For example, if a battery has a *c/d* ratio of 1.1 at a certain temperature, it means that the battery needs 10% more Ah charge than what was discharged in order to restore it to the fully charged state. The *charge efficiency*—also known as the *coulombic efficiency*—is the inverse of the *c/d* ratio.

12.3.3 ROUND TRIP ENERGY EFFICIENCY

The energy efficiency in one round trip of discharge and charge is defined as the ratio *Energy output/Energy input* at the terminals, which is given by

$$Round\,trip\,energy\,efficiency = \frac{Average\,discharge\,voltage \times Ah\,capacity}{Average\,charge\,voltage \times C/d\,ratio \times Ah\,capacity} \tag{12.5}$$

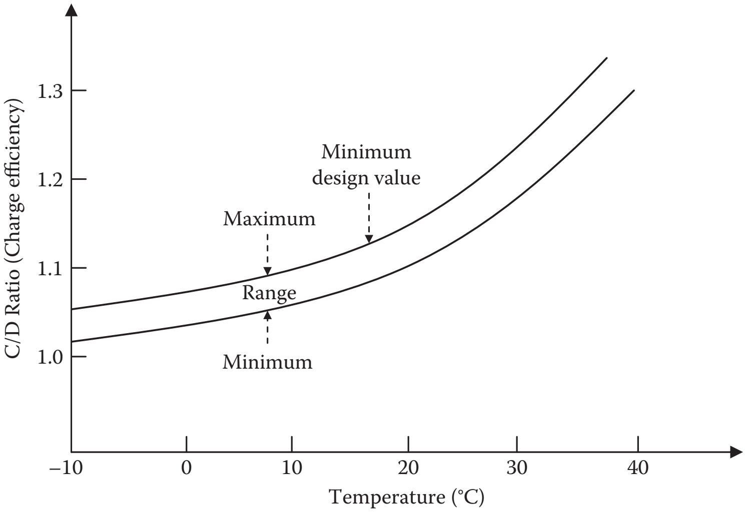


FIGURE 12.5 C/d ratio (charge efficiency) versus temperature.

For example, for a cell of capacity C with an average discharge voltage of 1.2 V, an average charge voltage of 1.45 V, and a c/d ratio of 1.1, the round trip energy efficiency at the cell terminals is

$$\eta_{energy} = \frac{1.2 \times C}{1.45 \times 1.1C} = 0.75 \text{ or } 75 \text{ percent} \tag{12.6}$$

Thus, in this battery, 25% of the energy is wasted (turned into heat) in a round trip of full discharge and then full charge. The battery room, therefore, is generally hot and hazardous with combustible fumes, especially if nonsealed cells are used.

Example 12.2

You have two options in procuring 100 Ah battery cells for a large battery assembly in an industrial plant, which requires frequent charging and discharging. Cell A has an average cell voltage of 1.2 V during discharge, 1.4 V during charge, and a *c/d* ratio of 1.1. Cell B has an average cell voltage of 1.3 V during discharge, 1.5 V during charge, and a *c/d* ratio of 1.15. Both cells A and B cost the same per Ah rating. Select the favorable cell.

SOLUTION

From the given data, both battery cells appear to be of the same electrochemistry. The small difference in the performance may be due to different cell design and construction methods—such as electrode thickness and electrolyte concentration—used by two manufacturers.

With equal cost, the decision has to be based on the round trip energy efficiency. Using Equation (12.15), we obtain

$$\text{Round trip energy efficiency of cell A} = \frac{100 \times 1.2}{100 \times 1.1 \times 1.4} = 0.779$$

$$\text{Round trip energy efficiency of cell B} = \frac{100 \times 1.3}{100 \times 1.15 \times 1.5} = 0.754$$

Cell A is preferred, since it has higher efficiency of 77.9% versus 75.4% for cell B.

12.3.4 SELF-DISCHARGE AND TRICKLE-CHARGE

The battery slowly self-discharges—even in open-circuit condition—at a rate that is typically less than 1% per day. To maintain full charge, the battery must be continuously trickle-charged to counter the self-discharge rate.

After the battery is fully charged, the energy storage stops rising, and any additional charge is converted into heat. If overcharged at a higher rate than the self-discharge rate for an extended period of time, the battery overheats, posing a safety hazard of potential explosion. Overcharging also produces internal gassing, which scrubs and wares the electrode plates, shortening the life. Therefore, the battery charger should have a regulator that detects full charge and cuts back the charge rate to trickle-rate after the battery is fully charged.

12.3.5 MEMORY EFFECT IN NICD

The NiCd battery remembers its *c/d* pattern and changes the performance accordingly. After it is repeatedly charged and discharged to say 25% of its full capacity to point M in Figure 12.6, it remembers point M. Subsequently, if it is discharged beyond point M,

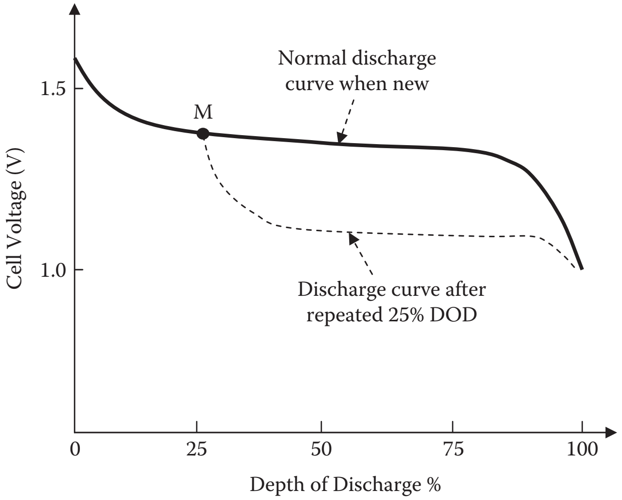


FIGURE 12.6 Memory effect in a NiCd battery.

the cell voltage drops much below its original normal value shown by the dotted line, resulting in a loss of full capacity. This phenomenon is like losing our body muscle due to lack of use over a long time. A remedy for restoring the full capacity after many shallow discharge cycles is to *exercise* the battery, also called *reconditioning* the battery. It means to fully discharge it to almost zero voltage once every few months and then fully charge it to full voltage per cell. The memory effect is unique to the NiCd battery; most other types of electrochemistries do not have such a memory effect.

12.3.6 TEMPERATURE EFFECTS

The operating temperature significantly influences the battery performance in three ways: (a) charge storage capacity and charge efficiency decrease with increasing temperature, (b) self-discharge rate increases with increasing temperature, and (c) internal resistance decreases with increasing temperature. Various battery characteristics affecting the thermal design are listed in Table 12.3.

12.4 BATTERY LIFE

The battery life is determined by its weakest cell in the long chain of cells. The battery cell can fail short or open. A shorted cell loses the voltage and Ah capacity completely, and works as a load on healthy cells in the battery. Charging the battery with a shorted cell may result in heat-related damage to the battery or to the charger. The cell can also fail open, which would disable the entire battery string.

Charging two parallel batteries by one common charger—especially with a shorted cell in one battery—can result in highly uneven current sharing, and may result in overheating both batteries and shortening the life. Even worse, the healthy battery can end up charging the defective battery, in addition to providing full load by itself. Two remedies to avoid this are: (1) charge both batteries with individual chargers at their own rated charge current, (2) replace the shorted cell immediately (this can sometimes be impractical), and (3) use an isolation diode in both batteries.

TABLE 12.3
Battery Characteristics Affecting Thermal Design

Electrochemistry	Op. Temperature. Range (°C)	Overcharge Tolerance	Heat Capacity (Wh/kg-K)	Mass Density (kg/liter)	Entropic Heating on Discharge (W/A-cell)
Lead-acid	−10 to 50	High	0.35	2.1	−0.06
Nickel-cadmium	−20 to 50	Medium	0.35	1.7	0.12
Nickel-metal hydride	−10 to 50	Low	0.35	2.3	0.07
Lithium-ion	+10 to 45	Very low	0.38	1.35	Negligible
Lithium-polymer	+50 to 70	Very low	0.40	1.3	Negligible

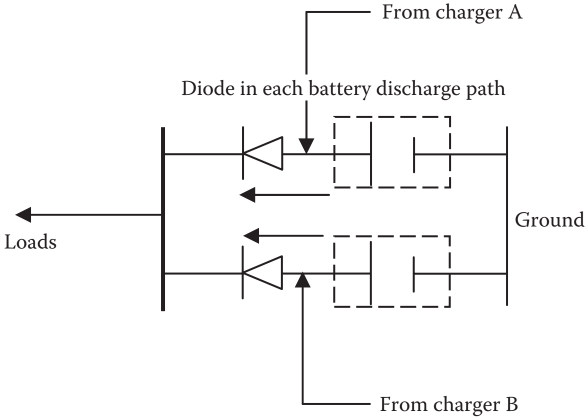


FIGURE 12.7 Two parallel batteries with separate chargers and isolation diodes.

In general, individual chargers for each battery is the best strategy, as shown in Figure 12.7. It incorporates isolation diodes to prevent one battery charging the other in case of an internal short in one of the batteries. It may also allow replacement of any one of many parallel batteries with a somewhat different age and load sharing characteristic. Batteries are usually replaced several times during the economic life of the plant.

Even without a random failure, the cell electrodes eventually wear out and fail due to repeated *c/d* cycles. The battery life is measured in number cycles it can be discharged and recharged before the electrodes wear out. The cycle life depends strongly on the electrochemistry, and also on the depth of discharge and temperature as shown in Figure 12.8 for the NiCd battery. Other electrochemistries will have

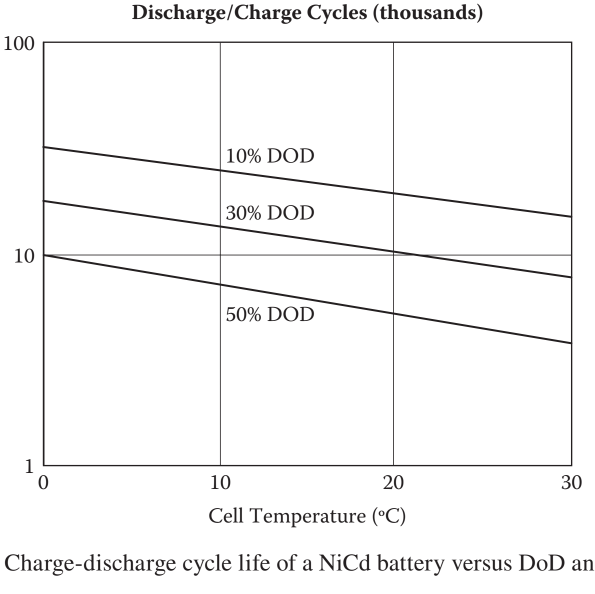


FIGURE 12.8 Charge-discharge cycle life of a NiCd battery versus DoD and temperature.

different but similar patterns. A very important observation from Figure 12.8 is that the life at a given temperature is inversely related to the depth of discharge. For example, at 20°C in Figure 12.8, the battery has 10,000 cycles life at 30% DoD, but only about 6000 cycles at 50% DoD. That is, the lower the DOD, the longer the cycle life, giving the following approximate relation:

$$\text{Number of cycles till failure} \times \text{DoD} = \text{Constant } K \tag{12.7}$$

Equation (12.7) holds well for most electrochemistries. However, the constant K decreases with increasing temperature. This means that the battery at a given temperature can deliver a certain number of equivalent full (100% DoD) cycles of Ah charge and discharge regardless of the depth of discharge. Expressed differently, the total Ah it can deliver over its life is approximately constant. Such an observation is useful in comparing the cost of various batteries for a given application.

Life consideration is a key design driver in determining the battery Ah rating. Even when the load may be met with a smaller capacity, the battery is oversized to meet the life requirement as measured in the number of *c*/*d* cycles. For example, with the same Wh load, a battery that must deliver twice as many cycles over its life must have double the Ah capacity for the same life in calendar years.

12.5 BATTERY TYPES COMPARED

The life and cost of various batteries are compared in Table 12.4. We note that the Li-ion battery has a relatively longer c/d cycle life and lower self-discharge rate, but higher cost. The Li-poly battery is still too new to meaningfully compare with other types at present. The lead-acid battery—because of its least cost per kWh delivered over life—has been the workhorse of the commercial industry for a long time. It is available in small to large Ah capacities in various terminal voltages, such as 6, 12, 24, 36 V, etc.

12.6 MORE ON THE LEAD-ACID BATTERY

This battery is most widely used for its overall performance at low cost. The nominal cell voltage of the lead-acid battery is 2.0 V, and the internal resistance

TABLE 12.4
Life and Cost Comparison of Various Batteries

Electrochemistry	Cycle Life in Full Discharge Cycles	Calendar Life in Years	Self-Discharge Percentage per Month at 25°C	Relative Cost ($/kWh)
Lead-acid	500–1000	5–8	3–5	200–300
Nickel-cadmium	1000–2000	10–15	20–30	1500
Nickel-metal hydride	1000–2000	8–10	20–30	400–600
Lithium-ion	1500–3000	8–10	5–10	500–800
Lithium-polymer	1000–1500	n/a	1–2	>2000

TABLE 12.5
Effects of SoC on Specific Gravity and Freezing Point of Lead-Acid Battery

State of Charge	Specific Gravity	Freezing Point	120 V battery Voltage
1 (fully charged)	1.27	−65°F (−54°C)	128
75%	1.23	−40°F (−40°C)	124
50%	1.19	−10°F (−23°C)	122
25%	1.15	+5°F (−15°C)	120
0 (fully discharged)	1.12	+15°F (−9°C)	118

of a 50 Ah cell is around 1 mΩ. The cycle life is 500–1000 full charge discharge cycles for medium-rate batteries. The self-discharge rate is typically less than 0.2% per day at normal room temperature. As in all batteries, its Ah capacity is sensitive to temperature. For example, its Ah capacity at −20°F is only about 20% of that at 100°F. For this reason, the car is hard to start in winter.

Table 12.5 shows the effect of the state of charge on the electrolyte specific gravity, freezing point, and cell voltage. The electrolyte in a fully charged battery has a high specific gravity and freezes at −65°F. On the other hand, a fully discharged battery freezes at +15°F. This explains the importance of keeping the battery fully charged on cold days. The table also shows that we can estimate the battery state of charge by measuring the specific gravity of the electrolyte. Since this is not possible in sealed cells, other means must be used, such as monitoring the voltage or the Ah going in and out (Ah book-keeping). The Ah book-keeping can be done numerically, or with an electronic integrating circuit. In either case, the error accumulated over a long time makes the end result less reliable, and often useless.

While the unsealed lead-acid battery is an old technology, it has one distinct advantage over modern sealed batteries. The measurement of specific gravity gives an accurate state of charge of each cell. Moreover, over a period of use, the engineer is able to feel the trend in each cell, and often can detect a weak cell before it fails. If a weak cell is dragging other healthy cells down, it can be bypassed from the battery. For this reason, many ship officers prefer the unsealed lead-acid battery. It is economical and offers an easy way to track the health of individual cells.

12.7 BATTERY DESIGN PROCESS

The battery is an essential part of marine systems to store energy for starting the engine or for emergency power for essential loads. Other alternative energy storage technologies are also used on ships. For example, the air tanks in submarines can blow the ballast tanks dry quickly for rising to the surface in a hurry. However, pneumatic systems, although more reliable, require much more space to store the same energy.

Battery design for a given application proceeds in the following eight steps:

1. Select the electrochemistry suitable for the overall system requirements.
2. Determine the number of series cells required to meet the voltage requirement.
3. Determine the Ah discharge required to power the load (i.e., load current × duration).
4. For the required number of *c/d* cycle life, determine the maximum allowable depth of discharge.
5. The Ah capacity of the battery is then determined by dividing the Ah discharge needed to power the load by the allowable depth of discharge (i.e., step 3 result ÷ step 4 result).
6. Determine the charge and discharge rates and necessary controls to protect against overcharging or overdischarging.
7. Determine the temperature rise and the thermal cooling requirement.
8. Determine the ventilation need of the battery room.

Being highly modular (i.e., built from numerous cells), there is no fundamental technological size limitation on the battery system that can be designed and operated. The world's largest 40 MW peak power battery was built and commissioned in 2003 at a $30 million cost. The system uses 14,000 sealed NiCd cells manufactured from recycled cadmium by Saft Corporation at a total cell cost of $10 million. The cells will be recycled again after its 20-year calendar life. The battery system is operated by Golden Valley Electric Association in Fairbanks for an Alaskan utility company. The spinning energy reserve of the battery provides continuous voltage support to the utility and lowers the possibility of blackout.

As stated earlier, when two or more batteries are placed in parallel, it is important that they be as identical as possible for equal load sharing for the reason explained in Section 3.8.1. Also, isolation diodes must be placed in each battery as shown in Figure 12.7 to avoid a stronger battery (with higher internal voltage) charging a weaker battery (with lower internal voltage).

Example 12.3

A battery lasts 2000 charge/discharge cycles of 100% DoD. Determine its approximate cycle life at 40% DoD and at 25% DoD.

SOLUTION

The battery's approximate cycle life is inversely related to the cyclic DoD; it lasts longer at a lower DoD. Therefore, we expect the battery to last longer than 2000 *c/d* cycles at 40% DoD, and even longer at 25% DoD.

Life at 40% DoD = 2000 × 100 ÷ 40 = 5000 cycles of charge and discharge

Life at 25% DoD = 2000 × 100 ÷ 25 = 8000 cycles of charge and discharge

We must remember that these are just the first-order approximations, and the exact life estimate comes from the manufacturer's extensive life test data.

Example 12.4

Design the battery for an industrial application that requires 10 kWh energy in 1.5 h for a dc load at 120 V three times daily for 250 days every year. The battery must be completely recharged in 2.0 hours after each discharge. Use Li-ion cells in this example, but the design process will be applicable to other electrochemistries as well. Use the following Li-ion cell parameters: average discharge voltage 3.6 V, average charge voltage 4.2 V, c/d ratio 1.1, trickle charge rate C/100, and average life 2000 cycles at 100% DoD. For reliability purposes, design the battery with two strings in parallel. To avoid one string charging the other, place an isolation diode with 1 V drop in the discharge path of each string as shown in the figure below. Assume that each cell has 10 mΩ internal resistance at the operating temperature. If the battery must last five calendar years before replacement, determine the (i) number of cells in series in each parallel string, (ii) discharge current, (iii) Ah ratings of each cell, (iv) charge current, (v) charge voltage, and (vi) trickle charge current.

SOLUTION

Total Ah discharged to load = 10,000 Wh ÷ 120 V = 83.33 Ah

For two strings in parallel, each string must deliver 83.33 ÷ 2 = 41.47 Ah

Current discharged from each strings = 41.47 Ah ÷ 1.5 h = 27.65 A for 1.5 h

If N = number of series cell in each strings, allowing for voltage drops in the internal resistance and the isolation diode, we must have

$120 = N\,(3.6 - 0.010 \times 27.65) - 1$, which gives $N = 121 \div 3.32 = 36.42$ cells

∴ We use 37 cells in series, each requiring 42 Ah capacity

Number of discharge and charge cycles in 5 years = 3 per day × 250 × 5 = 3750 cycles, which is more than 2000 cycles—its life at 100% DoD. To survive this life duty, the required Ah rating of the battery = 42 × 3750 ÷ 2000 = 78.75 Ah.

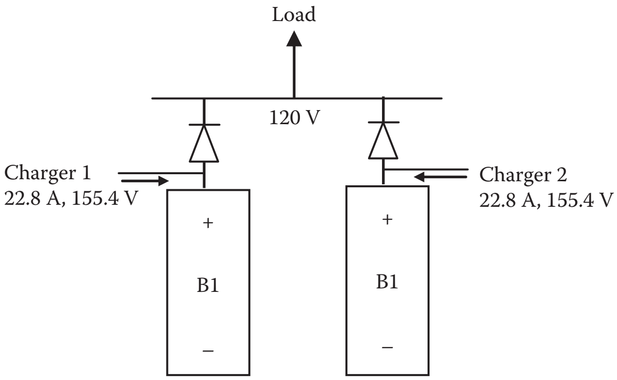
The next standard cell of 80 Ah is selected, which will last 3750 cycles at DoD = 41.47 ÷ 80 = 0.518 or 51.8%.

Charge Ah required to restore full state of charge after each discharge = 1.1 × 41.47 = 45.62 Ah in 2 hours, that is, at C/2 rate. ∴ Charge current = 45.62 ÷ 2 = 22.8 A

Required charger voltage = 37 series cells × 4.2 V/cell = 155.4 V

The trickle charge rate to counter the self-discharge = C/100, that is, 80/100 = 0.8 A.

The charger must be designed as a constant current source that delivers 22.8 A for 2 h and then shifts to 0.8 A for the rest of the time.



12.8 SAFETY AND ENVIRONMENT

Batteries use toxic chemicals such as lead, sulfuric acid, lithium, cadmium, etc. In unsealed batteries, the acidic fumes pose various hazards to the personnel working around. For example, the battery charging line officer aboard a nuclear submarine often has to crawl in a small space on top of a large unsealed lead-acid battery to take specific gravity readings during the charge until the battery is fully charged. The line officer often sees the clothes developing small holes, especially in the cotton undershirts, after a few washes. Also, the tools left in the battery room quickly corrode. This is undoubtedly due to sulfuric acid fumes. Therefore, eye and other protection must be worn when working closely with the batteries. It is more of an environmental safety and health issue than an environmental contamination issue.

Since the battery generates ignitable gas when overcharged, and some may leak out from unsealed or defective batteries, the battery room has to be provided with adequate air ventilation. As a guideline, the required fresh air ventilation Q in m^3/h is given by

$$Q_{\text{m3/hour}} = 0.0055 \times \text{Ah capacity/cell} \times \text{No. of elementary cells in room} \tag{12.8}$$

The 12 V lead-acid battery has 6 elementary cells of 2 V each inside the case. Therefore, if there are ten 12 V lead-acid batteries in the room, then the number of elementary cells is $10 \times 6 = 60$ in Equation (12.8).

Example 12.5

A large 240 V battery is assembled from 12 V, 110 Ah battery modules commercially available off the shelf. Determine the air ventilation requirement for each room.

SOLUTION

For the number of cells in the room in Equation (12.8), we assume a battery module of lead-acid electrochemistry, which has 2 V/cell, so 12 V module has 6 elementary cells in series inside the case.

Number of series cells in the battery room = 240 V battery voltage ÷ 12 V module voltage = 20 modules × 6 series cells/module = 120 elementary cells in series for the 240 V battery

Ah capacity of each series cell = 120 Ah

∴ Required air ventilation, $Q = 0.0055 \times 110 \times 120 = 72.6\ \text{m}^3/\text{h}$

Besides the personnel safety issues discussed earlier, the most important safety consideration for protecting the battery is not to overcharge. Any overcharge above the trickle charge rate is converted into heat and pressure buildup, which can explode the battery if allowed to overheat beyond a certain limit. For the lead-acid battery, the trickle charge rate is typically below C/20 amperes (5% of rated capacity). Another caution is never to overcharge above the float charge voltage of

2.25 V/cell at 25°C in a lead-acid battery. Above this voltage, all charging should be terminated.

Environmentally, the battery can be both safe and unsafe, depending on how we use it. It is safe until the time comes to dispose of it with many toxic materials inside. The battery is not *green* by itself, but it supports the energy sources that are safe for the environment. Proper disposal of toxic materials can minimize the environmental impact, but still leaves harmful waste products. Moreover, the batteries are costly to recycle. Not everyone recycles small cells at the consumer level. Lead-acid batteries use about 90% of the lead in the United States, and a large percentage of it is recycled. The recycled lead and plastic is used for new batteries, and the sulfuric acid converted to water or to sodium sulfate is used in detergent, textile, and glass manufacturing. Recycling NiCd batteries needs advanced recovery methods, where cadmium is reused for making new batteries, and the nickel is used in steel production. The liquid potassium sulfate may be used in some waste water treatment plants, but this use is not widespread as yet. Lithium batteries discarded from consumer products, cell phones, and personal electronic devices are recycled to a lesser extent at about 65%. The military recycles about 90% of the batteries. All these data suggest that the batteries pose some potential dangers, but are not necessarily unsafe if used and recycled properly.

PROBLEMS

Problem 12.1: A battery with an internal resistance of 0.015 Ω/cell needs to deliver 20 A current at 240 V to the load. Determine the number of cells in series if the electrochemical voltage is 2.0 V/cell.

Problem 12.2: Select the favorable cell from two options available in procuring 100 Ah battery cells. It is for a large battery assembly in an industrial plant, which requires frequent charging and discharging. Cell A has an average cell voltage of 1.25 V during discharge, 1.45 V during charge, and a c/d ratio of 1.15. Cell B has an average cell voltage of 1.3 V during discharge, 1.5 V during charge, and a *c/d* ratio of 1.2. Both cells A and B cost the same per Ah rating.

Problem 12.3: A battery lasts 3000 charge/discharge cycles of 100% DoD. Determine its approximate cycle life at 30% DoD and at 20% DoD.

Problem 12.4: Design the battery for an industrial application that requires 20 kWh energy in 2 hours for a dc load at 240 V four times daily for 300 days every year. The battery must be completely recharged in 2.5 hours after each discharge. Use NiMH cells with the following cell parameters: average discharge voltage 1.2 V, average charge voltage 1.45 V, *c/d* ratio 1.15, trickle charge rate C/80, and average life 1500 cycles at 100% DoD. For reliability purposes, design the battery with two strings in parallel. To avoid one string charging the other, place an isolation diode with a 1 V drop in the discharge path of each string. Assume that each cell has 15 mΩ internal resistance at the operating temperature. If the battery must last 7 calendar years before replacement, determine for each parallel string (1) the number of cells in series, (2) discharge current, (3) Ah ratings of each cell, (4) charge current, (5) charge voltage, and (6) trickle charge current.

Problem 12.5: A large 240 V battery is assembled from 2 V, 160 Ah lead-acid battery cells commercially available off the shelf. The maximum charging rate is *C*/4, and the charging rate during gas formation is *C*/20. Determine the air ventilation required in the battery room as per the marine industry standard covered in the next chapter Section 13.3, and also as per the commercial standards covered in this chapter.

Problem 12.6: A new electrochemical cell has an average charge voltage of 4.5 V, an average discharge voltage of 3.0 V, and a *c*/*d* ratio of 1.15. Determine its round trip energy efficiency.

Problem 12.7: The winter temperature of your car battery when parked outdoor on a street in Alaska can fall to −40°F at midnight. Determine the minimum SoC you must maintain in your battery to avoid electrolyte freezing.

Problem 12.8: A battery in your plant lasts 7 years when repeatedly discharged to 50% DoD. Determine its approximate life if the repeated DoD is increased to 75%.

Problem 12.9: Design a NiCd battery for 10-year calendar life, which is required to (i) power a 10 kW load at 240 V dc for 1 hour three times every day (once every 8 hours), and (ii) charge it back in 6 hours, allowing a 2 hour margin for the battery to cool down after being fully charged and before the next load cycle starts. Assume average discharge voltage 1.2 V/cell, internal resistance 0.02 Ω/cell, and operating temperature 20°C. Use Figure 12.8.

You must use three batteries in parallel to share full load and one dormant spare battery for reliability. With one failed battery, the other three must power full load. For each of the four batteries, determine the number of cells in series, cell capacity in Ah, and the required charge current if the c/d ratio is 1.1 and the charge voltage is 1.45 V. Use a diode with a 1 V drop in each battery to prevent a stronger battery charging a weaker battery in case of an internal cell short or for other reasons.

QUESTIONS

Question 12.1 If a 100 Ah battery is being charged at *C*/5 rate, how long will it take to get fully charged?

Question 12.2 If you have used or designed any type of battery, would you replace it with another electrochemistry, and for what reason?

Question 12.3 Identify the battery type that has the highest volumetric and gravimetric energy densities in a (i) secondary battery group, and (ii) in a primary battery group.

Question 12.4 What is the difference between charge efficiency and energy efficiency?

Question 12.5 Describe the memory effect in a NiCd battery, and the remedy for minimizing it.

Question 12.6 Discuss the key performance requirement for a battery for an electric car.

Question 12.7 When using two or more batteries in parallel, why should each battery have a separate charger and diode in the discharge path?

Question 12.8 Name alternatives to a battery for large-scale energy storage, say, for a wind farm or a solar park.

Question 12.9 Argue for and against the electric car in solving the (i) environment problem, and the (ii) energy problem.

Question 12.10 Share your thoughts on the impact of a battery on the energy shortage problem many nations face today. Will it solve it or aggravate it?

Question 12.11 Identify alternatives to the battery in the (i) marine industry, (ii) commercial industry, and (iii) consumer electronics.

FURTHER READING

Reddy, T.B. and D. Linden. 2010. *Linden's Handbook of Batteries*. New York: McGraw Hill.
Nazri, G.A. and G. Pistoia. 2009. *Lithium Batteries Science and Technology*. Kluwer Publishing.

13 Marine Industry Standards

The shipbuilding industry is guided by the recommended practices for electrical installations on ships formulated by various organizations around the world. Although they may be loosely called *standards*, they are not necessarily required unless specified in the procurement contract. The evolving recommendations basically define what are considered to be good present-day engineering practices for safety of the personnel and of the ship itself, as well as for reliability, availability, durability, and operational efficiency.

The shipboard electrical installation rules often come from the regulatory agencies and the classifying societies. Such rules are considered in the industry recommendations, but they may differ from the regulations. Attention should be given to the regulatory requirement where the recommended practice differs or is silent.

13.1 STANDARD-ISSUING ORGANIZATIONS

Among the many national and international standards related to the shipbuilding industry around the world, the following major ones have been established by the following organizations:

ABS	American Bureau of Shipping
IEEE	Institute of Electrical and Electronics Engineers
USCG	U.S. Coast Guard
ANSI	American Nationals Standard Institute
NEMA	National Electrical Manufacturers Association
IACS	International Association of Classification Societies
IEC	International Electrotechnical Commission
DVN.	Det Norske Veritas
USN	United Sates Navy

These standards give specific guidelines and requirements for system design, construction, and testing. In the United States, IEEE Std-45, ABS Rules (Part 4, Chapter 8), and USCG are major standards for electrical power systems and equipment for commercial ships. They all are recommended, but not required unless contractually agreed upon. The USN standards under the MIL-Std series are required for the Navy ships. The International Association of Maritime Universities (IAMU) is also active in research and development for bringing innovations to the industry. The IAMU's 2008 report on safety is an example of proposals for new recommendations and practices.

Various engineering disciplines have many similarities. An engineer well versed with a standard in one engineering field can draw parallels to another field. For example, from the top-level point of view, a fluid system is similar to an electrical system. The required pipe size is based on the operating pressure and allowable pressure drop; the cable size is based on the operating voltage and allowable voltage drop. The system pressure drops compare with the voltage drops; and the system protection is provided with relief valves in fluid systems and with fuses or circuit breakers in electrical systems. The check valves are like diodes. The water hammers in fluid systems are analogous to transient overvoltages in electrical systems. Thinking in terms of such analogies can leverage the transferable engineering skills, and also make it easier to understand the intent of the standard being applied. For this reason, some shipbuilding specifications and Navy technical manuals lay out requirements for fluid systems in a manner similar to IEEE-Std-45 for electrical systems.

13.2 CLASSIFICATION SOCIETIES

International Association of Classification Societies (IACS) establishes and applies technical standards related to the design, construction, and survey of marine-related facilities, including ships and offshore structures. These standards are issued by the classification societies as published rules. A ship that has been designed and built to the appropriate rules of a society may apply for a Certificate of Classification from that society. The members of IACS are:

ABS	American Bureau of Shipping
BV	Bureau Veritas
CCS	China Classification Society
DNV	Det Norske Veritas
GL	Germanischer Lloyd
KR	Korean Register of Shipping
LR	Lloyd's Register
NK	Nippon Kaiji Kyokai (ClassNK)
RINA	Registro Italiano Navale
RS	Russian Maritime Register of Shipping

The IACS's ten societies collectively classify about 94% of all commercial tonnage involved in the international trade. As an independent body, a classification society has no commercial interest related to the ship design, building, ownership, operation, management, maintenance, insurance, or chartering. In establishing its rules, each classification society may draw upon the advice and review of members of the industry having relevant expertise in their field.

The ship classification rules are developed for (1) the structural strength and integrity of essential parts of the ship hull and its appendages, (2) the reliability and the function of the propulsion and steering systems, (3) power generation and distribution, (4) pumps, engines, and other equipment vital to the ship's function, and (5) auxiliary systems and other features that have been built into the ship in order to maintain essential services on board. Classification rules are not intended as a design code and in fact cannot be used as such.

A classification survey before issuing a certificate is a visual examination that normally consists of (1) an overall examination of the items for survey, (2) detailed checks of selected parts, and (3) witnessing tests, measurements, and trials where applicable. The relevant certificate is then issued by a member of the IACS or EMSA (European Maritime Safety Agency) on behalf of the flag country. The old practice of assigning different classifications (e.g., A for the best hull and 1 for best interiors, etc.) has been abolished. Today, a ship either meets the relevant classification society's rules or it does not. As a consequence, it is either *in or out of class*. However, each of the classification societies has developed a series of notations that may be granted to a ship to indicate that it is in compliance with some additional criteria that may be either specific to that ship type or that are in excess of the standard classification requirements.

13.3 IEEE STANDARD-45

The first IEEE Std-45™ for electrical installations on ships was issued in 1920, and its latest revision was issued in 2002. The following are only several selected key requirements of the shipboard electrical power system as recommended by IEEE Std-45-2002. They are cited here merely for educational purposes. The full document in its current revision must be referred to in its entirety for the actual design.

Standard voltages: One or more of the following standard voltages are used on ship, depending on the total electrical power. The first number is the generation voltage, followed by the utilization voltage in the parentheses. The difference between the two is the voltage drop in the power distribution system, mainly in transformers and cables between the generator and the loads.

The standard ac voltages for generation (utilization) are 120 (115), 208 (200), 230 (220), 240 (230), 380 (350), 450 (440), 480 (460), 600 (575), 690 (600), 2400 (2300), 3300 (3150), 4160 (4000), 6600 (6000), 11,000 (10,600), 13,800 (13,460).

For dc, there are only two standard voltages at present: 120 (115) and 240 (230) volts.

Standard frequencies: 60 Hz in the United States, 50 Hz in Europe, and either 50 or 60 Hz in most other countries.

The electric propulsion loads are powered at high voltage, since they are several times greater than all the service loads combined. The total electrical load in electrically propelled ships is generally in tens of MW. For example, Queen Mary-2 cruise ship's electrical power plant is 80 MW_e (megawatt electric).

Grounding: Grounded low voltage (<600 V) systems should be solidly grounded to the hull at a suitable structural frame or longitudinal girder. The hull must not carry current as a conductor, except for the cathodic protection and for local ground for the battery in a 1-wire engine starting system.

The neutral of the generator and distribution system, if grounded, should be grounded at a single point near the generator switchboard. The switchboard should be positively grounded to the hull. Metal cases of instruments and other devices and secondary winding of instrument transformers should be grounded.

Voltage deviations: Steady-state voltage deviation < +5% and −6% to −10% for 460 V, and ±3% for 120 V loads. Transient voltage deviation < −15% and +20%.

Large motors can draw starting transient inrush current of 5 to 8 × full load current for several seconds, and transformers can draw an inrush magnetizing current of 10 to 12 × full load current for a few cycles. In both cases, the time-delay fuse or trip-delay circuit breaker can be used to avoid unwarranted circuit tripping. For motors, the starting current, and the resulting voltage drop, can be reduced by soft starting under reduced voltage, or power electronics soft-starters, or VFD.

Harmonic distortion: No specific limit is specified on the total harmonic distortion (THD) factor, but a generally acceptable limit is < 5% in voltage and < 15% in current. The THD limit is stringent for voltage, because voltage is the *community property* for all to use, as opposed to the individual load-specific current.

Emergency power: The emergency generator must be placed in a separate and remote space, with separate energy storage (battery) for six consecutive starts and a separate switchboard for the emergency power distribution system.

Fault current: The prospective maximum possible short circuit (fault) current should be determined from the aggregate contribution of all generators that can be simultaneously operated in parallel, and the maximum number of motors that can be in operation. It should be based on the circuit impedance, including the direct-axis subtransient reactance of the generator. The asymmetrical fault current can then be calculated by applying the offset factor given in the standard. The rms value is generally used to size the circuit breaker rating and for thermal design of the circuit breaker and the bus bars. The worst-case first peak value of the offset asymmetrical fault current is used to determine the peak mechanical force on the structural parts that must be designed to survive without mechanical damage under the bending stress and deflection between supports.

In the initial screening phase of the design, the fault current can be approximated as follows:

Maximum asymmetrical rms fault current at ½ cycle after the fault = 10 × Rated current of all generators +4 × Rated current of all motors that can be in operation.

Average asymmetrical rms fault current at ½ cycle after the fault = 8.5 × Rated current of all generators +3.5 × Rated current of all motors that can be in operation. This is the average of all three phases.

Circuit protection: Fuses are generally used in systems up to 690 V, and circuit breakers are used in higher-voltage systems.

Reversed power relay: In each parallel-connected generator, there must be a reverse power circuit breaker to protect against the generator drawing current inward, that is, working as a motor and overloading other generators.

Shore connection: Where the shore power port is provided for, it must also be protected by a dedicated circuit breaker.

Bus bars: The bus bars must be sized to carry rated current with a temperature rise < 50°C, and mechanically braced to withstand a repulsive force at the first peak of the worst-case asymmetrical fault current. The bus bars are generally made of copper, but aluminum bars may be used in special cases. All connection points must

TABLE 13.1
Ampacity of Copper Bus Bars Placed on Edge Based on 50°C (122°F) Rise in 50°C (122°F) Ambient Air

No. of Bars in Parallel per Phase	Bar Size (inches)	Ampacity 60 Hz (Cu)	Ampacity 60 Hz (Al)
One	1 × 1/8	330	245
	2 × ¼	900	685
	8 × ¼	2875	2250
Two	2 × ¼	1450	1100
(1/4 in. apart)	8 × ¼	4250	3450
Three	3 × ¼	2550	2025
(1/4 in. apart)	8 × ¼	5300	4450
Four	3 × ¼	3050	2375
(1/4 in. apart)	8 × ¼	7100	6225

be silver or tin-plated to avoid oxidation, and bus structural supports must be nonhygroscopic and of adequate mechanical strength.

The steady-state ampere rating of bus bars placed on edge for 50°C (122°F) rise in 50°C (122°F) ambient air is listed in Table 13.1. The bars are often placed on edge to minimize the mutual heating from other phases, although they result in higher leakage reactance per meter length.

Operating and ambient temperatures: Electrical equipment is generally designed and rated for industry-standard ambient temperature of 40°C. On shipboard, the actual ambient temperature in different areas can be quite different, as listed in Table 13.2. Allowance must be made to derate the nominal rating of the equipment accordingly.

TABLE 13.2
Ambient Temperature in Ships by Location

Area (space)	Ambient Temperature*	
	°C	°F
Accommodation space	40	104
Main and aux. machinery space	45	113
Prime mover and boiler space containing significant heat source	50	122
Uptake from prime mover and boiler space	65	149
Sea water cooling for podded propulsion motor or power electronics equipment	32	90

*All at relative humidity up to 95%.

For example, a motor with a design temperature rise of 80°C in 40°C ambient air, the design average operating temperature of the coils would be 40 + 80 = 120°C, so the insulation must be rated for 130°C (Class B) to allow a 10°C margin for local hotspot. If the motor is operating in the boiler room with 65°C ambient air, then its temperature rise must be limited to 55°C, so that the operating temperature does not rise above the 120°C design limit. For this, the motor horsepower must be appropriately selected and derated for higher ambient temperature.

Safety in arc welding and metal cutting: For manual arc welding, metal cutting, or other processes requiring hands-on operations, the industry standards require the open-circuit voltage below 100 V for personnel safety. Such a voltage is obtained by using a special high-impedance welding transformer that quickly drops voltage under high current caused by any reason, accidental short or human contact. When a special welding and cutting process requires an open-circuit voltage higher than 100 V, means must be provided to prevent the operator from making accidental contact with high voltage, by adequate insulation or other means. For ac welding under wet conditions or warm surroundings where perspiration is a factor, the use of a reliable automatic control for reducing no load voltage is recommended to reduce shock hazard. The ac reactor can induce high voltage spikes while interrupting high current, and autotransformer has no electrical isolation from the source to the load. For this reason, the auto transformer or ac reactor shall not be used to draw welding current directly from any arc power source having a voltage exceeding 80 V.

Oil-filled transformer and circuit breaker: The shipboard equipment is required to withstand the roll and pitch angles under static and dynamic conditions up to 5° to 15° in ship service equipment, 10° to 22.5° in emergency equipment, and 45° in switchgear equipment. They are also required to withstand peak accelerations for up to 5 to 10 sec due to ship motion in seaway of up to ±0.6 g in ships longer than 90 m, and ±1.0 g for shorter ships. Oil-filled electrical equipment may not meet these requirements. Moreover, they pose an unacceptable risk of oil spill and fire following an accidental rupture of the oil-filled enclosure. For these reasons, they are generally not used on ships.

Electromagnetic interference: To avoid radiated electromagnetic interference (EMI) from power electronics converters, the main switchboard and propulsion motor drive are installed in a separate compartment from the ship service switchboard and control console. The cables in electric propulsion motor drives require special attention due to harmonics and the resulting EMI due to corona in high-voltage (> 3.3 kV) systems. They are routed separately from ship service power cables with at least 2 feet (61 cm) distance between the two groups.

High-voltage insulation withstand test: The coil insulation when new at the factory or reconditioned in service must be designed and tested to withstand a high voltage of about $4 \times V_{\text{rated.rms}}$ (with a minimum of 1500 V) at 60 Hz for 1 minute as listed in Table 13.3. For testing the insulation strength of the whole coil to the ground, the two ends of the coil are shorted together and connected to the test generator, so that the voltage of the whole coil is raised above the ground for 1 min. The current drawn from the generator must be small, as expected, which is the normal capacitive charging current to the ground.

TABLE 13.3
60 Hz High Voltage Withstand Test Levels for Electrical Equipments

Equipment Rated Voltage V_{rms}	Applied V_{rms} 60 Hz for I Minute
Up to 120 V	1500 V
121–600	2200 V
601–1200	5000 V
1201–2400 V	15,000 V
2401–4760 V	19,000 V

Circuits around magnetic compass: The leakage (stray) magnetic flux can interfere with the performance of a ship's magnetic compass. For this reason, no single wire or no magnetic material—for which complete compensation cannot be made—can be installed near the compass. In the binnacle, only a single pair of lead and return conductors, twisted for full length, should be used for the binnacle light. For other wiring, the minimum distance from the compass should be approximately as given in Table 13.4. This requirement does not apply to multiple-conductor cables carrying ac, since it has no significant leakage flux outside the cable that can disturb the compass needle. This is due to the canceling effect of 1-phase or 3-phase currents with their phasor sum in the cable equal to zero at all times. A continuous steel deck or bulkhead between the compass and a heavy power equipment acts as a magnetic shield, making compliance with the table unnecessary.

Hazardous locations: The electrical equipment must be designed to perform safely as intended in the specified hazardous location. The IEC and NEC have developed hazardous location standards that have been adopted in many countries. Both have developed a classification scheme that defines the hazard that is or may be present as listed in Table 13.5. The details in the IEC and NEC requirements are similar

TABLE 13.4
Minimum Wiring Distance from Shipboard Magnetic Compass

Wire Type or Equipment	Minimum Distance of Compass from Conductor
Single conductor dc or ac	None should be used
Parallel twin conductor	2 ft (0.6 m) up to 1 A, 5 ft (1.5 m) for 1 A to 10 A, 8 ft (2.5 m) over 10 A wires
Twisted conductors	3 ft (1 m) up to 10 A, 5 ft (1.5 m) over 10 A
Motors	12 ft to 14 ft (3.7 m to 4.3 m)
Speakers, search lights, telephones, magnetic relay, etc.	5 ft to 12 ft (1.5 m to 3.7 m)

TABLE 13.5
Hazardous Location Designations per NEC

Hazard Class	Ignitability Group A (high) to D (low)
I: Flammable gases	A: Acetylene[a]
II: Combustible dust	B: Hydrogen
III: Ignitable fibers	C: Ethylene
	D: Propane

[a] Location IA means flammable gases of acetylene are present.

in many ways, but they differ at many places to the point that the two do not permit interchangeability of components in many instances. Appropriate regulatory agencies should be consulted in those cases.

Battery room: The battery room should be adequately ventilated by natural air or by exhaust fans, particularly recognizing that the battery gases are lighter than air and tend to accumulate in pockets at the top of the room. The battery room ventilation should be separate from other spaces, and each blower should have a nonsparking fan motor. The ventilation system should be interlocked with the battery charger such that the battery cannot be charged without operating the ventilation system.

The air circulation quantity in an unsealed battery room shall be at least as follows:

$$\text{Cubic feet of air per hour} = 3.9 \times \text{Number of cells in series} \times I_{ch} \qquad (13.1)$$

$$\text{Liters of air per hour} = 110 \times \text{Number of cells in series} \times I_{ch} \qquad (13.2)$$

where I_{ch} = Maximum charging current during gas formation, or 25% of the maximum possible charging current, whichever is greater. The sealed-gell electrolyte battery room ventilation rate may be 25% of the above for the unsealed batteries.

It is noteworthy that the foregoing ventilation rate recommended on ship is based on the charging rate, whereas Equation (12.8), recommended for general-purpose batteries on land, is based on merely the number of cells present in the room. When the air ventilation requirement from the two differ, we must use the higher of the two.

13.4 CODE OF FEDERAL REGULATIONS

The following are a few key requirements abridged from the *Code of Federal Regulations, Title 46 Shipping, Part 120*—Electrical Installations, Subpart C—Power sources and distribution systems, September 2009 issue:

Vital and essential loads must be arranged so that they can be energized from two sources.

A vessel of more than 65 feet (19.8 m) length carrying more than 600 passengers or with overnight accommodation for more than 49 passengers must have two generator sets, and the final emergency power source located outside the machinery space.

Each generator must have an independent prime mover.

Dual-voltage generators shall have a solidly grounded neutral.

A generator or motor designed for 40°C (104°F) ambient may be used on ships with 80% of rated load with the overcurrent device setting reduced accordingly.

Conductors in power and lighting circuits must be AWG # 14 or heavier, and conductors in control and indicator circuits must be AWG # 22 or heavier.

Batteries for engine starting must be located as close as possible to the engine served.

Vessel hull shall not carry current as a conductor except for cathodic protection and locally for the engine starting system.

Each nonmetallic mast and top mast must have a lightning ground conductor.

If a grounded distribution system is used, there must be only one single point ground, regardless of the number of power sources.

Each propulsion, power, lighting, or distribution system having a neutral conductor must have neutral grounded.

Each circuit breaker must indicate whether it is open or closed.

13.5 MILITARY-STD-1399

For military shipboard electrical power systems, the power quality standard for Type I power source at the user load terminals (440 V_{LL}, 3-phase, 60 Hz, ungrounded) as specified in MIL-Std-1399 are listed in Table 13.6. The requirements in IEEE-Std-45 for commercial ships are very comparable with this table. Similar standards at the generator terminals are relatively more stringent with narrower limits on deviations from the nominal values. The definitions of the terms used in these standards are as follows:

$$\textit{Frequency modulation \%} = \frac{\{f_{\max} - f_{\min}\}}{2f_{\text{nominal}}} \times 100 \tag{13.3}$$

$$\textit{Voltage modulation \%} = \frac{\{V_{\max} - V_{\min}\}}{2V_{\text{nominal}}} \times 100 \tag{13.4}$$

$$\textit{Line Volatge unbalance \%} = \frac{\{V_{\max} - V_{\min}\}}{V_{\text{nominal}}} \times 100 \tag{13.5}$$

TABLE 13.6
MIL-Std-1399 Type I Power

Characteristic parameter	MIL-Std-1399
Normal frequency tolerance from 60 Hz	±3%
Normal frequency modulation	½%
Transient frequency tolerance	±4%
Transient frequency recovery time	2 sec
Worst-case frequency excursion due to all reasons combined, except under emergency conditions	±5½%
Average of three line-to-line voltages (user voltage)	±5%
Any one line-to-line voltage	±7%
Line voltage unbalance	3%
Line voltage modulation	2%
Transient voltage tolerance	±16%
Transient voltage recovery time	2 sec
Voltage spikes (peak value, including fundamental)	See (a) and (b)
(a) ± 2500 V in 440 V system (b) ± 1000 V in 115 V system	In first column
Worst-case voltage excursion due to all reasons combined, except under emergency conditions	±20%
Insulation resistance test in surface ships	500 V_{dc} megger
Maximum total harmonic distortion factor	5%
Maximum single harmonic	3%
Emergency condition frequency excursion	−100% to +12%
Emergency condition duration of frequency excursion	< 2 min
Emergency condition voltage excursion	−100% to +35%
Emergency condition duration voltage excursion	< 2 min

Note: Ungrounded, 60 Hz, 440 V_{LL} 3-phase and 115 V_{LN} 1-phase user voltage. The IEEE-Std-45 for merchant ships is mostly similar to this table.

QUESTIONS

Question 13.1 Identify a few analogies between hydraulic and electrical power systems.

Question 13.2 Where are the neutral and chassis grounded in shipboard power systems?

Question 13.3 Give two reasons why oil-filled electrical equipment is not suitable on ships.

Question 13.4 Which space in the ship has generally the highest ambient air temperature that requires the greatest derating of the electrical equipment?

Question 13.5 Which safety features are incorporated in the design of the welding transformer?

Question 13.6 Why do multiple conductors in dc, 1-phase, and 3-phase power cables not disturb the magnetic compass needle on ships? Explain why the twisted pair of 1-ph wire is even better than multiple-conductor cables for the magnetic compass.

FURTHER READING

International Electrocommission Standard -IEC.92.350: *Electrical Installations on Ships.*

Military-Standard-461E (DoD Interface Standard). Requirement for the Control of EMI—Characteristics of Subsystems and Equipments.

Code of Federal Regulations, Title 46 Shipping, Part 120—Electrical Installations, Subpart C—Power sources and distribution systems.

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Appendix A: Symmetrical Components

The unsymmetrical operation of a 3-phase ac system—in steady state or during short circuit—is analyzed by resolving the unsymmetrical applied voltages and resulting currents into symmetrical components that are orthogonal in mathematical sense (not physical space). The method was first developed by Fortesque and later fully expanded by Wagner, Evans, and Clarke for applications in practical power systems. It is analogous to resolving any space vector force into three orthogonal components. It involves algebras of complex numbers and matrices. We study here a brief theory and its applications in power system analyses with unbalanced voltages applied in operation or during unsymmetrical short circuits faults. The L-G, L-L, and L-L-G short circuits are called *unsymmetrical faults*, since they do not involve all three lines symmetrically.

A.1 THEORY OF SYMMETRICAL COMPONENTS

The method of symmetrical components uses the operator $a = 1\angle 120°$ similar to the operator $j = 1\angle 90°$ we routinely use in ac circuits. We recall that the operator j was a short-hand notation for a 90° phase shift in the positive (counterclockwise) direction. Similarly, we use in this Appendix the operator α as a short-hand notation for a 120° phase shift in the positive direction. Although written as the Greek letter α here, we will hereafter write as the Latin letter "a" without confusion. With $a = 1\angle 120°$, we have $a^2 = 1\angle 240°$ and $a^3 = 1\angle 360° = 1\angle 0° = 1$. In exponential form, $a = e^{j2\pi/3}$. Obviously, the phasor sum of $1 + a + a^2 = 0$.

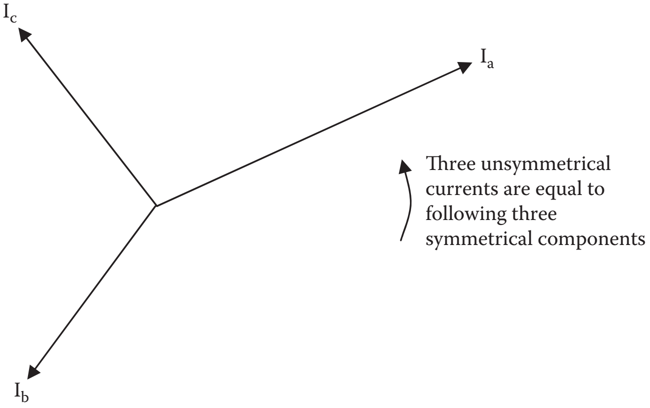
We can resolve any three unbalanced phase currents $\tilde{I}_a$, $\tilde{I}_b$, and $\tilde{I}_c$ into three symmetrical sets of balanced 3-phase currents $\tilde{I}_o$, $\tilde{I}_1$, and $\tilde{I}_2$ as follows:

$$\begin{aligned}
\tilde{I}_a &= \tilde{I}_o + \tilde{I}_1 + \tilde{I}_2 \\
\tilde{I}_b &= \tilde{I}_o + a^2\,\tilde{I}_1 + a\,\tilde{I}_2 \\
\tilde{I}_c &= \tilde{I}_o + a\,\tilde{I}_1 + a^2\,\tilde{I}_2
\end{aligned} \tag{A.1}$$

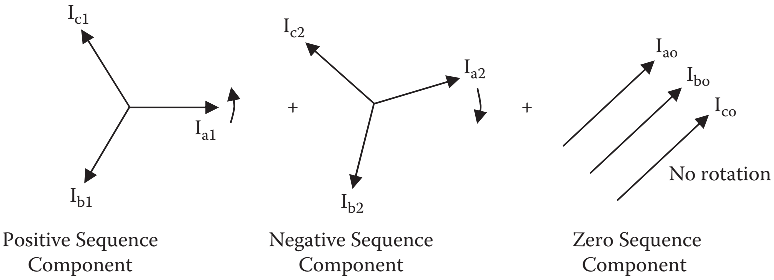
Where

$\tilde{I}_1$ = phase-a value of the positive sequence set of three-phase currents
$\tilde{I}_2$ = phase-a value of the negative sequence set of three-phase currents
$\tilde{I}_o$ = phase-a value of the zero sequence (in phase) set of three-phase currents

Figure A.1 depicts (a) three unsymmetrical currents and (b) their three symmetrical components. The symmetrical sets of balanced 3-phase zero, positive, and negative sequence currents $\tilde{I}_o$, $\tilde{I}_1$, and $\tilde{I}_2$ are also denoted by I^o, I^+, and I^-, respectively, in some books and literature.



(a) Unsymmetrical 3-phase currents



(b) Symmetrical 3-phase components

FIGURE A.1 Symmetrical components of three-phase unbalanced currents.

The argument Fortesque made was this. A vector has two degrees of freedom—in magnitude and in direction—so it can be resolved in two orthogonal components on *x*-*y* axes. A 3-dimensional vector has three degrees of freedom—magnitude, angle θ in *x*-*y* plane, and angle ϕ in *z*-plane—so it can be resolved into three orthogonal components on *x*-*y*-*z* axes. A set of 3-phase balanced currents has *two* degrees of freedom—magnitude and phase angle of any one phase current. The magnitudes and phase angles of the other two phase currents have no additional freedom, as they are fixed in magnitude and 120° phase shifts. However, a set of three unbalanced (unsymmetrical) currents has six degrees of freedom—two for each of the three phase currents—so it can be resolved into three balanced (symmetrical) sets (called symmetrical components), each having two degrees of freedom, making the total of six degrees of freedom. This is shown in Figure A.1, where the upper set is unbalanced 3-phase currents, and the three lower sets are the symmetrical components, which are 3-phase balanced sets.

In resolving any unsymmetrical 3-phase currents into three symmetrical components as in Equation (A.1), we recognize the following. Like all ac quantities, the components $\tilde{I}_o$, $\tilde{I}_1$, and $\tilde{I}_2$ are also phasors. By writing $\tilde{I}_a$ as in Equation (A.1), we imply that the component phase angles are with respect to $\tilde{I}_a$. The correct way of writing Equation (A.1) is $\tilde{I}_a = \tilde{I}_{oa} + \tilde{I}_{1a} + \tilde{I}_{2a}$. Instead of writing such long notations repeatedly, we just imply that $\tilde{I}_o$, $\tilde{I}_1$, and $\tilde{I}_2$ component phase angles are with respect to $\tilde{I}_a$, that is, $\tilde{I}_o = \tilde{I}_{oa}$, $\tilde{I}_1 = \tilde{I}_{1a}$, and $\tilde{I}_2 = \tilde{I}_{2a}$.

The set of Equation (A.1) can be written in the matrix form

$$\begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} I_o \\ I_1 \\ I_2 \end{pmatrix} = [A] \cdot \begin{pmatrix} I_o \\ I_1 \\ I_2 \end{pmatrix} \tag{A.2}$$

where $[A] = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$ is called the *operator matrix.*

The matrix Equation (A.2) can be solved for $\tilde{I}_o$, $\tilde{I}_1$, and $\tilde{I}_2$ by using the matrix inversion software built in many computers and even in advanced calculators. The solution for the symmetrical components can then be written in terms of the inverse of matrix [A], that is,

$$\begin{pmatrix} I_o \\ I_1 \\ I_2 \end{pmatrix} = \frac{1}{[A]} \cdot \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} = [A]^{-1} \cdot \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix} \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} \tag{A.3}$$

Thus, the inverse of the operator matrix [A] multiplied by the phase currents give the following set of component currents,

$$\tilde{I}_o = \tfrac{1}{3} (\tilde{I}_a + \tilde{I}_b + \tilde{I}_c)$$
$$\tilde{I}_1 = \tfrac{1}{3} (\tilde{I}_a + a\,\tilde{I}_b + a^2\,\tilde{I}_c)$$
$$\tilde{I}_2 = \tfrac{1}{3} (\tilde{I}_a + a^2\,\tilde{I}_b + a\,\tilde{I}_c) \tag{A.4}$$

In a *Y*-connected system with neutral, the neutral current must be a phasor sum of three phase currents. In a Δ-connected system, it must zero as there is no neutral wire. So, the neutral current $\tilde{I}_n$ is related to $\tilde{I}_o$ as follows:

In the 4-wire Y-n system,

$$\tilde{I}_n = -(\tilde{I}_a + \tilde{I}_b + \tilde{I}_c) = -3\,\tilde{I}_o, \text{ which gives } \tilde{I}_o = -\tfrac{1}{3}\,\tilde{I}_n \tag{A.5}$$

In the Δ system, the absence of neutral wire makes

$$\tilde{I}_n = 0, \text{ which gives } \tilde{I}_o = 0 \tag{A.6}$$

The symmetrical components of the 3-phase unbalanced voltages are found from relations exactly similar to Equations (A.1) through (A.4). It can be verified via actual plots of the symmetrical component phasors that the current $\tilde{I}_1$ in three phases have positive phase sequence (i.e, *a-b-c*, the same as $\tilde{I}_a$, $\tilde{I}_b$, *and* $\tilde{I}_c$), $\tilde{I}_2$ in three phases have negative phase sequence (i.e., *a-c-b*), and $\tilde{I}_o$ in three phases are all in phase (zero phase sequence). Since all three zero sequence current are in phase, they must have a return path through neutral wire. Therefore, in absence of the neutral wire in 3-wire ungrounded Y- and Δ-connected systems, $\tilde{I}_n = \tilde{I}_o = 0$.

A.2 SEQUENCE IMPEDANCES

Each symmetrical component current would have different impedance in rotating machines due to a different rotating direction of the resultant flux. They are called positive, negative, and zero sequence impedance, Z_1, Z_2, and Z_o, respectively. The positive sequence impedance is what we normally deal with in balanced 3-phase systems. For example, the positive sequence 3-phase currents in a generator set up flux in the positive direction (same as the rotor direction), and the positive sequence impedance Z_1 = (synchronous reactance + armature resistance). On the other hand, the negative sequence 3-phase currents in a generator set up flux rotating backward, and the rotor has slip of –2.0 with respect to the negative sequence flux. The rotor, therefore, works like an induction motor with rotor slip –2.0, offering very low impedance that will be similar to the subtransient impedance we discussed in Section 9.5.3. Since the negative sequence flux alternately sweeps the *d*- and *q*-axis of the rotor, $Z_2 = ½\,(Z''_d + Z''_q)$. In static equipment like transformers and cables, Z_1 and Z_2 are equal, since the flux rotation sequence does not matter in the equipment performance. As for Z_o, it is different than Z_1 and Z_2 in both the generator and the transformer depending on the flux pattern of the zero sequence current.

In general, $Z_2 << Z_1$ in the rotating machines, $Z_2 = Z_1$ in transformers and cables, and Z_o depends on the equipment connection and neutral grounding method. When the phasor sum of 3-ph currents cannot return back, such as in ungrounded *Y*- and Δ-connected systems, I_o cannot flow, hence $Z_o = \infty$ (effectively open circuit). The values of $\tilde{Z}_1$, $\tilde{Z}_2$, and $\tilde{Z}_o$ can be derived from the machine configuration. In many cases where the sequence impedances cannot be calculated, they are best derived from tests. First, a purely positive sequence voltage $\tilde{V}_1$ is applied to the equipment, and the resulting positive sequence current $\tilde{I}_1$ is measured. Then, the positive sequence impedance of the equipment, $\tilde{Z}_1 = \tilde{V}_1/\tilde{I}_1$. Similarly, the negative and zero sequence impedances are determined separately. In practice, they are usually determined by applying simulated unbalanced faults at low voltages and measuring the currents. For the system analysis under unsymmetrical conditions, the sequence components are added to make the total sequence impedance of the system.

The symmetrical component analysis is often used to determine the fault currents in unsymmetrical faults for circuit breaker sizing and for designing the protective relaying schemes. In such analyses, the system's equivalent Thevenin network up to the fault location is established for each sequence voltage with the sequence impedance. The three sequence networks are then connected as dictated by the nature of the unsymmetrical fault and the correspondingly imposed boundary conditions. The classical circuit solution then gives the sequence currents, from which the actual phase currents are determined.

With this introduction to the theory of symmetrical components, we now examine its applications in the fault current analysis, starting with our familiar symmetrical fault, and then moving into unsymmetrical faults.

A.3 SYMMETRICAL FAULT CURRENT

As such, the symmetrical component analysis is not required for symmetrical faults, since the fault current calculations follow the analysis as we covered in Chapter 9. However, we apply the symmetrical components theory to see what we already know about symmetrical faults.

Equation (A.3) for 3-phase symmetrical fault current gives

$$\begin{aligned}
\tilde{I}_o &= \tfrac{1}{3}\,(\tilde{I}_a + \tilde{I}_b + \tilde{I}_c) = 0\\
\tilde{I}_1 &= \tfrac{1}{3}\,(\tilde{I}_a + a\,\tilde{I}_b + a^2\,\tilde{I}_c\,) = \tfrac{1}{3}\,(\tilde{I}_a + \tilde{I}_a + \tilde{I}_a) = \tilde{I}_a\\
\tilde{I}_2 &= \tfrac{1}{3}\,(\tilde{I}_a + a^2\,\tilde{I}_b + a\,\tilde{I}_c) = \tfrac{1}{3}\,(\tilde{I}_a + a\,\tilde{I}_a + a^2\,\tilde{I}_a) = 0
\end{aligned} \tag{A.7}$$

Similarly, assuming symmetrical 3 ph voltages before the fault, only a positive sequence is involved, that is, $\tilde{V}_1 = \tilde{V}_a$, and $\tilde{V}_1 = \tilde{V}_2 = 0$. Then,

$$\tilde{I}_1 = \tilde{V}_1 \,/\, \tilde{Z}_1\,,\ \tilde{I}_2 = 0\,,\ \tilde{I}_o = 0 \tag{A.8}$$

And since $\tilde{I}_a$, $\tilde{I}_b$, and $\tilde{I}_c$ are all balanced,

$$\tilde{I}_a = \tilde{I}_1,\ \tilde{I}_b = a^2\,\tilde{I}_1\,,\ \tilde{I}_c = a\,\tilde{I}_1 \text{ and } \tilde{I}_n = 0 \tag{A.9}$$

This gives only the positive sequence current with the other two components zero, as expected.

A.4 L-G FAULT CURRENT

This is the most frequent fault in practical power systems, and it can be analyzed only using the theory of symmetrical components. For a grounded system shown in Figure A.2, $\tilde{Z}_f$ = ground fault impedance (called *soft fault* or *arcing fault impedance*, as opposed to hard dead fault of zero impedance), $\tilde{Z}_g$ = actual earth impedance in the return path, and $\tilde{Z}_n$ = impedance intentionally placed in the neutral. For a ground fault on any one phase, say Phase A, we know the boundary conditions imposed by the circuit, which are, by inspection, $\tilde{I}_a \neq 0$ and $\tilde{I}_b = \tilde{I}_c = 0$.

Resolving the phase currents into their symmetrical components leads to $\tilde{I}_1 = \tilde{I}_2 = \tilde{I}_o = \tfrac{1}{3}\,\tilde{I}_a$. The equal sequence currents indicate that all sequence networks are in series. Moreover, at the point of fault, $\tilde{V}_a = \tilde{I}_a\,(\,\tilde{Z}_f + \tilde{Z}_g + \tilde{Z}_n) = \tfrac{1}{3}\,\tilde{I}_a\,(3\,\tilde{Z}_f + 3\,\tilde{Z}_g + 3\,\tilde{Z}_n)$. This indicates that the total $(3\,\tilde{Z}_f + 3\,\tilde{Z}_g + 3\,\tilde{Z}_n)$ is in series with the sequence currents.

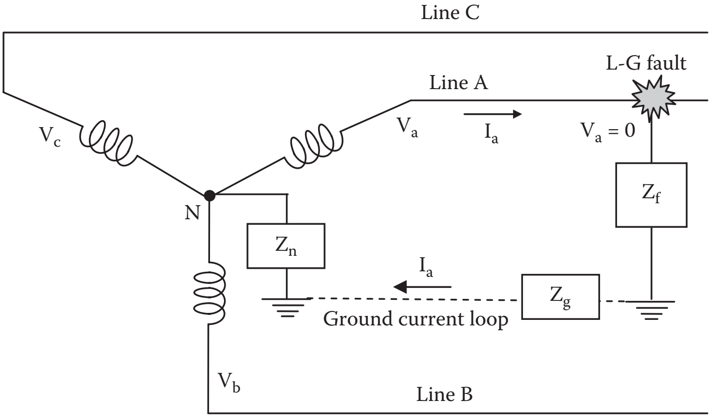


FIGURE A.2 Single-line to ground fault in 3-phase power system.

These two indications lead us to the sequence network shown in Figure A.3. For this circuit, therefore, the total driving voltage divided by the total impedance gives the current. The total driving voltage is the sum of three sequence voltages (that is, $\tilde{V}_1 + \tilde{V}_2 + \tilde{V}_o$), and the total impedance is $\tilde{Z}_1 + \tilde{Z}_{2+20} + (3\,\tilde{Z}_f + 3\,\tilde{Z}_g + 3\,\tilde{Z}_n)$.

$$\therefore\ \tilde{I}_1 = \tilde{I}_2 = \tilde{I}_o = (\tilde{V}_1 + \tilde{V}_{2+}\,\tilde{V}_o) \div (\tilde{Z}_1 + \tilde{Z}_2 + \tilde{Z}_o + 3\,\tilde{Z}_f + 3\,\tilde{Z}_g + 3\,\tilde{Z}_n) \tag{A.10}$$

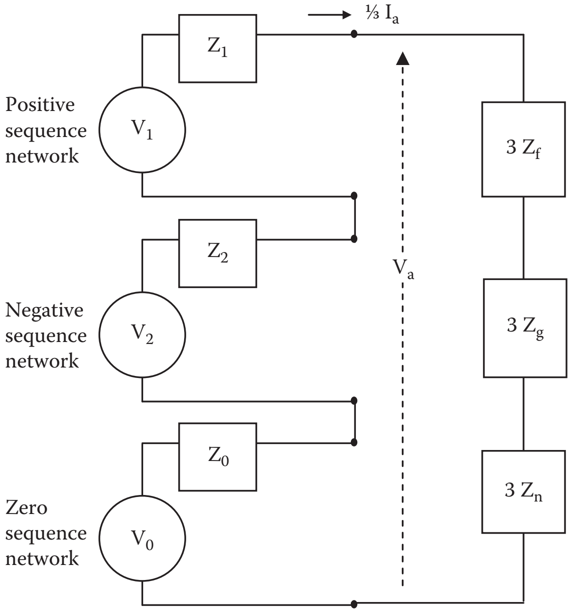


FIGURE A.3 Sequence networks connected in series for L-G fault (depends on type of fault).

Assuming that the prefault voltages are balance voltages, the sequence voltages are

$$\tilde{V}_1 = \tilde{V}_a \text{ and } \tilde{V}_2 = \tilde{V}_o = 0 \tag{A.11}$$

$$I_1 = I_2 = I_3 = \frac{V_1}{Z_1 + Z_2 + Z_0 + 3Z_f + 3Z_g + 3Z_n} \tag{A.12}$$

$$\therefore I_a = 3I_1 = \frac{V_a}{\frac{1}{3}(Z_1 + Z_2 + Z_0) + Z_f + Z_g + Z_n} \tag{A.13}$$

For this fault, we already know that $\tilde{I}_b = \tilde{I}_c = 0$ and $\tilde{I}_n = -\tilde{I}_a$.

For a special case of dead fault with solidly grounded neutral, $\tilde{Z}_f = \tilde{Z}_g = \tilde{Z}_n = 0$ in Equation (A.13).

A.5 L-L-G FAULT

For L-L-G fault from phase B to C to solid ground ($\tilde{Z}_f = \tilde{Z}_g = \tilde{Z}_n = 0$), we know the boundary conditions, which are $\tilde{V}_b = \tilde{V}_c = 0$ and $\tilde{V}_a \neq 0$. For these phase voltages, we derive the sequence voltages, that lead to $\tilde{V}_o = \tilde{V}_1 = \tilde{V}_2 = \frac{1}{3}\,\tilde{V}_a$, indicating that the three sequence networks are connected in parallel, from which we derive the sequence currents.

For this fault, a similar analysis as we discussed for L-G fault would lead to the sequence currents, where the // sign indicates the parallel connected impedances.

$$\tilde{I}_1 = \tilde{V}_1 \div (\tilde{Z}_1 + \tilde{Z}_2 /\!/ \tilde{Z}_o), \qquad \tilde{I}_2 = -\,\tilde{I}_1 \times \tilde{Z}_o \div (\tilde{Z}_2 + \tilde{Z}_o)$$

$$\text{and} \qquad \tilde{I}_o = -\,\tilde{I}_1 \times \tilde{Z}_2 \div (\,\tilde{Z}_2 + \tilde{Z}_o) \tag{A.14}$$

From Equation (A.14) values of $\tilde{I}_1$, $\tilde{I}_2$, and $\tilde{I}_o$, we can derive $\tilde{I}_b$ and $\tilde{I}_c$ using Equation (A.1), which are

$$\tilde{I}_b = \tilde{I}_o + a^2\,\tilde{I}_1 + a\,\tilde{I}_2 \quad \text{and} \quad \tilde{I}_c = \tilde{I}_o + a\,\tilde{I}_1 + a^2\,\tilde{I}_2 \tag{A.15}$$

We know that $\tilde{I}_a = 0$, so the neutral current

$$\tilde{I}_n = -(\tilde{I}_b + \tilde{I}_c\,) = -3\,\tilde{I}_o \tag{A.16}$$

A.6 L-L FAULT

For L-L fault from phase B to C, the boundary conditions are $\tilde{V}_b = \tilde{V}_c$ and $\tilde{I}_a = 0$ Applying the same methods as above, except that now $\tilde{I}_b$ must return via phase C, that is, $\tilde{I}_b = -\tilde{I}_c$ and $\tilde{I}_a = 0$, which gives $3\,\tilde{I}_o = \tilde{I}_n = 0$. Since $\tilde{I}_o = 0$, we deduce that the zero sequence network is not involved at all in the circuit. We would then get the positive and negative sequence currents equal to

$$I_1 = -I_2 = \frac{V_1}{Z_1 + Z_2} \tag{A.17}$$

Using these values of $\tilde{I}_1$, $\tilde{I}_2$, and $\tilde{I}_o$ in Equation (A.1) would give us $\tilde{I}_b$, and then $\tilde{I}_c = -\tilde{I}_b$. We already know that $\tilde{I}_a = 0$ and $\tilde{I}_o = 0$. These results can also be derived from a simple view point that L-L fault (no ground involved) is a special case of L-L-G fault with $\tilde{Z}_o$ = infinity (open), since $\tilde{I}_o = 0$.

Then, $\tilde{I}_1 = -\tilde{I}_2 = \tilde{V}_1 \div (\tilde{Z}_1 + \tilde{Z}_2)$, which is the same result as in Equation (A.17)

FURTHER READING

Clark, E., *Circuit Analysis of ac Power Systems, Volume I Symmetrical and Related Components*, John Wiley & Sons, New York, 1943.
Wagner, C. F. and R. D. Evans. *Symmetrical Components*, McGraw Hill, New York, 1933.
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Appendix B: Operating Ships Power System Data

This appendix presents the as-built and operating electrical power systems in ships of various sizes for advance study project the students or the instructor can use as needed. The three example ships are as follows:

B.1 Cable laying ship: 10 MW$_e$, 4.16 kV, 60 Hz with dc propulsion (from Derrick Kirsch, U.S. Merchant Marine Academy)
B.2 Large cruise ship: 88 MW$_e$, 11 kV, 60 Hz with ac propulsion
B.3 Small Navy ship: 2.4 kW$_e$, 600 V, 60 Hz with dc propulsion (training ship of U.S. Merchant Marine Academy)

B.1 CABLE-LAYING SHIP WITH DC PROPULSION

(From Derrick Kirsch, U.S. Merchant Marine Academy)

The power system of this ship consists of the following:

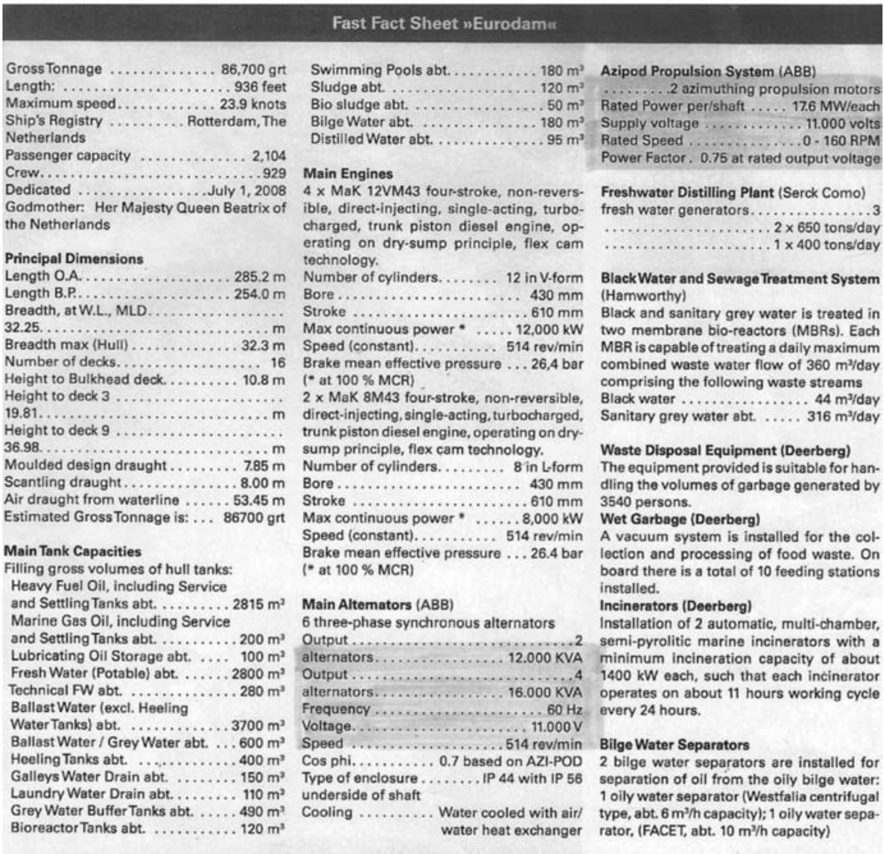
(3) AC generators 4160 V, 60 Hz
(2) DC propulsion motors (3.5 MW each) driving fixed pitch propeller via gear box
(2) Propulsion voltage 460 V via 4160/600 V transformers
(2) Ship service voltage 450 V via 4160/450 V transformer
(4) AC-DC power conversion using thyristors
Port/Starboard thrusters (1.3 MW each) with Azipods

Propulsion motors speed control scheme has the following features.

Motor power and torque is proportional to armature and field currents ($T = KI_aI_f$)
Armature voltage adjusted by adjusting thyristor firing angle
Motor field current maintained constant by adjusting field resistance

B.2 Large cruise ship with electric propulsion

(From published Eurodam data)



Fast Fact Sheet »Eurodam«

Gross Tonnage	86,700 grt
Length:	936 feet
Maximum speed	23.9 knots
Ship's Registry	Rotterdam, The Netherlands
Passenger capacity	2,104
Crew	929
Dedicated	July 1, 2008

Godmother: Her Majesty Queen Beatrix of the Netherlands

Principal Dimensions

Length O.A.	285.2 m
Length B.P.	254.0 m
Breadth, at W.L., MLD	32.25 m
Breadth max (Hull)	32.3 m
Number of decks	16
Height to Bulkhead deck	10.8 m
Height to deck 3	19.81 m
Height to deck 9	36.98 m
Moulded design draught	7.85 m
Scantling draught	8.00 m
Air draught from waterline	53.45 m
Estimated Gross Tonnage is:	86700 grt

Main Tank Capacities

Filling gross volumes of hull tanks:

Heavy Fuel Oil, including Service and Settling Tanks abt.	2815 m³
Marine Gas Oil, including Service and Settling Tanks abt.	200 m³
Lubricating Oil Storage abt.	100 m³
Fresh Water (Potable) abt.	2800 m³
Technical FW abt.	280 m³
Ballast Water (excl. Heeling Water Tanks) abt.	3700 m³
Ballast Water / Grey Water abt.	600 m³
Heeling Tanks abt.	400 m³
Galleys Water Drain abt.	150 m³
Laundry Water Drain abt.	110 m³
Grey Water Buffer Tanks abt.	490 m³
Bioreactor Tanks abt.	120 m³
Swimming Pools abt.	180 m³
Sludge abt.	120 m³
Bio sludge abt.	50 m³
Bilge Water abt.	180 m³
Distilled Water abt.	95 m³

Main Engines

4 x MaK 12VM43 four-stroke, non-reversible, direct-injecting, single-acting, turbocharged, trunk piston diesel engine, operating on dry-sump principle, flex cam technology.

Number of cylinders	12 in V-form
Bore	430 mm
Stroke	610 mm
Max continuous power *	12,000 kW
Speed (constant)	514 rev/min
Brake mean effective pressure	26,4 bar

(* at 100 % MCR)

2 x MaK 8M43 four-stroke, non-reversible, direct-injecting, single-acting, turbocharged, trunk piston diesel engine, operating on dry-sump principle, flex cam technology.

Number of cylinders	8 in L-form
Bore	430 mm
Stroke	610 mm
Max continuous power *	8,000 kW
Speed (constant)	514 rev/min
Brake mean effective pressure	26.4 bar

(* at 100 % MCR)

Main Alternators (ABB)

6 three-phase synchronous alternators

Output	2
alternators	12.000 KVA
Output	4
alternators	16.000 KVA
Frequency	60 Hz
Voltage	11.000 V
Speed	514 rev/min
Cos phi	0.7 based on AZI-POD
Type of enclosure	IP 44 with IP 56 underside of shaft
Cooling	Water cooled with air/water heat exchanger

Azipod Propulsion System (ABB)

	2 azimuthing propulsion motors
Rated Power per/shaft	17.6 MW/each
Supply voltage	11.000 volts
Rated Speed	0 - 160 RPM
Power Factor	0.75 at rated output voltage

Freshwater Distilling Plant (Serck Como)

fresh water generators	3
	2 x 650 tons/day
	1 x 400 tons/day

Black Water and Sewage Treatment System (Hamworthy)

Black and sanitary grey water is treated in two membrane bio-reactors (MBRs). Each MBR is capable of treating a daily maximum combined waste water flow of 360 m³/day comprising the following waste streams

Black water	44 m³/day
Sanitary grey water abt.	316 m³/day

Waste Disposal Equipment (Deerberg)

The equipment provided is suitable for handling the volumes of garbage generated by 3540 persons.

Wet Garbage (Deerberg)

A vacuum system is installed for the collection and processing of food waste. On board there is a total of 10 feeding stations installed.

Incinerators (Deerberg)

Installation of 2 automatic, multi-chamber, semi-pyrolitic marine incinerators with a minimum incineration capacity of about 1400 kW each, such that each incinerator operates on about 11 hours working cycle every 24 hours.

Bilge Water Separators

2 bilge water separators are installed for separation of oil from the oily bilge water: 1 oily water separator (Westfalia centrifugal type, abt. 6 m³/h capacity); 1 oily water separator, (FACET, abt. 10 m³/h capacity)

B.3 SMALL NAVY SHIP WITH DC PROPULSION

The *Kings Pointer* is presently the U.S. Merchant Marine Academy's training ship, which was formerly a navy–military surveillance ship collecting data on foreign ships. It was originally built in the early 1980s. It is 224 ft long with 2250 tons dead weight. It has diesel–electric propulsion designed for cruising at 10 knots.

The main features of the *Kings Pointer*'s electric propulsion are:

(4) Diesel engines, each rated 970 HP.
(4) Electric generators, each rated 600 kW, 600 Vac, 3-phase, 60 Hz.
(2) Propulsion motors, each rated 800 hp, 750 Vdc.
(2) Propellers, each 4 blades, 8 feet diameter.
Emergency power system: 400 hp diesel engine +250 kW generator.
The diesel fuel storage capacity is 228,600 gal, and the fuel consumption rate at 10 knots speed is 1850 gallons per day.

Mechanical Engineering

SHIPBOARD ELECTRICAL POWER SYSTEMS

Mukund R. Patel

Shipboard Electrical Power Systems addresses new developments in this growing field. Focused on the trend toward electrification to power commercial shipping, naval, and passenger vessels, this book helps new or experienced engineers master cutting-edge methods for power system design, control, protection, and economic use of power.

PROVIDES BASIC TRANSFERABLE SKILLS FOR MANAGING ELECTRICAL POWER ON SHIPS OR ON LAND

This groundbreaking book is the first volume of its kind to illustrate optimization of all aspects of shipboard electrical power systems. Applying author Mukund Patel's rare combination of industrial and educational work experiences and insight, it offers solutions to meet the increasing demand for large, fast, efficient, and reconfigurable ships to compete in international markets.

For 30 years, Professor Patel was an engineer for companies including General Electric, Lockheed Martin, and Westinghouse Electric, and in the past 15 years he has been an engineering professor at the U.S. Merchant Marine Academy. That varied experience helped him zero in on the specialized multidimensional knowledge an engineer requires—and that is what sets his book apart.

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